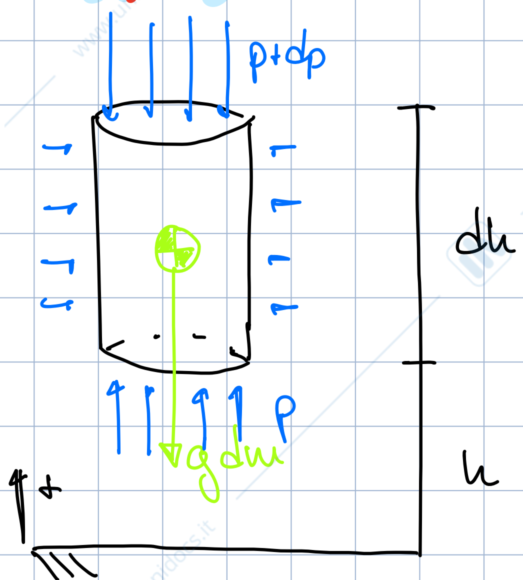
M

Legge di Stevino

Appunti di meccanica del volo di cui l'eli potrà usufruire in un futuro molto prossimo



$$p\,dA - (p+dp)\,dA - g\,dm = 0$$

$$\cancel{p\,dA} - \cancel{p\,dA} - dp\,dA - g\,dm = 0$$

$$dA\,dp = -g\,dm \qquad \rho = \frac{dm}{dV} \qquad dV = dA\,dh$$

$$dA\,dp = -g\rho\,dV$$

$$\cancel{dA}\,dp = -g\rho\,\cancel{dA}\,dh \qquad \boxed{dp = -g\rho\,dh}$$

$$\begin{cases} g = g(h) \\ \rho = \rho(h) \end{cases}$$

introduco h_g = quota geo-potenziale $= \dfrac{R_\oplus h}{R_\oplus + h}$

$$\frac{dh_g}{dh} = \frac{R_\oplus(R_\oplus + h) - R_\oplus h}{(R_\oplus + h)^2} = \left(\frac{R_\oplus}{R_\oplus + h}\right)^2 \qquad g(h) = g_0\left(\frac{R_\oplus}{R_\oplus + h}\right)^2$$

$$\frac{dh_g}{dh} = \frac{g(h)}{g_0} \qquad dh\,g = dh_g\,g_0 \qquad \boxed{dp = -g_0\,\rho\,dh_g}$$

MODELLI ATMOSFERICI

- Troposfera

$$dp = -\rho g\,dh \qquad \rho = \frac{p}{RT} \qquad dp = -\frac{p}{RT}g_0\,dh$$

$$\frac{dT}{dh} = \lambda \qquad \int_{T_0}^{T} dT = \int_{h_0}^{h}\lambda\,dh \qquad T - T_0 = \lambda h \qquad T = T_0 + \lambda h$$

$$\int_{p_0}^{p}\frac{dp}{p} = -\frac{g_0}{R}\int_{h_0}^{h}\frac{dh}{T_0 + \lambda h} \qquad \ln\frac{p}{p_0} = -\frac{g_0}{R\lambda}\ln\left(\frac{T_0 + \lambda h}{T_0}\right)$$

$$\boxed{\frac{P}{P_0} = \left(1 + \frac{\lambda h}{T_0}\right) e^{-\frac{g_0}{R\lambda}}}$$

$P = \rho R T \qquad \frac{P}{P_0} = \frac{\rho}{\rho_0}\frac{T}{T_0}$

$P_0 = \rho_0 R T_0$

$$\boxed{\frac{\rho}{\rho_0} = \left(1 + \frac{\lambda h}{T_0}\right) e^{-\frac{g_0}{R\lambda} - 1}} \qquad \boxed{\frac{T}{T_0} = 1 + \frac{\lambda h}{T_0}}$$

- Tropopausa

$\frac{dh}{dT} = 0 \qquad dp = -g_0 \rho \, dh \qquad p = \rho R T$

$\frac{dp}{p} = -\frac{g_0}{RT} dh \qquad \int \frac{dp}{p} = -\frac{g_0}{RT}\int dh \qquad \Delta h = h - h_{11k}$

$$\boxed{\frac{P}{P_{11}} = e^{-\frac{g_0}{RT_{11}}\Delta h}} \qquad \boxed{\frac{\rho}{\rho_{11}} = e^{-\frac{g_0}{RT_{11}}\Delta h}} \qquad \boxed{\frac{T}{T_0} = 1}$$

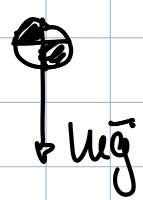
- Atmosfera esponenziale

$$\boxed{\frac{P}{P_{0ISA}} = e^{-\frac{h}{\beta_{opt}}}} \qquad \boxed{\frac{\rho}{\rho_{0ISA}} = e^{-\frac{h}{\beta_{all}}}}$$

AZIONI SUI VELIVOLI

- Gravitazionali

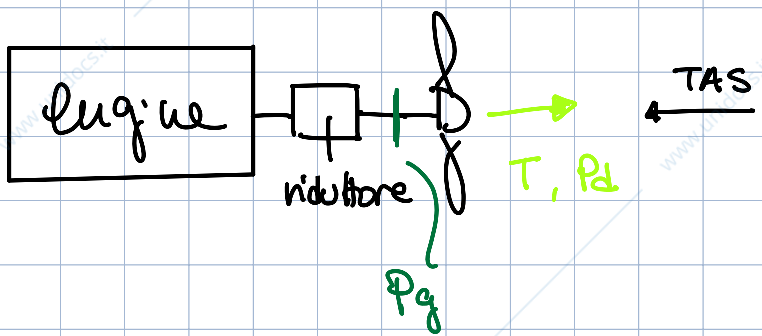
Forza di gravità



$g(h) = g_0 \left(\frac{R_\oplus}{R_\oplus + h}\right)^2 \qquad g_0 = 9{,}80665 \ \frac{m}{s^2}$

• Propulsive

ELICA
- aspirato
- sovralimentato
  - geared supercharger
  - turbocharger
- elettrico
- turbina
  - turboelica
  - turboshaft



$$P_d = P_g \, \eta_{prop} \quad [BHP]$$

$$P_d = T \cdot TAS$$

ELICHE
- Passo fisso
- Passo variabile $\quad \dfrac{\partial \eta_p}{\partial TAS} \simeq 0 \Rightarrow \eta_p \simeq cost \simeq \eta_p^{max}$
- Giri costanti $\quad$ CSU - regolatore dei giri

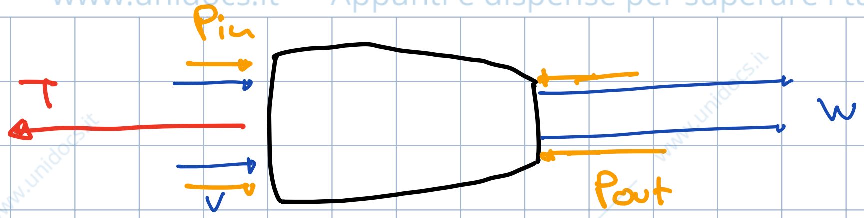
$$\boxed{P_g^{max}(h) = P_g^{max}(0) \, \frac{\rho}{\rho_0}}$$

$$\boxed{\frac{dw}{dt} = -K_c P_g} \qquad K_c = \left[\frac{N}{Sw}\right]$$

N.B.: motori turbo

$$\boxed{\begin{cases} P_g^{max}(h) = P_g^{max}(0) & h \leq 20'000 \text{ ft} \\ P_g^{max}(h) = P_g^{max}(20_k) \, \dfrac{\rho}{\rho_{20k}} & h > 20'000 \text{ ft} \end{cases}}$$

