Terna intrinseca

$$\bar{t}_g = \frac{\bar{v}}{|\bar{v}|}$$

$$\bar{b} = \frac{\bar{v} \wedge \bar{a}}{|\bar{v} \wedge \bar{a}|} \Rightarrow \bar{n} = \bar{b} \wedge \bar{t}_g$$

direz: $\operatorname{arctg}\left(\frac{v_{py}}{v_{px}}\right)$

$$W_{in} = -m\, \bar{a}^T \cdot \bar{v}$$

Cinematica

- Terne relative

Velocità $\quad \bar{v}^{ASS} = \bar{v}^{TR} + \bar{v}^{REL}$

terna rotante $\quad \bar{v}^{TR} = \bar{\omega} \wedge (P-O)$

terna traslante $\quad \bar{v}^{TR} = \bar{v}_{TERNA}$

Due metodi: Grafico: se ho tutte le direzioni compongo il triangolo delle velocità

Analitico: esplicito ogni termine come v^{TR} e v^{REL}

Accelerazione: $\bar{a}^{ASS} = \bar{a}^{TR} + \bar{a}^{REL} + \bar{a}^{COR}$

$$= \bar{a}^{TR\,tg} + \bar{a}^{TR\,n} + a^{REL\,tg} + a^{REL\,n} + a^{COR}$$

$$\bar{a}^{tg} = \dot{\bar{\omega}} \wedge (P-O)$$

$$\bar{a}^{n} = -\omega^2 (P-O)$$

$$\bar{a}^{COR} = 2\bar{\omega} \wedge \bar{v}_{REL}$$

Complessi

Vettori posizione $\quad (P-O) = \rho e^{i\theta} \quad \Rightarrow$ derivo e ottengo v e a

Da CIR a velocità

$$v_A = v_{CIR} + \bar{\omega} \wedge (A - CIR) = \bar{\omega} \wedge (A - CIR)$$

Equazioni cardinali

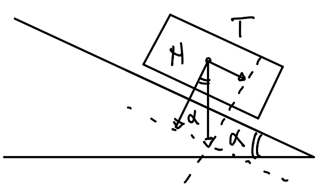
$$\begin{cases} R^{EXT} = \dfrac{dQ_{TOT}}{dt} = m_{TOT}\, a_G \\ M_O^{EXT} = \dfrac{d\bar{\Gamma}_G}{dt} + (G-O) \wedge m\bar{a}_G = J_G \dot{\omega} \end{cases}$$

$$\begin{cases} \dfrac{d\bar{Q}}{dt} = \bar{R} \\ \dfrac{d\bar{\Gamma}_O}{dt} + \bar{v}_O \wedge \bar{Q} = \bar{M}_O \end{cases}$$

D'Alembert

$$\begin{cases} \Sigma F_x - F_{IN\,x} = 0 \\ \Sigma F_y - F_{IN\,y} = 0 \\ \Sigma M_O - J\ddot{\theta} = 0 \end{cases}$$

Piano inclinato



$$T = mg \sin\alpha$$

$$N = mg \cos\alpha$$

Calcolo del J

$$J_G = \int_V \rho (x^2 + y^2)\, dV \xrightarrow{\text{notevoli}}$$

Asta omogenea $J_G = M \frac{L^2}{12}$

Disco omogeneo $J_G = \frac{MR^2}{2}$

Anello $J_G = MR^2 = M \frac{R_e^2 + R_i^2}{2}$

Mezzo anello $J_o = \rho\, S_p\, \pi \left(\frac{R_e^4 - R_i^4}{4} \right)$

Rettangolo $J_G = m \left(\frac{a^2}{12} + \frac{b^2}{12} \right)$

Trasporto $J_o = J_G + m (G-o)^2$

Cinghia di trasmissione

Condizione di attrito $T_1 = T_2\, e^{f_s \alpha_1}$

Angolo di avvolgimento $I \sin\alpha = (R - r)$
$\alpha_1 = \pi - \alpha$
$\alpha_2 = \pi + \alpha$

BdP
$C_m W_m (1-\eta) = C_r W_r$

Attrito volvente

$u = f_v R \quad T = N f_v \quad W = N f_v v$
$= N f_v W_R R$
$= N_u W_R$

Metodi energetici

$W = F \cdot v = C \cdot w$

$$W_{IN} + W_{OUT} = \frac{dE_c}{dt} \qquad E_c = \frac{1}{2} m v^2 + \frac{1}{2} J w^2$$

Lavori virtuali

$$\sum L^* = 0 \qquad \mathcal{L}_{el} + \mathcal{L}_p + \mathcal{L}_c = 0$$

SEGNO PER TENERE SISTEMA IN EQ.

