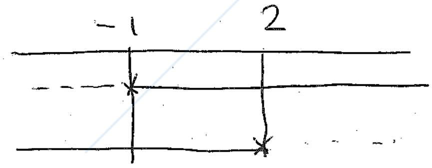
Esercizi assegnati il 13 novembre 2019

1) Risolvere la seguente disequazione:

$$\frac{3-|x+1|}{|2-x|-4}\le 0$$

Si determinano gli intervalli:

$$\begin{cases} x+1\ge 0\\ 2-x\ge 0\end{cases} \qquad \begin{cases} x\ge -1\\ x\le 2\end{cases}$$



Si scompone la disequazione in tre sistemi:

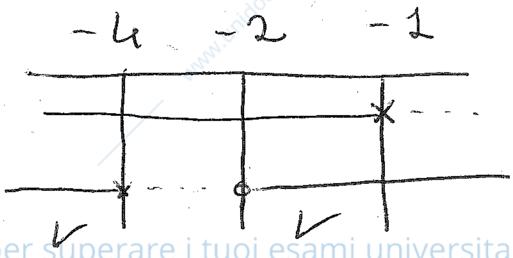
A $\begin{cases} x\le -1\\ \dfrac{3+x+1}{2-x-4}\le 0\end{cases}$ B $\begin{cases} -1< x< 2\\ \dfrac{3-x-1}{2-x-4}\le 0\end{cases}$ C $\begin{cases} x\ge 2\\ \dfrac{3-x-1}{x-2-4}\le 0\end{cases}$

$\begin{cases} x\le -1\\ \dfrac{4+x}{-2-x}\le 0\end{cases}$ $\begin{cases} -1< x< 2\\ \dfrac{2-x}{-2-x}\le 0\end{cases}$ $\begin{cases} x\ge 2\\ \dfrac{2-x}{x-6}\le 0\end{cases}$

$\begin{cases} x\le -1\\ \dfrac{x+4}{x+2}\ge 0\end{cases}$ $\begin{cases} -1< x< 2\\ \dfrac{x-2}{x+2}\le 0\end{cases}$ $\begin{cases} x\ge 2\\ \dfrac{x-2}{x-6}\ge 0\end{cases}$

A $\begin{cases} x\le -1\\ \begin{cases} x\ge -4\\ x> -2\end{cases}\end{cases}$

$\begin{cases} x\le -1\\ x\le -4 \vee x>-2\end{cases}$

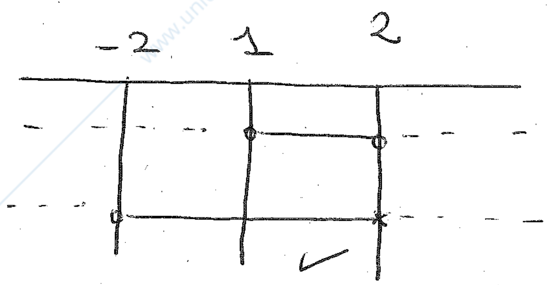


$Sol_{(A)}$: $x \le -4 \ \vee \ -2 < x \le 1$

(B) $\begin{cases} -1 < x < 2 \\ \begin{cases} x-2 \geq 0 \\ x+2 > 0 \end{cases} \end{cases}$

$\begin{cases} -1 < x < 2 \\ \begin{cases} x \geq 2 \\ x > -2 \end{cases} \end{cases}$

$\begin{cases} -1 < x < 2 \\ -2 < x \le 2 \end{cases}$

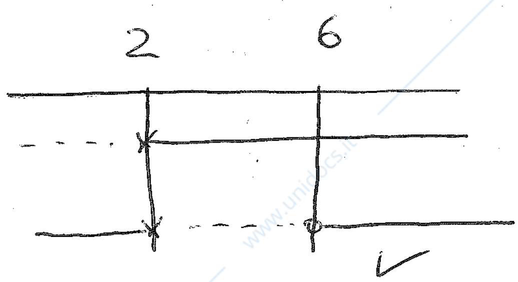


$Sol_{(B)}$: $-1 < x < 2$

(C) $\begin{cases} x \geq 2 \\ \begin{cases} x-2 \geq 0 \\ x-6 > 0 \end{cases} \end{cases}$