GETTO $\qquad T = \dot{m}(w - v) - A(P_{out} - P_{in})$



turbo jet
turbo fan

$$\begin{cases} T^{max}(h) = T^{max}(0)\left(\dfrac{\rho}{\rho_0}\right)^{0,7} \\ T^{max}(h) = T^{max}(11k)\dfrac{\rho}{\rho_{11k}} \end{cases}$$

troposfera
tropopausa

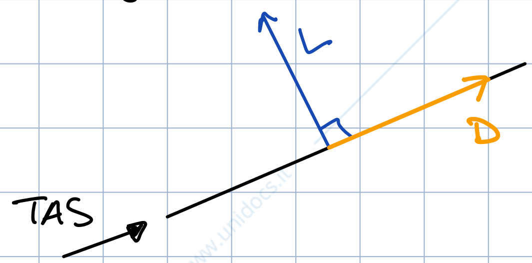
$$\boxed{\frac{dW}{dt} = -cT}$$

$C = [s^{-1}]$

• Aerodinamiche

PORTANZA $\quad L = \frac{1}{2}\rho S V^2 C_L$ ~ coeff di portanza

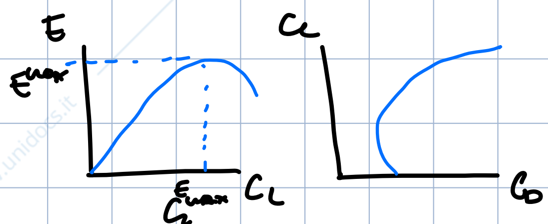
RESISTENZA $\quad D = \frac{1}{2}\rho S V^2 C_D$ ~ coeff di resistenza



Polare di resistenza $\quad C_D = \underbrace{C_{D0}}_{\text{coeff r parassita}} \underbrace{- K_1 C_L + K_2 C_L^{\,4}}_{\text{coeff r indott}}$

Polare di Prandtl $\quad \boxed{C_D = C_{D0} + K C_L^2}$

EFFICIENZA $\quad E = \dfrac{L}{D} = \dfrac{C_L}{C_D}$



Prandtl

$$\frac{\partial E}{\partial C_L} = \frac{\partial}{\partial C_L}\left(\frac{C_L}{C_{D0}+KC_L^2}\right) = \frac{C_{D0}+KC_L^2 - C_L(2KC_L)}{(\quad)^2} = 0$$

$$C_{D0} + \cancel{KC_L^2} - \cancel{2}KC_L^2 = 0 \qquad KC_L^2 = C_{D0} \qquad \boxed{C_L^{Emax} = \sqrt{\frac{C_{D0}}{K}}}$$

$$E^{max} = \frac{C_L^{Emax}}{C_D^{Emax}} = \frac{\sqrt{\frac{C_{D0}}{K}}}{C_{D0} + K\frac{C_{D0}}{K}} = \sqrt{\frac{C_{D0}}{K}}\,\frac{1}{2C_{D0}}\,\frac{\sqrt{C_{D0}}}{\sqrt{C_{D0}}}$$

$$= \frac{\cancel{C_{D0}}}{2\cancel{C_{D0}}\sqrt{KC_{D0}}} = \boxed{\frac{1}{2\sqrt{KC_{D0}}} = E^{max}}$$

$$K = \frac{1}{\pi AR\, e} \qquad AR = \frac{b^2}{S}$$

EQUAZIONI DEL MOTO

$$\bar{F}^{(e)}\Big|_i = \frac{d}{dt}(m\bar{V})\Big|_i$$

$$\vec{F}^{(e)}\Big|_i = m\frac{d\bar{V}}{dt}\Big|_i \qquad i = \text{assi terra}$$

$$F^{(e)}\Big|_e = m\frac{d}{dt}(\overline{TAS} + \overline{Wind})\Big|_e \qquad \text{ora passo al SdR mobile}$$

$$T_{we}\bar{F}^{(e)}\Big|_e = m\,T_{we}\frac{d}{dt}(\overline{TAS} + \overline{Wind})\Big|_e$$

$$\bar{F}^{(e)}\Big|_w = m\frac{d}{dt}(\overline{TAS} + \overline{Wind})\Big|_w$$

$$\frac{d}{dt}(\overline{TAS} + \overline{Wind})\Big|_w = \frac{\partial}{\partial t}(\overline{TAS} + \overline{Wind})_w + \bar{\Omega}_w \wedge (\overline{TAS} + \overline{Wind})_w$$

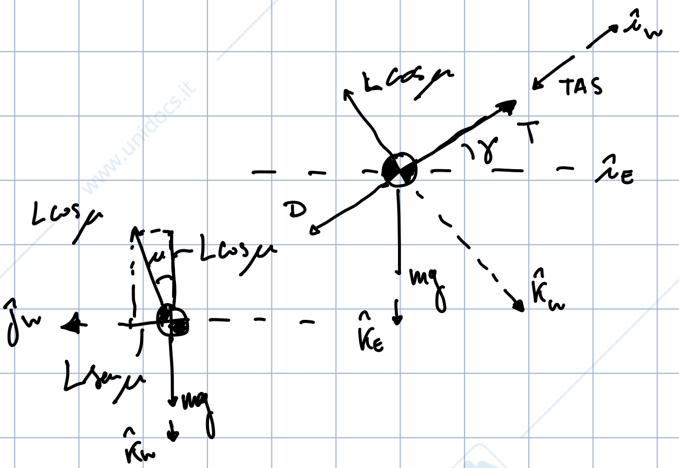
$$\overline{TAS}_w = \begin{bmatrix} TAS \\ 0 \\ 0 \end{bmatrix} \qquad \overline{Wind}_E = \begin{bmatrix} u \\ v \\ w \end{bmatrix} \qquad \bar{\Omega}_w = \begin{bmatrix} -\dot{\chi}\,\mathrm{sen}\gamma \\ \dot{\gamma} \\ \dot{\chi}\cos\gamma \end{bmatrix}$$

$$\dot{\overline{TAS}}_w = \begin{bmatrix} \dot{TAS} \\ 0 \\ 0 \end{bmatrix} \qquad \dot{\overline{Wind}}_E = \begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{bmatrix}$$

$$\overline{Wind}_W = T_{WE}\,\overline{Wind}_E$$

$$\frac{d}{dt}\left(\overline{TAS}+\overline{Wind}\right)_W = \left[\dot{\overline{TAS}}_W + \dot{T}_{WE}\,\overline{Wind}_E + T_{WE}\,\dot{\overline{Wind}}_E + \Omega_W \wedge \left(\overline{TAS}_W + T_{WE}\,\overline{Wind}_E\right)\right.$$

$$T_{WE} = \begin{bmatrix} \cos\gamma\cos\chi & \cos\gamma\,\mathrm{sen}\,\chi & -\mathrm{sen}\,\gamma \\ - & - & - \\ - & - & \cdot \end{bmatrix}$$



$$\hat{i}_w)\ \overline{F}^e_{x_w} = T - D - mg\,\mathrm{sen}\,\gamma$$

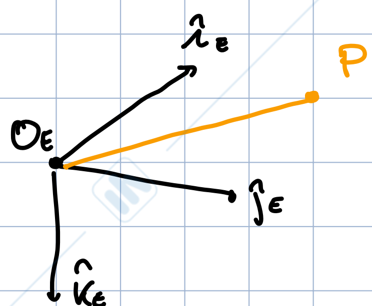
$$\hat{j}_w)\ \overline{F}^e_{y_w} = L\,\mathrm{sen}\,\mu$$

$$\hat{k}_w)\ \overline{F}^e_{z_w} = mg\cos\gamma - L\cos\mu$$

$$\begin{bmatrix} T - D - mg\,\mathrm{sen}\,\gamma \\ L\,\mathrm{sen}\,\mu \\ mg\cos\gamma - L\cos\mu \end{bmatrix} = \overline{F}^e_w$$

<u>eq dinamica</u>

→ cinematica



$$\frac{dP}{dt} = \overline{V} \qquad P_E = [x, y, z]^T$$

$$\frac{dP_E}{dt} = \overline{V}_E = \overline{TAS}_E + \overline{Wind}_E = T_{WE}^{-1}\,\overline{TAS}_W + \overline{Wind}_E$$

$$T_{WE}\,\overline{TAS}_E = \overline{TAS}_W$$

$$T_{WE}^{-1} = \begin{bmatrix} \cos\gamma\cos\chi & - & - \\ \cos\gamma\,\mathrm{sen}\,\chi & - & \cdot \\ -\mathrm{sen}\,\gamma & \cdot & \cdot \end{bmatrix}$$

$$\boxed{\frac{dP_E}{dt} = \begin{bmatrix} TAS\cos\gamma\cos\chi + u \\ TAS\cos\gamma\,\mathrm{sen}\,\chi + v \\ -TAS\,\mathrm{sen}\,\gamma + w \end{bmatrix} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix}}$$