$\mathcal{L}_{el} = k \Delta l\, \delta S_p \qquad \Delta l = l - l_0$
$\mathcal{L}_c = C \delta\alpha$
$\mathcal{L}_p = mg\, \delta S_p$

CASO MOLLA NON VINCOLATA
l: spostamento max. di uno dei vincoli tenendo fermo l'altro
→ poi uso $\delta \Delta l$ come spostamento

Lagrange

$$\frac{\delta Q}{\delta q_k} = \frac{d}{dt}\left(\frac{\partial E_c}{\partial \dot q_k}\right) - \frac{\partial E_c}{\partial q_k} + \frac{\partial V}{\partial q_k} + \frac{\partial D}{\partial \dot q_k}$$

con

$E_c = \frac{1}{2} m v^2 + \frac{1}{2} J w^2$

$V = \frac{1}{2} k \Delta x^2$

$D = \frac{1}{2} r \dot x^2$

Sistemi vibranti

$$\boxed{\begin{gathered} \alpha = h w_0 \\ w_r = w_0 \sqrt{1 - h^2} \\ h = \frac{r}{r_c} = \frac{r}{2 m w_0} \\ w_0 = \sqrt{\frac{k}{m}} \end{gathered}}$$

omogenea non smorzata

$$x_{0A} = C \cos(w_0 t + \varphi)$$

Forzante costante

$$x_{1P} = \frac{F_0}{k} = X_{ST}$$

$Q = \frac{\Omega}{w_0} > 1$ sismo

$$\left| \frac{X_0}{X_{ST}} \right| = \frac{1}{1 - Q^2}$$

omogenea smorzata

$$x_{0A} = e^{-\alpha t} c(\cos \omega_d + \varphi)$$

Forzante tempovariante

$$F(t) = F_0 \cos \Omega t$$

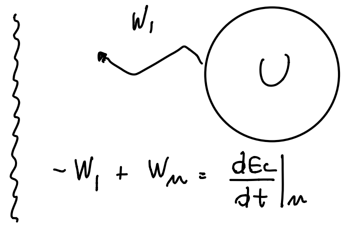
$$Q = \frac{\Omega}{\omega_0}$$

$$\frac{z_0}{x_{ST}} = \frac{1}{(1-Q^2) + i(Qh)}$$

MTU

$$W_m + W_p + W_n = \frac{dE_c}{dt} \qquad \tau = \frac{W_n}{W_m} \qquad W_p = -(1-\eta) W_{1M}$$

$$C_m = C_0 \left(1 - \frac{\omega_m}{\omega_S}\right)$$



$$-W_1 + W_m = \left.\frac{dE_c}{dt}\right|_m$$

Periodico

$$i = \frac{\omega_{max} - \omega_{min}}{\omega_{media}} \qquad \Delta E_c = J_{eq}\, i\, \omega^2$$

$\Delta E_c =$ COPPIA FRENANTE × ANGOLO DI APPLICAZIONE

$$t_{ARR} = \frac{\omega^{REG}}{|\dot{\omega}_m|}$$

$\Delta E_{c\,max} =$ [Somma delle coppie dove si sovrappongono] × [angolo di sovrapposizione]

es. $= [C_m - C_r]\pi$

Eq. differenziali

Polinomio caratteristico

$\lambda^2 + A\lambda + B = 0$

$\Delta > 0 \Rightarrow y(t) = c_1 e^{\lambda_1 t} + c_2 e^{\lambda_2 t}$

$\Delta = 0 \Rightarrow y(t) = c_1 e^{\lambda_1 t} + t c_2 e^{\lambda_1 t}$

$\Delta < 0$ con $\alpha + i\beta$, $\alpha - i\beta$ $\quad y(t) = c_1 e^{\alpha t} \cos \beta t + c_2 e^{\alpha t} \sin \beta t$

