$$(x+2)y = 7x+1$$
$$2y + xy = 7x+1$$
$$y(2+x) = 7x+1$$
$$2y-1 = (7-y)x$$
$$x = \frac{2y-1}{7-y} \quad \text{se } y \neq 7$$

$$7 = \frac{7x+1}{x+2}$$

$$7x+14 = 7x+1$$
$$14 = 1 \qquad \text{Im} f = \mathbb{R} \setminus \{7\}$$

Determinare il dominio naturale di

$$f(x) = \frac{3x+1}{x^2+x-6}$$

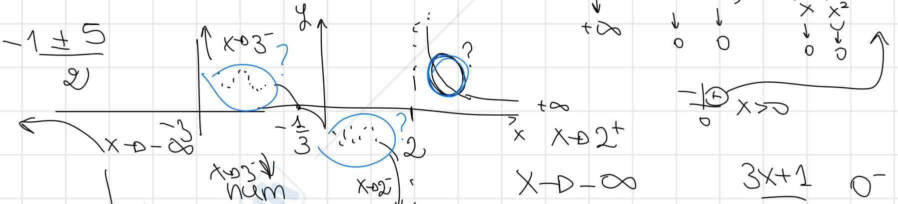
$$X = \mathbb{R} \setminus \{\text{radici denominatore}\} = \mathbb{R} \setminus \{x_+, x_-\}$$
$$p(x) = x^2+x-6$$
$$\Delta = b^2 - 4ac =$$

$$x_\pm = \frac{-b \pm \sqrt{\Delta}}{2a}$$
$$= \frac{-1 \pm 5}{2}$$

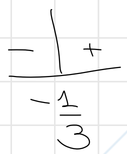
$x \to +\infty$

$$\frac{3x+1}{x^2+x-6} = \frac{3x+1}{x^2\left(1+\frac{1}{x}-\frac{6}{x^2}\right)} \sim \frac{3x+1}{x^2} =$$

$\frac{3}{x}+\frac{1}{x^2} \to 0^+$



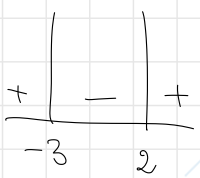
$x^2+x-6>0$

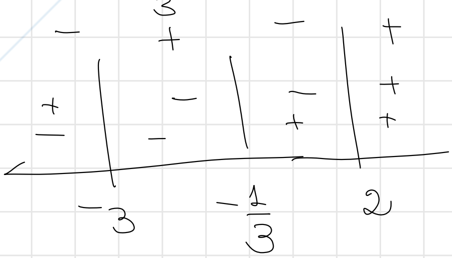


$x > -\frac{1}{3}$

$x \to -\infty$

$$\frac{3x+1}{x^2} \quad 0^-$$





Qual'è l'immagine?

$$y = \frac{3x+1}{x^2+x-6}$$

$$yx^2+yx-6y = 3x+1$$

$$\underbrace{y}_{a}x^2 + \underbrace{(y-3)}_{b}x \underbrace{-6y-1}_{c}=0$$

$$\Delta_p = (y-3)^2+4y(6y+1) = y^2-6y+5+24y^2+4y$$

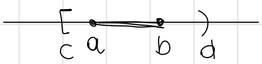
ho una sola x quando $\Delta \geq 0$ $\quad = 25y^2-2y+9$

$$\Delta_{\Delta_p} = 4-4\cdot 25\cdot 9 < 0$$

Un sottoinsieme $A \subset \mathbb{R}$
si dice "connesso" se $a,b \in A$, $a<b$ implica $[a,b] \subset A$ (non ci sono buchi)

TEO: gli insiemi connessi sono precisamente gli intervalli
- $[c,d]$ chiuso
- (c,d) aperto
- $[c,d)$
- $(c,d]$

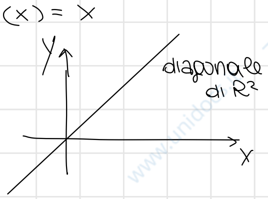
l'infinito non app. ai reali
$[c,+\infty) = \{x \in \mathbb{R} : x \geq c\}$
$(-\infty,+\infty) = \mathbb{R}$

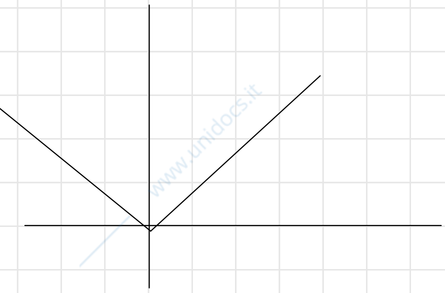


TEO: l'immagine continua di un connesso è un connesso
$f((-3,2))$ è un intervallo perché f è continua
$(-\infty,+\infty) = \mathbb{R}$