<u>eq. cinematica</u>

N.B.: caso in assenza di vento

$$\frac{d}{dt}\left(\overline{TAS}+\cancel{\overline{Wind}}^{=0}\right)_w = \dot{\overline{TAS}}_w + \Omega_w \wedge \overline{TAS}_w$$

$$\begin{bmatrix} \dot{TAS} \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -\dot{\chi}\,\text{sen}\,\gamma \\ \dot{\gamma} \\ \dot{\chi}\cos\gamma \end{bmatrix} \wedge \begin{bmatrix} TAS \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \dot{TAS} \\ TAS\,\dot{\chi}\cos\gamma \\ -TAS\,\dot{\gamma} \end{bmatrix}$$

$$\overline{F}^e_w = m\frac{d}{dt}\overline{TAS}_w$$

$$\begin{bmatrix} T-D-mg\,\text{sen}\,\gamma \\ L\,\text{sen}\,\mu \\ mg\cos\gamma - L\cos\mu \end{bmatrix} = \begin{bmatrix} m\dot{TAS} \\ m\,TAS\,\dot{\chi}\cos\gamma \\ -m\,TAS\,\dot{\gamma} \end{bmatrix}$$

eq dinamica NO vento

$$\boxed{\begin{cases} \dot{TAS} = \dfrac{T-D}{m} - g\,\text{sen}\,\gamma \\ \dot{\chi} = \dfrac{1}{m\,TAS} L \dfrac{\text{sen}\,\mu}{\cos\gamma} \\ \dot{\gamma} = \dfrac{1}{m\,TAS}\left(L\cos\mu - mg\cos\gamma\right) \end{cases}}$$

eq cinematica NO vento

$$\boxed{\begin{cases} \dot{x} = TAS\cos\gamma\cos\chi \\ \dot{y} = TAS\cos\gamma\,\text{sen}\,\chi \\ \dot{z} = -TAS\,\text{sen}\,\gamma \end{cases}}$$

FATTORE DI CARICO

$\overline{n}$ = e' il rapporto fra le forze di massa e il peso

$$\overline{n} = \frac{\overline{F}_{massa}}{\|\overline{W}\|}$$ ~ forze che il corpo subisce per il fatto di avere una massa m

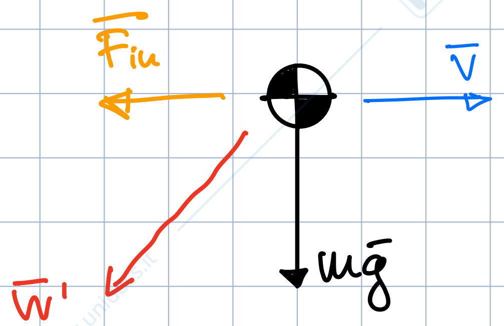
$\overline{F}_{massa} = \overline{W} + \overline{F}_{inerziali}$ $\quad \overline{F}^e = m\overline{a}$ $\quad \overline{F}^e \underbrace{-m\overline{a}}_{F_{inerziali}} = 0$

$\overline{F}_{inerziali} = -m\overline{a}$ $\quad \overline{F}_i = -\overline{F}_e$

1) $\bar{n} = \dfrac{m\bar{g} - m\bar{a}}{mg} = \boxed{\dfrac{\bar{g}-\bar{a}}{g}}$

Forze aerodinamiche e propulsive

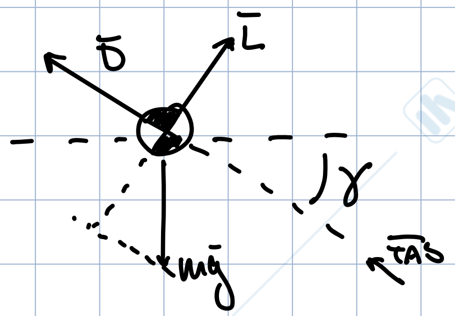
2) $\bar{n} = \dfrac{\bar{W} - \bar{F}_e}{W} = \dfrac{\bar{W} - (\bar{W} + \bar{F}^{a,p})}{W} = \boxed{-\dfrac{\bar{F}^{ap}}{W}}$



$\bar{F}^e = \bar{W} + \bar{F}^{ap}$

$\bar{n} = \dfrac{\bar{W}'}{W}$

VOLO IN ASSENZA DI FORZE PROP

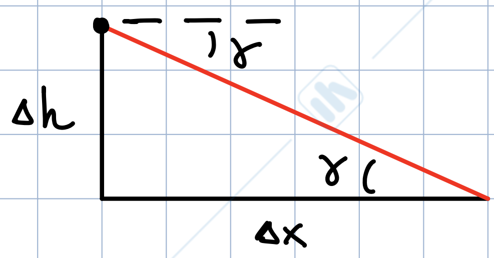


$$\begin{cases} mg \operatorname{sen}\gamma + D = 0 \\ 0 = 0 \\ L = mg\cos\gamma \end{cases}$$ dinamica

$$\begin{cases} \dot{x} = TAS\cos\gamma \\ \dot{y} = 0 \\ \dot{z} = -TAS \operatorname{sen}\gamma \end{cases}$$ cinematica

$\dfrac{\operatorname{sen}\gamma}{\cos\gamma} = \dfrac{-D}{W} = \dfrac{-D\cos\gamma}{L\cos\gamma}$

$\boxed{\tan\gamma = -\dfrac{1}{E}}$



$\Delta h = \Delta x \tan\gamma = \Delta x \dfrac{1}{E}$

$\Delta x = \Delta h E$

$\boxed{\Delta x^{max} = \Delta h E^{max}} \rightarrow \underline{C_L^{OR} = C_L^{Emax}}$

$\dot{z} = -TAS \sin\gamma \qquad \dot{h} = TAS \sin\gamma \qquad \frac{dh}{dt} = TAS \sin\gamma$

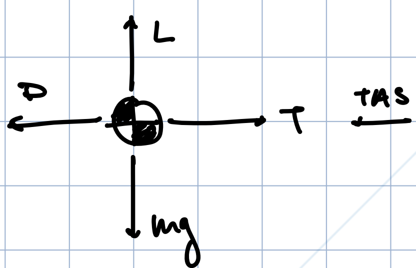
$\sin\gamma \simeq \tan\gamma \qquad TAS = IAS\sqrt{\frac{\rho_0}{\rho}} \qquad \frac{dh}{dt} = IAS\sqrt{\frac{\rho_0}{\rho}}\tan\gamma = -\frac{IAS}{E}\sqrt{\frac{\rho_0}{\rho}}$

$dt = -\frac{E}{IAS}\sqrt{\frac{\rho}{\rho_0}}\,dh \qquad \sqrt{\frac{\rho}{\rho_0}} = \sqrt{\frac{\cancel{\rho_0} e^{-\frac{h}{\beta}}}{\cancel{\rho_0}}} = e^{-\frac{h}{2\beta}}$

$\int_0^t dt = -\frac{E}{IAS}\int_{h_i}^{h_f} e^{-\frac{h}{2\beta}}\,dh \qquad \Delta t = \frac{2\beta E}{IAS}\left(e^{-\frac{h_i}{2\beta}} - e^{-\frac{h_f}{2\beta}}\right)$

$$\boxed{\Delta t = \frac{2\beta E}{IAS}\left(e^{-\frac{h_f}{2\beta}} - e^{-\frac{h_i}{2\beta}}\right)} \rightarrow \underline{C_L^{OE} = \sqrt{3}\, C_L^{E\max}}$$

VOLO IN CROCIERA



$\begin{cases} T = D \\ L = W \end{cases}$ dinamica

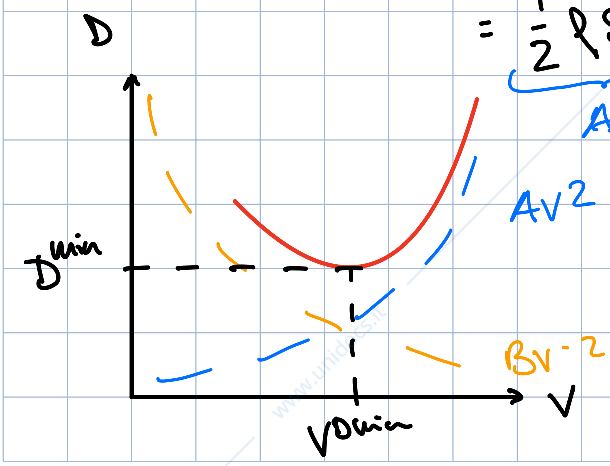
$\begin{cases} \dot{X} = TAS \end{cases}$ cinematica