Integrale particolare

polinomio $\rightarrow At^n + Bt^{n-1} \dots$ con n uguale a forzante

exp $\rightarrow y(t) = Ae^{bt}$

sin/cos $\rightarrow y(t) = A_1 \cos bt + A_2 \sin bt$

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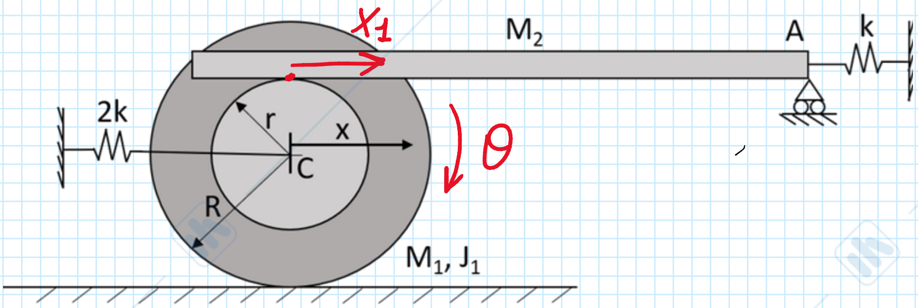
Svolgimento Prova 29/06

martedì 30 giugno 2020 15:11

(1)



keq?

$$x_1 = \theta \cdot (R+r)$$

$$\left(x = \theta \cdot R \Rightarrow \theta = \frac{x}{R}\right)$$

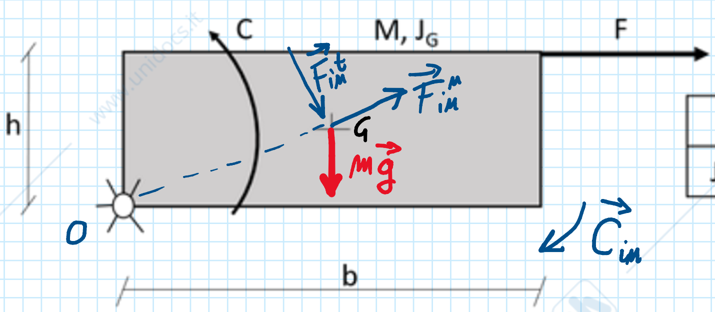
$$x_1 = \frac{x}{R}(r+R)$$

$$V = \frac{1}{2} \cdot 2k(x)^2 + \frac{1}{2}k(-x_1)^2$$

$$V = \frac{1}{2} \cdot 2k\, x^2 + \frac{1}{2}k\, x^2 \cdot \frac{(r+R)^2}{R^2}$$

$$V = \frac{1}{2} \cdot \underbrace{\left(2k + k\frac{(r+R)^2}{R^2}\right)}_{k_{eq}} x^2$$

(2)



M = 10 kg	h = 1 m	b = 3 m
J_G=8,33 kgm^2	C = XX*200 Nm	F = 100 N

$\Sigma M_O = 0$ (+)

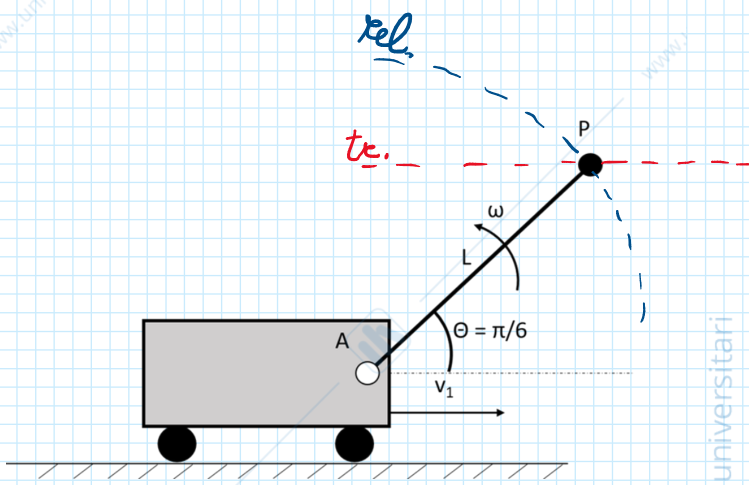
$$C - F_{in}^t \cdot \overline{OG} - mg \cdot \frac{b}{2} - F \cdot \frac{h}{2} = 0$$