Funzioni notevoli

$f(x) = x$

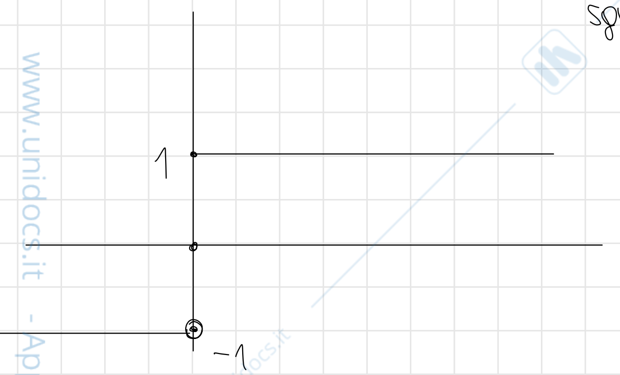




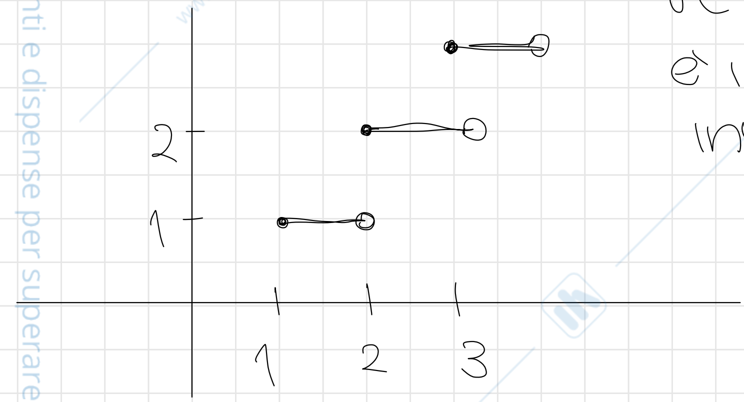
$$f(x) = |x| \quad \begin{cases} x & \text{per } x \geq 0 \\ -x & \text{per } x < 0 \end{cases}$$

nota $|0| = 0$

$$f(x) = \text{segno}(x) = \text{sgn}(x) = \begin{cases} 1, & x > 0 \\ 0 & x = 0 \\ -1 & x < 0 \end{cases}$$

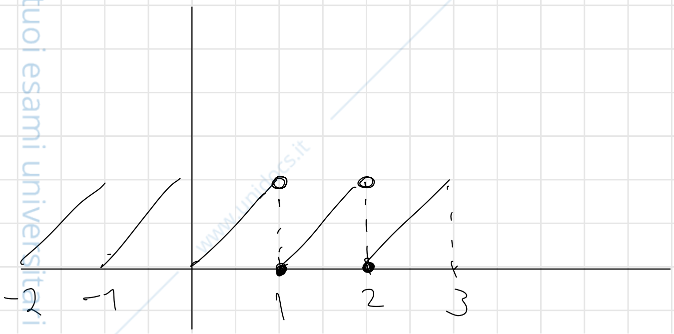


$f(x) = [x]$ dove $[x]$ è la più grande intero tale che $\underbrace{0 \leq x - [x]}_{M(x)} < 1$



mantissa

$$M(x) = x - [x]$$



$[2;3] = 2$

$[3,34] = 3$

$M(2,3) = 0.3$

$M(3,54) = 0.54$

Funzioni pari e dispari

$f: D \to \mathbb{R}$ $\quad D \subset \mathbb{R}$

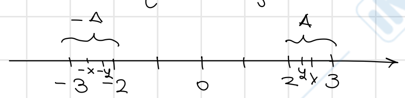
diciamo che è pari se $D = -D$ e $f(-x) = f(x)$ $\forall x \in D$

(se $x \in D$, allora $-x \in D$ ovvero D è invariante per riflessione)