$\begin{cases} x \geq 2 \\ \begin{cases} x \geq 2 \\ x > 6 \end{cases} \end{cases}$

$\begin{cases} x \geq 2 \\ x \le 2 \ \vee \ x > 6 \end{cases}$



$Sol_{(C)}$: $x = 2 \ \vee \ x > 6$

Procediamo con l'unione delle soluzioni:

$$Sol = Sol_{(A)} \cup Sol_{(B)} \cup Sol_{(C)}:$$

$$x \leq -4 \vee -2 < x \leq 2 \vee x > 6$$

2) Determinare chiusura, apertura, frontiera, inf, sup (e max e min se esistono) dei seguenti insiemi:

$$A = \{x : x \in \mathbb{R} \quad 3x+2 \geq -1\}$$

$$B = \{x : x \in \mathbb{R} \quad \frac{\log_{\frac{1}{2}} x - 2}{1-x} \geq 0\}$$

$$C = (-\infty; 1) \cup (1;2) \cup (3,4) \cup \{5\}$$

A.

$$3x+2 \geq -1$$

$$3x \geq -3$$

$$x \geq -1$$

Possiamo quindi scrivere:

$$A = [1; +\infty).$$

Quindi:

$$\bar{A} = A = [1; +\infty)$$

$$\mathring{A} = (1; +\infty)$$

$$\mathcal{F}(A) = \{1\}$$

$$\inf(A) = \min(A) = 1$$

$$\sup(A) = +\infty$$

max(A) non esiste

B

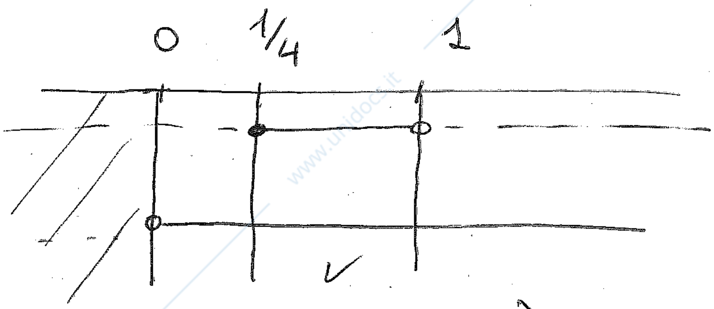
$$\frac{\log_{\frac{1}{2}} x - 2}{1-x} \le 0$$

$$\begin{cases} \dfrac{\log_{\frac{1}{2}} x - 2}{1-x} \le 0 \\ x > 0 \end{cases}$$

$$\begin{cases} \begin{cases} \log_{\frac{1}{2}} x \ge 2 \\ 1-x > 0 \end{cases} \\ x > 0 \end{cases}$$

$$\begin{cases} \begin{cases} x \le \frac{1}{4} \\ x < 1 \end{cases} \\ x > 0 \end{cases}$$

$$\begin{cases} \frac{1}{4} \le x < 1 \\ x > 0 \end{cases}$$



Possiamo scrivere $B \equiv [1/4; 1)$

Quindi:

$\overline{B} = [1/4; 1]$

$\mathring{B} = (1/4; 1)$

$\mathcal{F}(B) = \{1/4; 1\}$

$\inf(B) = \min(B) = \frac{1}{4}$

$\sup(B) = 1$

$\max(B)$ non esiste

$\underline{e}$

$\mathcal{C} = (-\infty; 1) \cup (1; 2) \cup (3; 4) \cup \{5\}$

$\overline{\mathcal{C}} = (-\infty; 2] \cup [3; 4] \cup \{5\}$

$\mathring{\mathcal{C}} = (-\infty; 1) \cup (1; 2) \cup (3; 4)$

$\mathcal{F}(\mathcal{C}) = \{1, 2, 3, 4, 5\}$

$\inf(\mathcal{C}) = -\infty$

$\min(\mathcal{C})$ non esiste

$\sup(\mathcal{C}) = \max(\mathcal{C}) = 5$