$\Downarrow$

1 eq. in 1 inc. $\rightarrow \dot{\omega}$

- $\overline{OG} = \sqrt{\left(\frac{h}{2}\right)^2 + \left(\frac{b}{2}\right)^2}$
- $F_{in}^t = \dot{\omega} \cdot \overline{OG}$

③ terna traslante in A



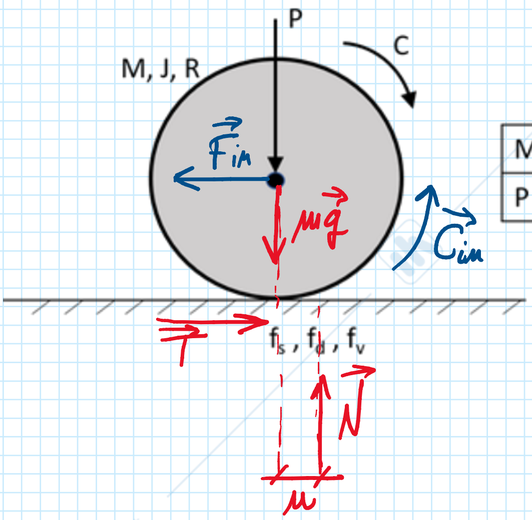
$$\vec{v}_P = \vec{v}_{P_{tr}} + \vec{v}_{P_{rel}}$$

$$\vec{v}_P = \vec{v} + \omega\vec{k} \wedge (P-A) \rightarrow l\cos\theta\,\vec{i} + l\,\text{sen}\,\theta\,\vec{j}$$

$$\vec{v}_P = v\vec{i} + \omega l\cos\theta\,\vec{j} - \omega l\,\text{sen}\,\theta\,\vec{i}$$

$$\vec{a}_P = \vec{a}^{\,n}_{P_{rel}} = -\omega^2 l\cos\theta\,\vec{i} - \omega^2 l\,\text{sen}\,\theta\,\vec{j}$$

Verifica di aderenza



M = 10 kg	R = 1 m	J = 5 kgm^2	C = XX*10 Nm
P = 30 N	fs = 0.9	fd =0.4	fv = 0.02

$$|T| \le f_s|N|$$

$$\boxed{\Sigma F_y = 0}$$

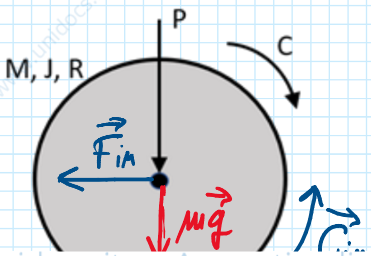
$$N - mg - P = 0 \Rightarrow N = mg + P$$

$$\boxed{\Sigma F_x = 0}$$

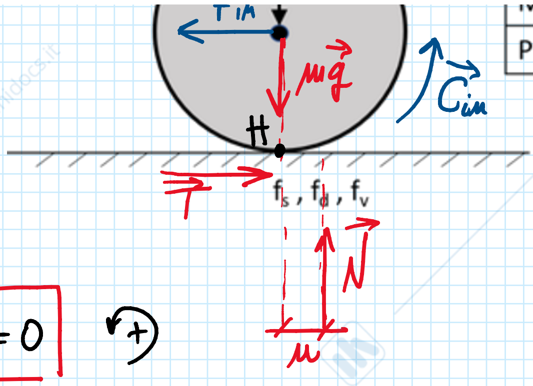
$$T - F_{in} = 0 \rightarrow m a \rightarrow \text{Risposta alla 5} \Rightarrow T = ma$$

$|T| \le f_s|N|$ → SI → ADEREN
NO → STRISCIAME

⑤ a del disco?