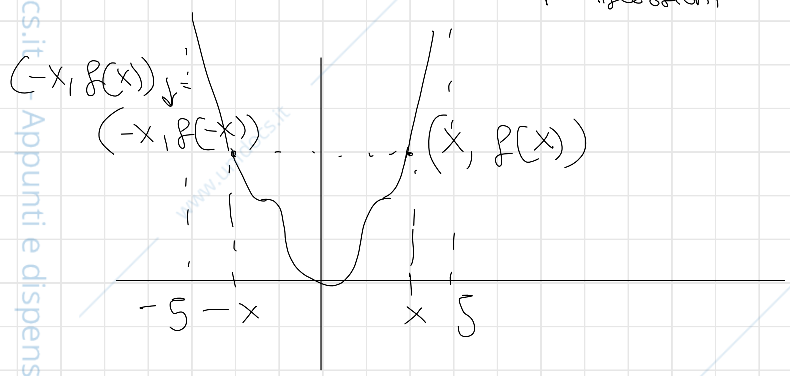
se $A \subset \mathbb{R}$

$-A := \{-x : x \in A\}$

$= \{x : -x \in A\}$

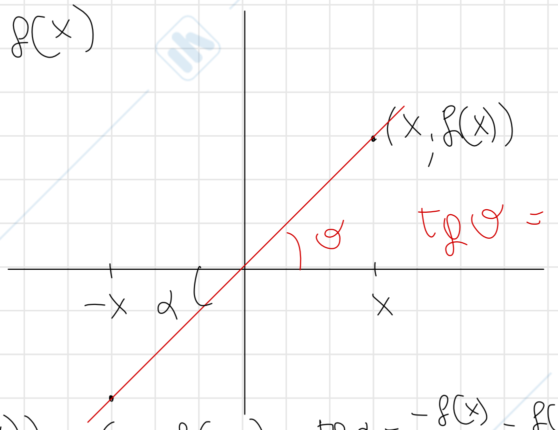


$[-3,-2] \cup [2,3)$ è invariante per riflessioni



f è dispari se $D = -D$

$f(-x) = -f(x)$



$\operatorname{tg}\theta = \dfrac{f(x)}{x}$

$(-x, -f(x)) = (-x, f(-x))$ $\quad \operatorname{tg}\alpha = \dfrac{-f(x)}{-x} = \dfrac{f(x)}{x} = \operatorname{tg}\theta$

$0 \in D$

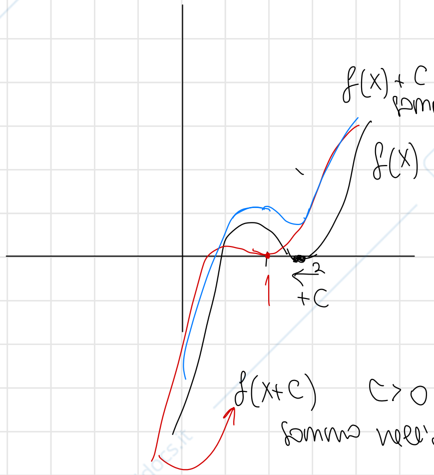
$f(-0) = -f(0)$

$f(0) = -f(0)$

$2f(0) = 0$

$f(0) = 0$

$f(x) + C \quad C > 0$
sommando C verticalmente

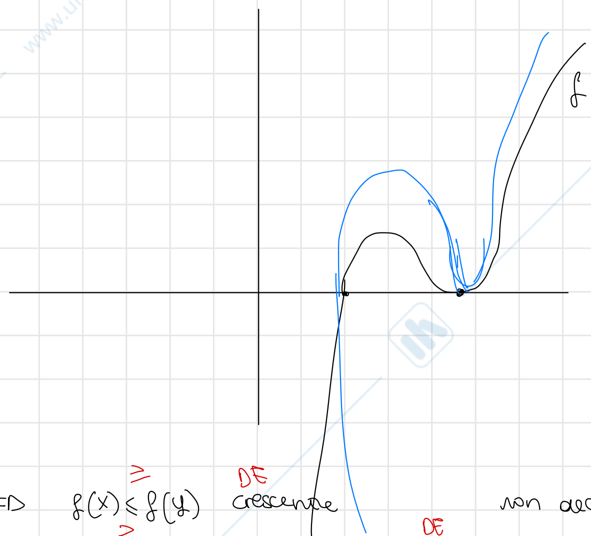


$f(x+C) \quad C > 0$
sommo nell'argomento

supponiamo $f(2) = 0$
$C = 1$

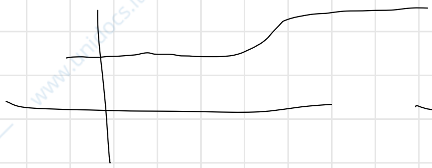
$f(x) = f(x+1)$
per $x = 1$
fa
$f(2) = 0$

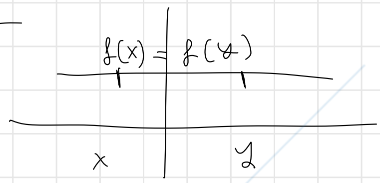
$c f(x) \quad c > 1$

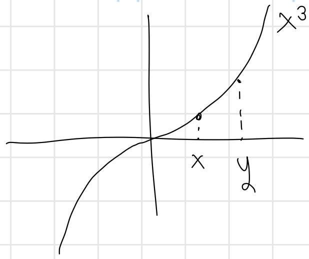


$x \leq y \Rightarrow f(x) \overset{\geq}{\leq} f(y)$ $\overset{\text{DE}}{\text{crescente}}$ non decrescente