Resistenza

$D = \frac{1}{2}\rho S V^2 C_D = \frac{1}{2}\rho S V^2 (C_{D0} + K C_L^2)$

$= \frac{1}{2}\rho S C_{D0} V^2 + \frac{1}{\cancel{2}}\cancel{\rho} S K \frac{\cancel{2}^{2} W^2}{\rho^{\cancel{2}} S^{\cancel{2}} V^{\cancel{4}2}}\cancel{V^2}$

$= \underbrace{\frac{1}{2}\rho S C_{D0}}_{A} V^2 + \underbrace{\frac{2KW^2}{\rho S}}_{B}\frac{1}{V^2} = \boxed{AV^2 + BV^{-2}}$



$D^{min} = \frac{L}{E^{max}} \Rightarrow \underline{V^{Dmin} = V^{E\max}}$

$\underline{C_L^{Dmin} = C_L^{E\max}}$

$$\frac{\partial D}{\partial V} = 0 \qquad 2AV - 2BV^{-3} = 0 \qquad \cancel{2}AV^4 - \cancel{2}B = 0 \qquad V = \sqrt[4]{\frac{B}{A}} = V^{Dmin}$$

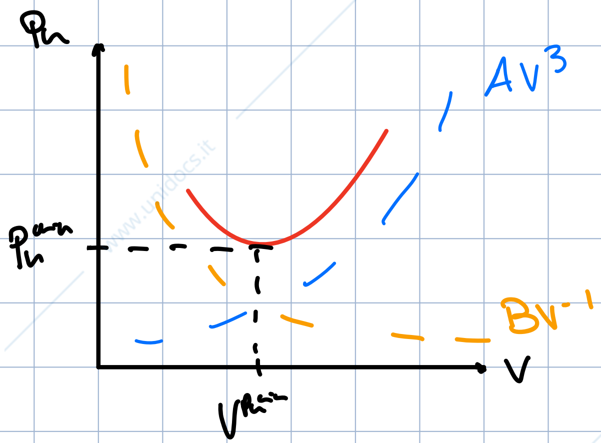
Potenza

$P_n = D \cdot TAS$

$P_d = T \cdot TAS$

in crociera $P_n = P_d$ $\quad D\cancel{TAS} = T\cancel{TAS}$

$$P_n = \frac{1}{2}\rho S V^3 C_D = \boxed{AV^3 + BV^{-1}}$$



$$\frac{\partial P_n}{\partial V} = 0 \qquad 3AV^2 - BV^{-2} = 0$$

$$3AV^4 - B = 0$$

$$V^{P_n min} = \frac{1}{\sqrt[4]{3}}\sqrt[4]{\frac{B}{A}} = \frac{1}{\sqrt[4]{3}} V^{Emax}$$

$$C_L^{P_n min} = \sqrt{3}\, C_L^{Emax}$$

ANALISI PRESTAZIONI

Autonomia oraria getto

• $C_L = cost$ $\quad \frac{dW}{dt} = -cT \quad E = \frac{L}{D} = \frac{W}{T} \quad T = \frac{W}{E} \quad \frac{dW}{dt} = -c\frac{W}{E}$

$$-\int \frac{E\,dW}{c\,W} = \int dt \qquad \boxed{\Delta T = \frac{E}{c}\ln\frac{W_i}{W_f}}$$

• $TAS = cost$ $\quad \frac{dW}{dt} = -cT \quad -\frac{dW}{cT} = dt \quad T = D \quad -\frac{dW}{cD} = dt$

$$D = \frac{1}{2}\rho S V^2\left[C_{D0} + k\left(\frac{2W}{\rho S V^2}\right)^2\right] = \frac{1}{2}\rho S V^2 C_{D0} + \frac{1}{\cancel{2}}\cancel{\rho S V^2}\,k\,\frac{\overset{2}{\cancel{4}}W^2}{\rho^{\cancel{2}}S^{\cancel{2}}V^{\cancel{4}2}}$$

$$= \frac{1}{2}\rho S C_{D0} V^2 + \frac{2kW^2}{\ldots}\frac{1}{\ldots} = a_1 + a_2 W^2$$

$$\underbrace{\tfrac{1}{2}\rho S C_{D0} V^2}_{a_1} \quad \underbrace{\rho S \quad V^2}_{a_2}$$

$$-\frac{1}{C}\frac{dw}{a_1 + a_2 w^2} = dt \qquad dt = -\frac{1}{Ca_1}\frac{dw}{1 + \frac{a_2}{a_1}w^2}$$

$$dy = \sqrt{\frac{a_2}{a_1}}\,dw \qquad -\frac{1}{Ca_1}\frac{\sqrt{\frac{a_1}{a_2}}}{1+y^2}\,dy = dt \qquad -\sqrt{\frac{a_1}{a_2}}\frac{1}{Ca_1}\int\frac{dy}{1+y^2} = \int dt$$

$$\Delta t = -\sqrt{\frac{\cancel{a_1}}{a_2}}\frac{\cancel{\sqrt{a_1}}}{\cancel{\sqrt{a_1}}}\frac{1}{C\cancel{a_1}}\left(\operatorname{atan} y_f - \operatorname{atan} y_i\right)$$

$$\boxed{\Delta t = \frac{1}{C\sqrt{a_1 a_2}}\left(\operatorname{atan}\sqrt{\frac{a_2}{a_1}}w_i - \operatorname{atan}\sqrt{\frac{a_2}{a_1}}w_f\right)}$$

$$\sqrt{a_1 a_2} = \sqrt{\frac{1}{\cancel{2}}\cancel{\rho S}C_{D0}\cancel{V^2}\,\frac{\cancel{2}K}{\cancel{\rho S V^2}}} = \sqrt{C_{D0}K}$$

$$\sqrt{\frac{a_2}{a_1}} = \sqrt{\frac{2K}{\rho S V^2}\cdot\frac{2}{\rho S C_{D0} V^2}} = \sqrt{\frac{4K}{\rho^2 S^2 V^4 C_{D0}}} = \frac{2}{\rho S V^2}\sqrt{\frac{K}{C_{D0}}}$$

$$= \frac{2}{\rho S V^2}\frac{1}{C_L^{Emax}}$$

$$\boxed{\Delta t = \frac{2E^{max}}{C}\left(\operatorname{atan}\frac{C_{Li}}{C_L^{Emax}} - \operatorname{atan}\frac{C_{Lf}}{C_L^{Emax}}\right)}$$

- $T = cost$ $\qquad \dfrac{dw}{dt} = -CT \qquad \displaystyle\int dt = -\int\frac{dw}{CT}$

$$\boxed{\Delta t = \frac{W_{fuel}}{CT}}$$

Autonomia chilometrica getto

- $G = \text{cost}$

$$\frac{dW}{dx}\frac{dx}{dt} \cdot TAS = -CT \qquad \frac{dW}{dt} = \frac{-CT}{TAS} = -\frac{CW}{E\,TAS}$$

$$dx = -\frac{E\,TAS\,dW}{CW} = -\frac{E\sqrt{\frac{2W}{\rho S C_L}}\,dW}{C\;W} = -\frac{E\sqrt{\frac{2}{\rho S C_L}}}{C}\frac{W^{1/2}}{W}dW$$

$$\int dx = -\frac{E\sqrt{\frac{2}{\rho S C_L}}}{C}\int W^{-1/2}dW \qquad \Delta x = -\frac{E\sqrt{\frac{2}{\rho S C_L}}}{C}\,2W^{1/2}\Big|_{W_i}^{W_f}$$

$$\boxed{\Delta x = \frac{2E\sqrt{\frac{2}{\rho S C_L}}}{C}\left(W_i^{1/2} - W_f^{1/2}\right)}$$