M = 10 kg	R = 1 m	J = 5 kgm^2	C = XX*10 Nm
P = 30 N	fs = 0.9	fd =0.4	fv = 0.02



P = 30 N	fs = 0.9	fd = 0.4	fv = 0.02

$$\boxed{a = \dot{\omega} R}$$

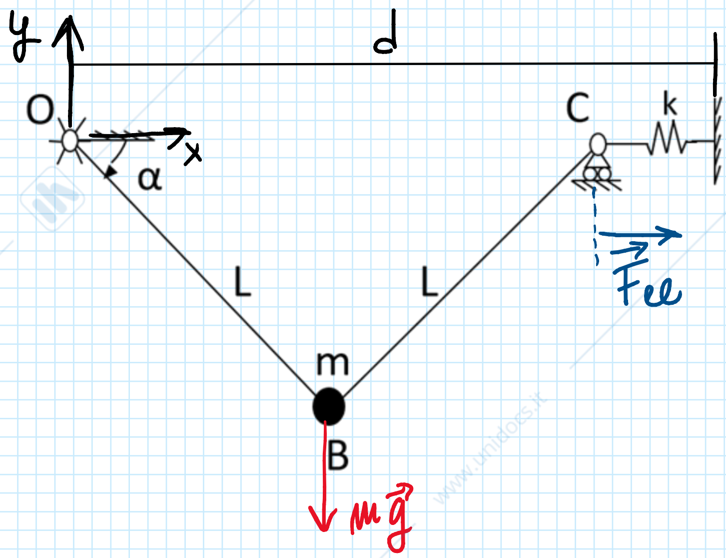
$$\boxed{\sum M_H = 0} \quad (+)$$

$$\overbrace{N u}^{Mg+P} + \overbrace{J\dot{\omega}}^{C_{in}} + \overbrace{m a}^{F_{in}} \cdot R - C = 0$$

$$N \cdot f_v \cdot R + J\dot{\omega} + m\dot{\omega}R^2 - C = 0$$

$$\dot{\omega} = \frac{C - N \cdot f_v \cdot R}{J + mR^2} \Rightarrow \boxed{a = \dot{\omega} R}$$

6 k?



m = XX*10 kg	
L = 2 m	
$\alpha_{eq} = \pi/6$	

$$\vec{F}_p = -mg\vec{j}$$

$$\vec{F}_{el} = k\Delta l\,\vec{i}$$

$$\Delta l = l - l_0$$

$$l = d - 2l\cos\alpha$$

$$l_0 = d - 2l$$

$$\Delta l = d - 2l\cos\alpha - d + 2l = 2l - 2l\cos\alpha$$

$$\begin{cases} \vec{P}_C = 2l\cos\alpha\,\vec{i} \\ \vec{P}_B = l\cos\alpha\,\vec{i} - l\sin\alpha\,\vec{j} \end{cases} \Rightarrow \begin{cases} \delta\vec{P}_C = -2l\sin\alpha\,\delta\alpha\,\vec{i} \\ \delta\vec{P}_B = -l\sin\alpha\,\delta\alpha\,\vec{i} - l\cos\alpha\,\delta\alpha\,\vec{j} \end{cases}$$

[EQ. STATICO]

$$\sum \mathcal{L}^* = \mathcal{L}^*_p + \mathcal{L}^*_{el} = 0$$

$$\mathcal{L}^*_p = \vec{F}_p \cdot \delta\vec{P}_B = mgl\cos\alpha\,\delta\alpha$$

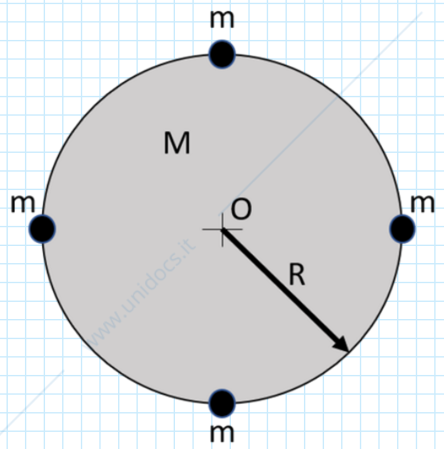
$$\mathcal{L}^*_{el} = \vec{F}_{el} \cdot \delta\vec{S}_C = -k\Delta l\,2l\sin\alpha\,\delta\alpha$$