$x < y \rightarrow f(x) \overset{>}{<} f(y)$ strettamente $\overset{\text{DE}}{\text{crescente}}$ crescente







OSS

se f è strettamente monotona allora è iniettiva

Dim devo dimostrare $x \neq y \Rightarrow f(x) \neq f(y)$

caso 1 $x < y \Rightarrow f(x) < f(y)$
$y < x \Rightarrow f(y) < f(x)$ } caso crescente $f(x) \neq f(y)$

$f(x) = f(y)$ $x = y$ iniettiva

$f(x) \neq f(y) \Leftarrow x \neq y$

$x \neq y \rightarrow f(x) \neq f(y)$

$x \neq y \Rightarrow x < y$ o $y < x$

mi metto nel caso strettamente crescente (l'altro è simile

CASO 1

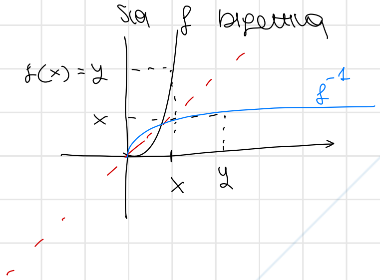
$x < y \Rightarrow f(x) \overset{>}{<} f(y) \Rightarrow f(x) \neq f(y)$

CASO 2

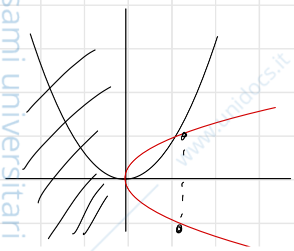
$y < x \Rightarrow f(y) \overset{>}{<} f(x) \Rightarrow f(x) \neq f(y)$

Grafico funzione inversa

Sia f biiettiva

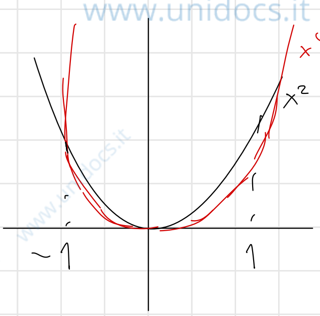


$f^{-1} = \sqrt{y}$



$x^2 : [0, +\infty) \rightarrow [0, +\infty)$

$\sqrt{x} : [0, +\infty) \rightarrow [0, +\infty)$



elevamento a potenza pari

$(0,1)^2 = 0,01$

$(0,1)^4 = 0,0001$

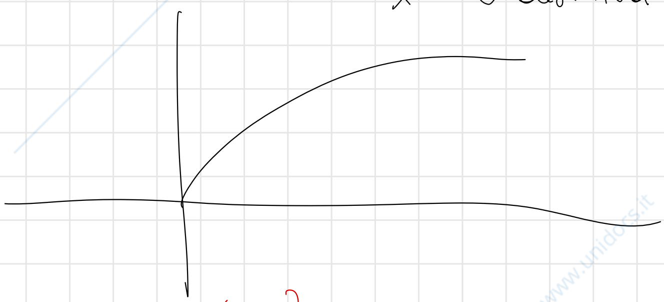
la funzione è pari

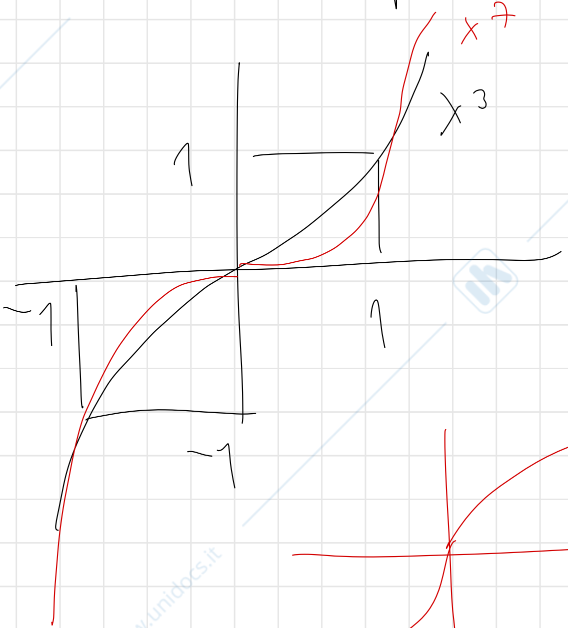
$f(x) = f(-x)$

$x, -x$ danno lo stesso valore quindi non iniettiva

$x^{\frac{1}{n}}$ $f^{-1}(x)$ con $f(y) = y^n$

$x^{\frac{1}{n}}$ è definita su $[0, +\infty)$





n dispari

$x^{\frac{1}{n}} : \mathbb{R} \to \mathbb{R}$