- $TAS = \text{cost}$ $\qquad \Delta x = V\Delta t$

$$\boxed{\Delta x = \frac{2E^{max}\,TAS}{C}\left(\operatorname{atan}\frac{G_i}{G^{Em}} - \operatorname{atan}\frac{G_f}{G^{Em}}\right)}$$

- $T = \text{cost}$ $\qquad \frac{dW}{dx} = -\frac{CT}{TAS}$

$$\int dx = -\frac{1}{CT}\int\frac{dW}{TAS}$$

Autonomia oraria elica

- $G = \text{cost}$ $\qquad \frac{dW}{dt} = -K_c P_a = -K_c\frac{P_d}{\eta_p}$

$$dt = -\frac{\eta_p\,dW}{K_c P_d} = -\frac{\eta_p\,dW}{K_c\,T\,TAS} = -\frac{E\,\eta_p\,dW}{K_c\,TAS\,W}$$

$$dt = -\frac{E\eta_p}{K_c\sqrt{\frac{2}{\rho S a}}}\frac{dw}{w^{3/2}} = -\frac{E\eta_p}{K_c\sqrt{\ }} w^{-3/2}\,dw \qquad \int w^{-3/2} = -2w^{-1/2}$$

$$\boxed{dt = \frac{2E\eta_p}{K_c\sqrt{\ }}\left(w_f^{-\frac{1}{2}} - w_i^{-\frac{1}{2}}\right)}$$

- $TAS = cost$ $\qquad \frac{dw}{dt} = -K_c\frac{P_d}{\eta_p} = -K_c\frac{D\,TAS}{\eta_p}$

$$dt = -\frac{\eta_p\,dw}{K_c\left(\frac{1}{2}\rho S V^3 C_{D0} + \frac{2kw^2}{\rho S V^3}\right)} = -\frac{\eta_p\,dw}{K_c\left(a_1 + a_2 w^2\right)}$$

$$dt = -\frac{\eta_p}{K_c}\frac{dw}{a_1 + a_2 w^2} = -\frac{\eta_p}{K_c a_1}\frac{dw}{1+\frac{a_2}{a_1}w^2} \qquad dy = \sqrt{\frac{a_2}{a_1}}\,dw$$

$$dt = -\frac{\eta_p}{K_c a_1}\frac{\sqrt{\frac{a_1}{a_2}}}{1+y^2}dy = -\frac{\cancel{\sqrt{a_1}}\sqrt{\frac{a_1}{a_2}}\,\eta_p}{\sqrt{a_1}\,K_c\,\cancel{a_1}}\left(\operatorname{atan} y_f - \operatorname{atan} y_i\right)$$

$$\Delta t = \frac{\eta_p}{K_c\sqrt{a_1 a_2}}\left[\operatorname{atan}\left(\sqrt{\frac{a_2}{a_1}}\,w_i\right) - \operatorname{atan}\left(\sqrt{\frac{a_2}{a_1}}\,w_f\right)\right]$$

$$\boxed{\Delta t = \frac{2E^{max}\eta_p}{K_c\,TAS}\left[\operatorname{atan}\frac{C_{L_i}}{C_L^{E_{max}}} - \operatorname{atan}\frac{C_{L_f}}{C_L^{E_{max}}}\right]}$$

- $P_d = cost$ $\qquad \frac{dw}{dt} = -K_c\frac{P_d}{\eta_p} \qquad dt = -\frac{\eta_p\,dw}{K_c P_d}$

$$\boxed{\Delta t = \frac{\eta_p W_{fuel}}{K_c P_d}}$$

Autonomia chilometrica elica

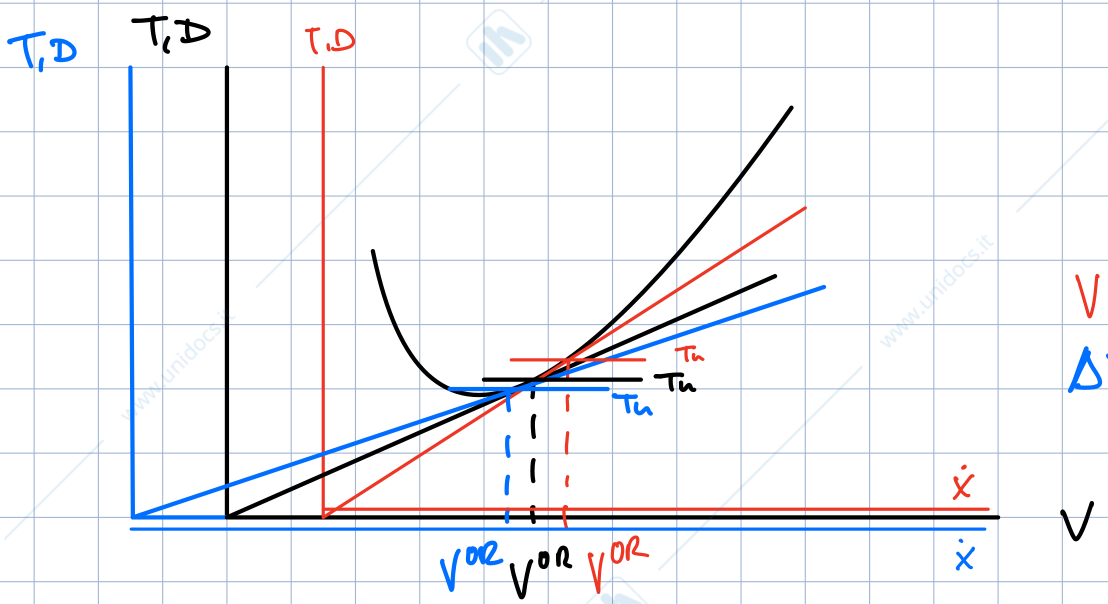
- $C_L = \text{cost}$ $\qquad \dfrac{dW}{dx} = -\dfrac{K_c P_d}{TAS\,\eta_p} = -\dfrac{K_c\, W\, \cancel{TAS}}{\cancel{TAS}\, E\, \eta_p} = -\dfrac{K_c\, W}{E\, \eta_p}$

$$dx = -\frac{E\eta_p}{K_c W}\,dW \qquad \boxed{\Delta x = \frac{E\,\eta_p}{K_c}\ln\frac{W_i}{W_f}}$$

- $TAS = \text{Cost}$ $\qquad V = \dfrac{s}{t} \qquad \Delta x = \Delta t\; TAS$

$$\boxed{\Delta x = \frac{2E^{max}\eta_p}{K_c}\left[\operatorname{atan}\frac{C_{L_i}}{C_L^{E\,max}} - \operatorname{atan}\frac{C_{L_f}}{C_L^{E\,max}}\right]}$$

EFFETTO DEL VENTO



$u = 0$

$u < 0$ contrario

$u > 0$ a favore

$V^{OR} > V^{OR} > V^{OR}$

$\Delta x > \Delta x > \Delta x$

PROGRAMMA SECONDO BREGUET

$C_L = \text{Cost} \quad TAS = \text{cost} \implies L = \text{cost}$

motore endotermico $\implies W \downarrow \implies L > W$

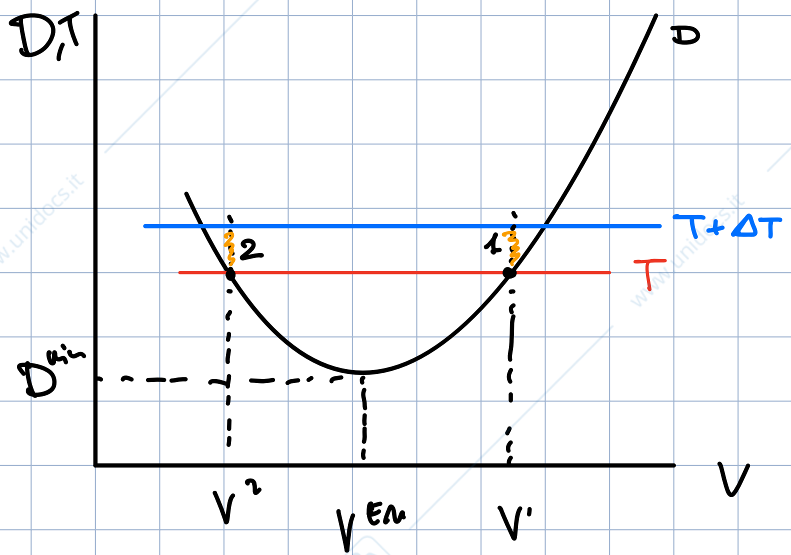
$\dfrac{dW}{dt}\dfrac{dx}{\ldots} = -cT \qquad \dfrac{dW}{dx} = -\dfrac{cT}{TAS\ldots} \qquad dx = -\dfrac{(TAS+u)\,dW}{\ldots}$