$\delta \mathcal{L}_{el} = F_{el} \cdot \delta \theta_c = - k \Delta l \, 2l \, \text{sen}\, \alpha \, \delta\theta$

$$K = \frac{mgl \cos \alpha_{eq}}{\Delta l_{eq} \cdot 2l \, \text{sen} \alpha_{eq}} = \frac{mgl \cos \alpha_{eq}}{(2l - 2l \cos \alpha_{eq}) \cdot 2l \, \text{sen} \alpha_{eq}}$$

7

J_0 ?



M = XX*100 kg	
m = XX kg	
R = 10 m	

$$J_{0_{disco}} = \frac{MR^2}{2}$$

$$J_{0_{masse}} = m R^2$$

$$J_0 = \frac{MR^2}{2} + 4 m R^2$$

8

$$\begin{cases} x(t) = -4t \\ y(t) = \frac{t^2}{4} \end{cases}$$

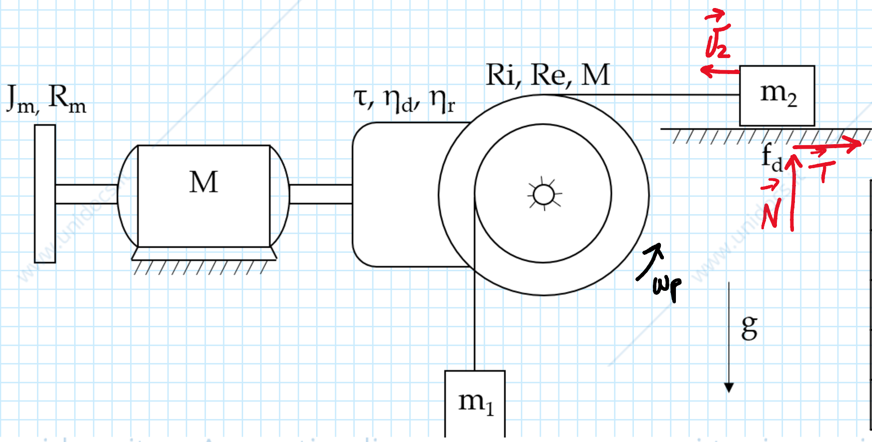
traiettoria: $t = -\frac{x}{4} \Rightarrow y = \frac{x^2}{64}$

versore tangente: $(t = 2s)$

$\vec{v} = \dot{x}\vec{i} + \dot{y}\vec{j} = -4\vec{i} + \frac{t}{2}\vec{j} \Rightarrow \vec{t} = -\frac{4}{\sqrt{17}}\vec{i} + \frac{1}{\sqrt{17}}\vec{j}$

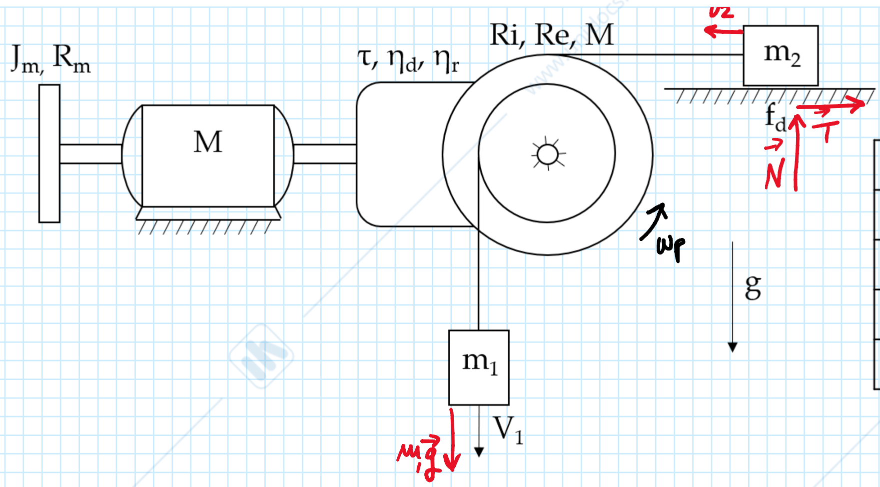
$|\vec{v}| = \sqrt{17}$

9



M1 = 10*XX kg	
M2 = 100 kg	
Ri = 2 m	
Re = 4 m	
fd = 0.4	

9



M1 = 10*XX kg	
M2 = 100 kg	
Ri = 2 m	
Re = 4 m	
fd = 0.4	

Moto Diretto o Retrogrado?

