

Mathematics 11/27 Exam

Group - 1

- 1) State the definition and the properties of the *inverse matrix*.
- 2) Show that

$$\begin{bmatrix} 5 & 6 \\ 5 & 10 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 1/2 & -3/10 \\ -1/4 & 1/4 \end{bmatrix}$$

are inverse of each other.

- 3) Express the vector $v_1 = \begin{bmatrix} 8 \\ 9 \end{bmatrix}$ as a linear combination of the vectors $v_2 = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$ and $v_3 = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$.

- 4) State *Euler's Theorem*.

- 5) Show that the following function, defined for all x and y is homogeneous of degree 3

$$f(x, y) = 3x^2y - y^3.$$

- 6) Consider the function

$$f(x, y) = (x^2 - y^2),$$

show that $(0,0)$ is a critical point; is it a maximum, a minimum or a saddle point?

- 7) Find the equation of the tangent plane with respect to the surface

$$f(x, y) = 4x^2 + y^2 - 6x,$$

at the point $P(2, 3, 13)$.

- 8) Given the following first order difference equation:

$$x_{t+1} = f(x_t),$$

state the condition for the *local stability* of a *steady state* x^* of $f(x_t)$.

- 9) Given the following first order difference equation

$$x_{t+1} = x_t^3$$

find the steady states and discuss their stability.