$$dx = -\frac{(TAS+u)\,E\,dw}{c\,W}$$

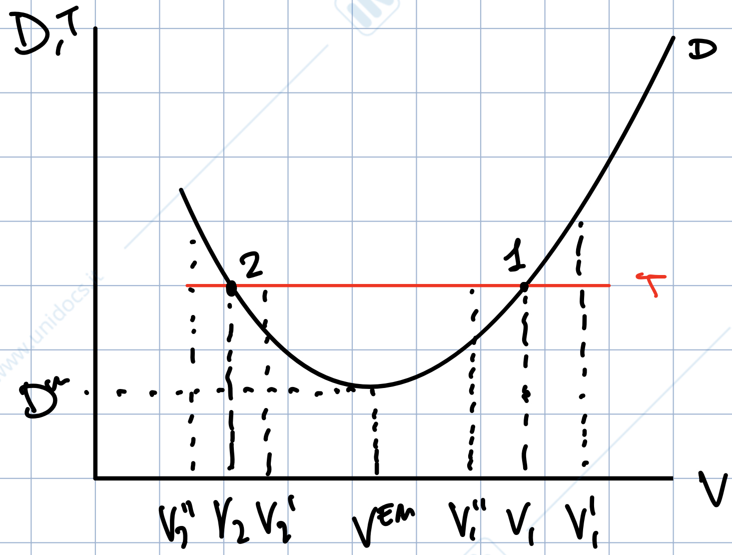
$$\boxed{\Delta x = \frac{(TAS+u)\,E}{c}\ln\frac{W_i}{W_f}}$$

STABILITA' PROPULSIVA

Getto



Sia in 1 che in 2 se aumento la manetta ho un esubero di spinta e quindi guadagno quota



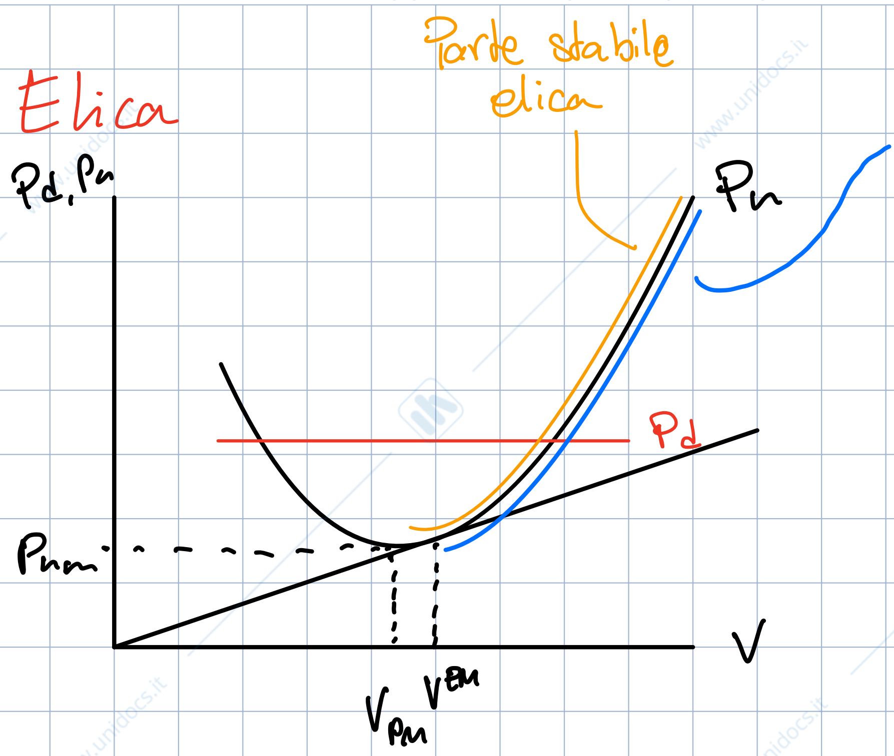
punto 1) STABILE

- picchio: aumento V e ho $D > T$ quindi diminuisco di quota (come volevo)
- cabro: diminuisco V e ho $T > D$ quindi aumento di quota (come volevo)

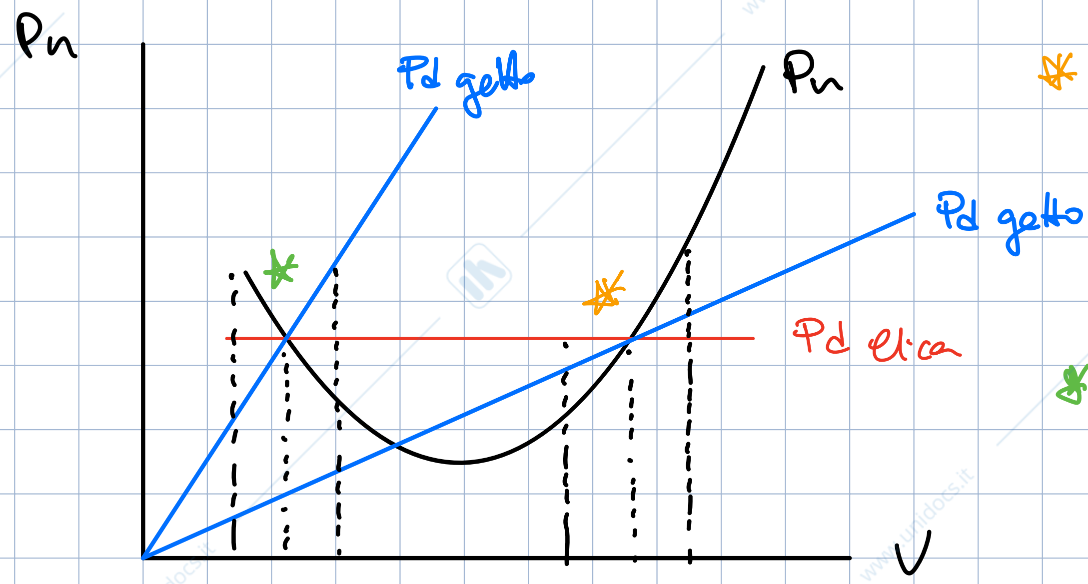
punto 2) INSTABILE

- picchio: aumento V ma ho $T > D$ e quindi aumento la quota
- cabro: diminuisco V ma ho $D > T$ e quindi diminuisco di quota

Elica



Stesse considerazioni del getto ma la vel minima di stabilità è quella di $P_n^{min} < V^{Emax}$ e quindi la curva stabile è maggiore



✱ Nella parte stabi[le] l'elica ha esuberi e difetti maggiori del getto e quindi è più reattivo

✱ Nella parte instabile l'elica ha esuberi e difetti minori e quindi è meno reattivo ma essendo nella parte instabile è meglio così.

⇒ Elica > Getto in stabilità e reattività.

