$f(x)$

- C.E.
- SEGNO $f(x) > 0$ $f(x) \leq 0$ $f(x) < 0$ da risolvere nel C.E.
- LIMITI
- DERIVATA PRIMA → pt. staz. e loro natura
  → cresc / decr.
- (DER. SECONDA)
- GRAFICO $(x, y) : y = f(x)$

(1)

$$\lim_{x \to -2} \frac{x^2 - x - 6}{x^2 + 5x + 6} = \lim_{x \to -2} \frac{(x+2)(x-3)}{(x+2)(x+3)} = -5$$

$x^2 - x - 6 \to 0$, $x^2 + 5x + 6 \to 0$, $(x-3) \to -5$, $(x+3) \to 1$

$$f(x) = \frac{p(x)}{q(x)} \rightsquigarrow \text{polinomi}$$

C.E $\quad x \in \mathbb{R} : q(x) \neq 0$

$$x^2 + 5x + 6 \neq 0 \qquad x^2 + 5x + 6 = 0$$

$$\Rightarrow x = \frac{-5 \pm \sqrt{25 - 24}}{2} = \frac{-5 \pm 1}{2} = \begin{cases} -2 \\ -3 \end{cases}$$

C.E $\quad (-\infty, -3) \cup (-3, -2) \cup (-2, +\infty)$

-3^-, -3^+, -2^-, -2^+

$$(x^2 - x - 6) = (x - (-2))(x + \alpha) = (x+2)(x-3)$$

$$(x^2 + 5x + 6) = (x - (-2))(x + \beta) = (x+2)(x+3)$$

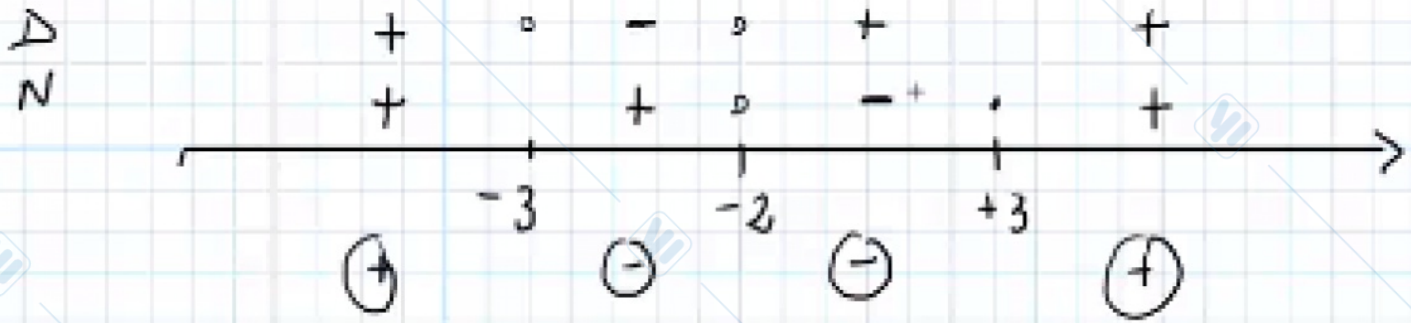
SEGNO $f(x) = \dfrac{x^2-x-6}{x^2+5x+6}$

$f(x) \geq 0$

$$\frac{x^2-x-6}{x^2+5x+6} \geq 0$$

N: $x^2-x-6 \geq 0$ $\quad x \leq -2 \ \vee \ x \geq 3$

D: $x^2+5x+6 > 0$ $\quad x < -3 \ \vee \ x > -2$



$f(x) > 0$ per $x < -3 \ \vee \ x > +3$

$f(x) = 0$ per $x = 3$

$f(x) < 0$ per $-3 < x < -2 \quad \vee \quad -2 < x < +3$

DERIVATA PRIMA

$$f(x) = \frac{x^2 - x - 6}{x^2 + 5x + 6} = \frac{p(x)}{q(x)}$$

$$q(x) = x^2 + 5x + 6 = (x+2)(x+3)$$

$$f'(x) = \frac{p'(x) \cdot q(x) - p(x) \cdot q'(x)}{(q(x))^2} = \frac{(2x-1)(x^2+5x+6) - (x^2-x-6)\cdot(2x+5)}{(q(x))^2} =$$

$$= \frac{\cancel{2x^3} + 10x^2 + 12x - x^2 - 5x - 6 \cancel{- 2x^3} + 2x^2 + 12x - 5x^2 + 5x + 30}{} =$$

$$= \frac{6x^2 + 24x + 24}{(q(x))^2} = \frac{6(x^2+4x+4)}{(q(x))^2} =$$

$$= \frac{6\cancel{(x+2)^2}}{\cancel{(x+2)^2}(x+3)^2} = \frac{6}{(x+3)^2}$$

$f'(x) \neq 0 \quad \forall x \in C.E.$

$f'(x) > 0 \quad \forall x \in C.E.$

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