BdP parz. lato utilizz.

$W_2 \swarrow \; U$

$$\begin{cases} v_1 = \omega_p R_i = \tau \omega_m R_i \\ v_2 = \omega_p R_e = \tau \omega_m R_e \end{cases}$$

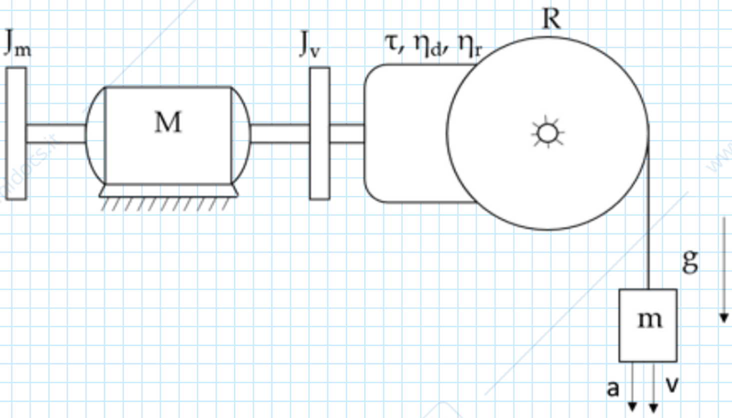
$-W_2 + W_u = 0$

$$W_2 = W_u = m_1 g v_1 - \overset{f_d \cdot N \,\nearrow\, m_2 g}{T}\, v_2 = m_1 g \tau \omega_m R_i - f_d\, m_2 g \tau \omega_m R_e = \underbrace{(m_1 g R_i - f_d\, m_2 g R_e)}\tau \omega_m$$

> 0 retrograd

< 0 diretto

10



m = XX*100 kg	J_m = 20 kgm^2
C_f = -50 Nm	η_r = 0.85
η_r = 0.9	R = 2 m
a = 2 m/s^2	τ = 0.4

J_v?

BdP tot

$$W_m + W_p + W_u = \frac{dE_c}{dt}$$

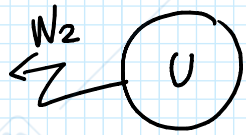
$$W_m = C_F\, \omega_m$$

$$W_u = m g v$$

$$\frac{dE_c}{dt} = J_m \dot{\omega}_m \omega_m + J_v \dot{\omega}_m \omega_m + m v a$$

W_p ? $\longrightarrow$ BdP parziale utilizzatore

$$-W_2 + W_u = \frac{dE_c}{dt}$$



$$W_2 = W_u - \frac{dE_c}{dt}$$

$$W_2 = mgv - mav = \underbrace{(mg - ma)}_{>0 \Rightarrow \text{moto Retrogrado}} v$$

$$W_p = -(1 - \eta_r) W_2$$

BdP tot

$$C_F \omega_m - (\cancel{1} - \eta_r)(mgv - mva) + \cancel{mgv} = (J_m + J_v)\dot{\omega}_m \omega_m + \cancel{mva}$$

$$\left[\omega_m = \frac{v}{\tau R} \; ; \; \dot{\omega}_m = \frac{a}{\tau R} \right]$$

$$C_F \frac{\cancel{v}}{\tau R} + \eta_r m g \cancel{v} - \eta_r m \cancel{v} a = \frac{J_m}{\tau^2 R^2} \cancel{v} a + \frac{J_v}{\tau^2 R^2} \cancel{v} a$$

$$J_v = \left(\frac{C_F}{\tau R} + \eta_r m g - \eta_r m a - \frac{J_m}{\tau^2 R^2} a \right) \frac{\tau^2 R^2}{a}$$