SPEED PARAMETER

$$\boxed{V = b\, V^{Emax}}$$

$$C_L = \frac{2W}{\rho S V^2} = \frac{2W}{\rho S V_{Em}^2 b^2}$$

$\sim C_L^{Emax}$

$$C_L = \frac{1}{b^2} C_L^{Emax}$$

SOLO IN CROCIERA

$$E = \frac{C_L}{C_D} = \frac{\frac{1}{b^2} C_L^{Emax}}{C_{D0} + K \frac{C_L^{Em\,2}}{b^4}} = \frac{\frac{1}{b^2}\sqrt{\frac{C_{D0}}{K}}}{C_{D0} + K \frac{C_{D0}}{K} \frac{1}{b^4}}$$

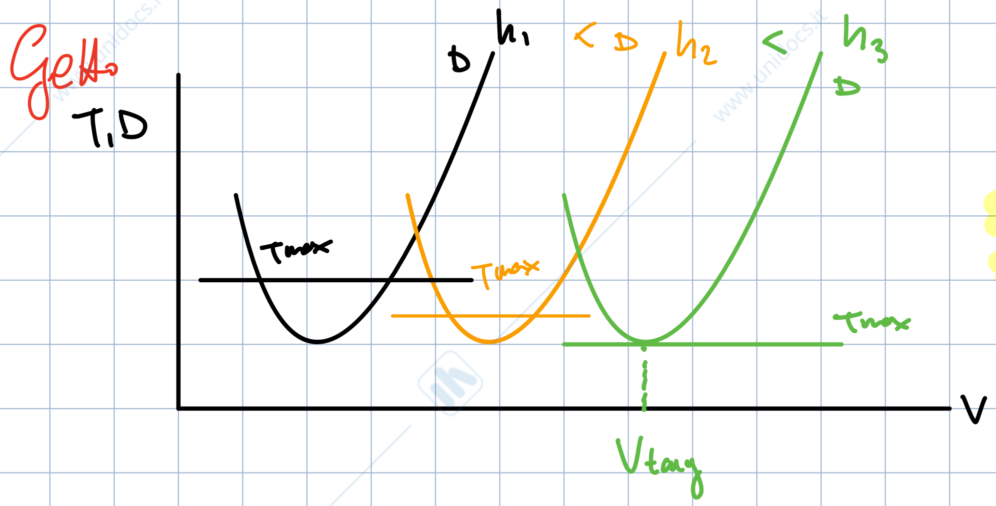
$$= \frac{\frac{1}{b^2}\sqrt{\frac{C_{D0}}{K}}\,\frac{\sqrt{C_{D0}}}{\sqrt{C_{D0}}}}{\cancel{C_{D0}}\left(1+\frac{1}{b^4}\right)} = \frac{\frac{1}{b^2}\,\frac{2}{2\sqrt{C_{D0}K}}}{1+\frac{1}{b^4}} = \frac{\frac{2}{b^2}\,b^4\,2\,E^{max}}{b^4\left(1+\frac{1}{b^4}\right)}$$

$$= \frac{2b^2}{b^4+1}E^{max} = E$$

solo in crociera

QUOTA DI TANGENZA

Getto



quota di tangenza: massima quota a cui posso fare crociera

$T^{max} = D^{min}$ $\quad D^{min} = \frac{W}{E^{max}}$

$T^{max}_{troposfera} = T_0\left(\frac{\rho_{11}}{\rho_0}\right)^{0,7}$

Se $T^{max}_{troposfera} > D^{min} \Rightarrow h_c \in$ tropopausa

Se $T^{max}_{troposfera} \leq D^{min} \Rightarrow h_c \in$ troposfera

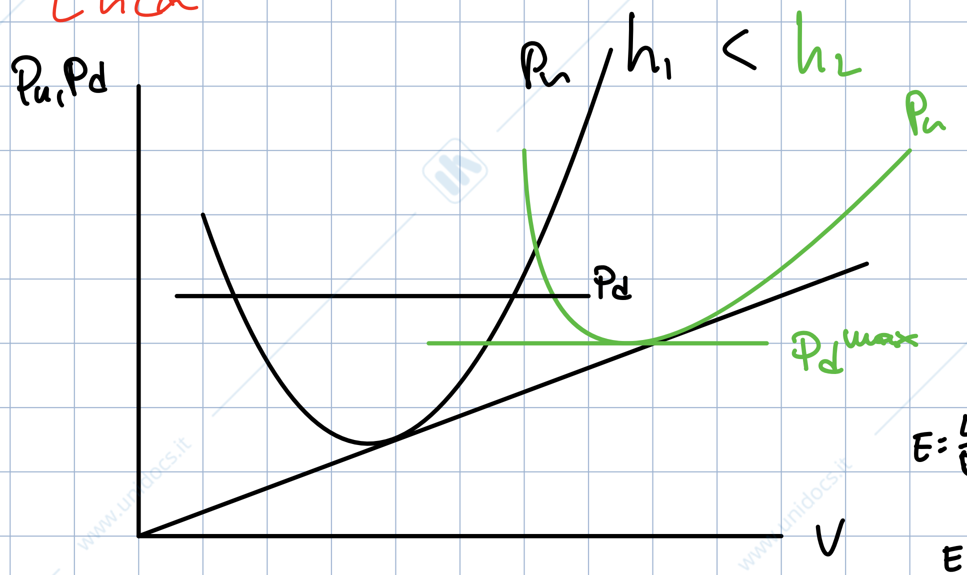
troposfera

$T^{max} = T_0\left(\frac{\rho}{\rho_0}\right)^{0,7} = \frac{W}{E^{max}}$ $\quad \frac{\rho}{\rho_0} = \left(\frac{W}{T_0 E^{m}}\right)^{\frac{1}{0,7}}$

$$\boxed{\rho_c = \rho_0\left(\frac{W}{T_0 E^{m}}\right)^{1/0,7}}$$

Tropopausa $\boxed{\rho_c = \rho_{11k}\left(\frac{W}{T_{11k} E^m}\right)}$

Elica



$P_d^{max} = P_u^{min}$

$P_u = D\,TAS = D\,b\,TAS^{Em}$

$= \frac{W}{E}\,b\,TAS^{Em}$

$E = \frac{L}{D}$ $\quad D = \frac{W}{E}$

$E = \frac{2b^2}{1+b^4}E^m$

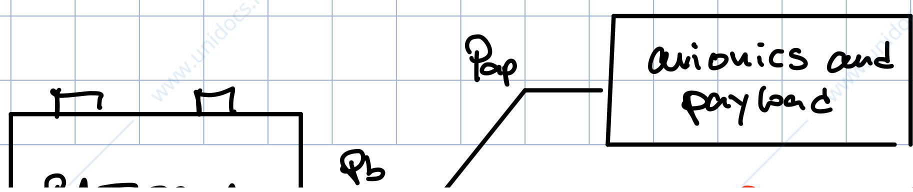
$$P_u = W\underbrace{\frac{1+b^4}{2b^2E^m}\,TAS^{Em}\sqrt{\rho_0}}_{\xi}\,\frac{1}{\sqrt{\rho}} \qquad V^{P_{u,min}} = \overset{\approx b}{\frac{1}{\sqrt{3}}}\,V^{E_{max}}$$

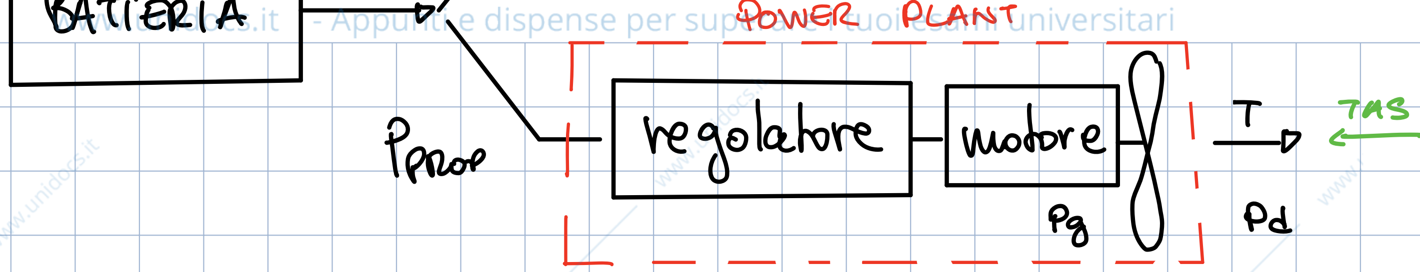
$$P_u = \xi\frac{1}{\sqrt{\rho}} \qquad P_d = \eta_p P_g$$

$$\eta_p P_{g_0}\frac{\rho_c}{\rho_0} = \frac{\xi}{\sqrt{\rho_c}} \qquad \xi = \eta_p P_{g_0}\left(\frac{\rho_c}{\rho_0}\right)^{3/2}$$

$$\boxed{\rho_c = \left(\frac{\xi\,\rho_0}{\eta_p P_{g_0}}\right)^{2/3}}$$

VELIVOLI ELETTRICI





$$P_b = P_{ap} + P_{PROP} \qquad \eta_{TOT} = \frac{P_d}{P_{PROP}} \qquad P_{PROP} = \frac{P_d}{\eta_T}$$

$$P_b = P_{ap} + \frac{P_d}{\eta_T} = \boxed{P_{ap} + \frac{ATAS^3 + BTAS^{-1}}{\eta_T} = P_b}$$

$$\boxed{t = \delta P_b^{\varepsilon} C^{\beta}}$$

autonomia oraria

$$t = \delta \Big(\underbrace{P_{ap} + \frac{ATAS^3 + BTAS^{-1}}{\eta_T}}_{P_b}\Big)^{\varepsilon} C^{\beta}$$

$\max(\Delta t) \Rightarrow \min(P_b)$

perche' elevato per $\varepsilon < -1$

$$P_{ap} \simeq cost \Rightarrow \min\left(\frac{A}{\eta_T} TAS^3 + \frac{B}{\eta_T} TAS^{-1}\right) \qquad \bar{A} = \frac{A}{\eta_T} \qquad \bar{B} = \frac{B}{\eta_T}$$

$$\frac{\partial P_b}{\partial TAS} : \frac{\partial \bar{A} TAS^3 + \bar{B} TAS^{-1}}{\partial TAS} = 3\bar{A}TAS^2 - \bar{B}TAS^{-2} = 0 \qquad 3\bar{A}TAS^4 = \bar{B}$$

$$TAS^{OE} = \frac{1}{\sqrt[4]{3}} \sqrt{\frac{\bar{B}}{\bar{A}}} = \frac{1}{\sqrt[4]{3}} \sqrt{\frac{B}{A}} = \underbrace{\frac{1}{\sqrt[4]{3}}}_{b_{OE}} TAS^{Em}$$

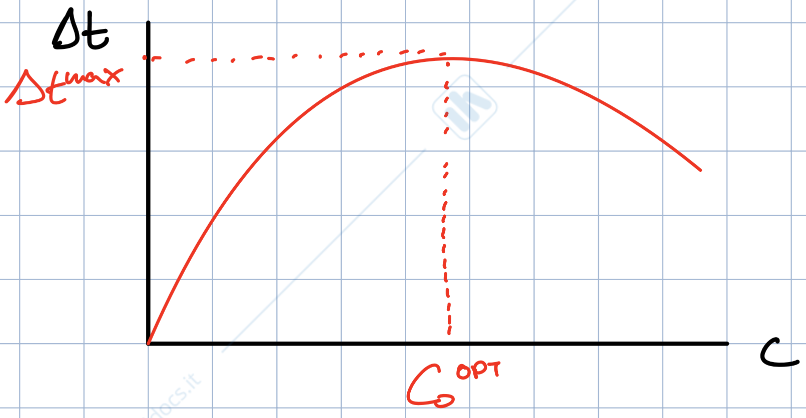
Autonomia chilometrica

$$\Delta X = \Delta t \, TAS = \delta \left(P_{ap} + \frac{ATAS^3 + BTAS^{-1}}{\eta_T}\right)^{\varepsilon} C^{\beta} TAS$$

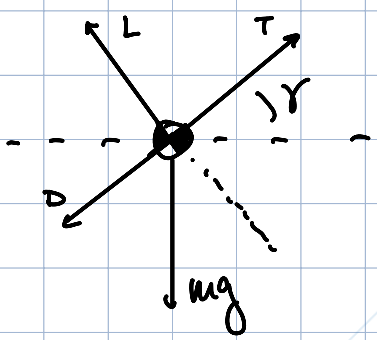
$\max(\Delta t) \Rightarrow \min(P_b^{\varepsilon} TAS)$

$$TAS_{OR} = \sqrt[4]{\frac{\varepsilon-1}{1+3\varepsilon}}\sqrt[4]{\frac{B}{A}} = \underbrace{\sqrt[4]{\frac{\varepsilon-1}{1+3\varepsilon}}}_{b_{OR}} TAS^{EM}$$

batteria ideale $\varepsilon = -1$ $\quad \beta = 1$ $\quad t = \frac{\delta C}{P_b}$ $\quad TAS^{OR} \simeq TAS^{EM}$



SALITA (stazionaria)



$$\begin{cases} T = D + mg \operatorname{sen}\gamma \\ L = mg\cos\gamma \end{cases}$$

$$\operatorname{sen}\gamma = \frac{T-D}{mg}$$

salita ripida $\gamma = \gamma^{max}$

salita rapida $h = h^{max}$

Getto

• SAL RIPIDA

$\operatorname{sen}\gamma = \frac{T-D}{W}$ $\quad \gamma^{max} \Rightarrow (T-D)_{max}$

$T = T^{max}$

$D = D^{min} = A\,TAS^2 + B\,TAS^{-2}$

$$TAS_{\substack{sal\\rip}} : \frac{\partial}{\partial T} D^{min} = 2A\,TAS - 2B\,TAS^{-3} = \cancel{2}A\,TAS^4 = \cancel{2}B$$

$$TAS_{\substack{sal\\rip}} = \sqrt[4]{\frac{B}{A}} = TAS^{Emax}$$

- SAL RAPIDA

$$\dot{h} = TAS \operatorname{sen}\gamma = \frac{T-D}{W} TAS$$

$$\dot{h} = \frac{T\,TAS - D\,TAS}{W}$$

$$\max(\dot{h}) \Rightarrow \max(T\,TAS - D\,TAS)$$

$$TAS_{\substack{sal\\rap}} = \frac{\partial}{\partial TAS}\left(T\,TAS - A\,TAS^3 - B\,TAS^{-1}\right) = 0$$

$$T - 3A\,TAS^2 + B\,TAS^{-2} = 0$$

$$T\,TAS^2 - 3A\,TAS^4 + B = 0$$

$$3A\,TAS^4 - T\,TAS^2 - B = 0 \qquad y = TAS^2$$

$$TAS = \sqrt{y}$$

$$3Ay^2 - Ty - B = 0$$

$$y = \frac{T \pm \sqrt{T^2 + 12AB}}{6A}$$

$$TAS_{\substack{sal\\rap}} = \sqrt{\frac{T \pm \sqrt{T^2 + 12AB}}{6A}}$$

Elica

- Salita ripida

$$\operatorname{sen}\gamma = \frac{T-D}{W} \qquad P_d = T\,TAS \qquad \frac{T-D}{W} = \frac{\frac{P_d}{TAS} - D}{W}$$

$$\max(\gamma) = \max\left(\frac{P_d}{TAS} - D\right) = \frac{P_d}{TAS} - A\,TAS^2 - B\,TAS^{-2}$$

$$V_{\substack{sal\\rip}} = \frac{\partial}{\partial TAS}\left(\frac{Pd}{TAS} - ATAS^2 - BTAS^{-2}\right) = 0$$

$$\frac{\partial}{\partial TAS} Pd\,TAS^{-1} = Pd\left(-TAS^{-2}\right)$$

$$-Pd\,TAS^{-2} - 2ATAS + 2BTAS^{-3} = 0$$

$$-Pd\,TAS - 2ATAS^4 + 2B = 0$$

$$2ATAS^4 + Pd\,TAS - 2B = 0$$

- Salita rapida

$$h = TAS\,sen\gamma = \frac{T-D}{W}TAS = \frac{\left(\frac{Pd}{TAS} - D\right)TAS}{W} = \frac{Pd - DTAS}{W}$$

$$max(h) = max(Pd - DTAS) = (Pd - Pu)$$

$$V_{\substack{sal\\rap}} = V_{Pumin} = \frac{1}{\sqrt[4]{3}} V^{Emax}$$

SALITA NON STAZIONARIA

$$\dot{TAS} = \frac{T-D}{m} - g\,sen\gamma$$

$$\dot{TAS} = \frac{dTAS}{dt} = \frac{dTAS}{dh}\frac{dh}{dt} = \frac{dTAS}{dh} TAS\,sen\gamma$$

$$\frac{1}{2}\cdot\frac{d}{dh} TAS\,TAS = \frac{dTAS}{dh}TAS + TAS\frac{dTAS}{dh} = \cancel{2}\frac{dTAS}{dh}TAS \cdot \frac{1}{\cancel{2}}$$

$$\frac{dTAS}{dh}TAS = \frac{1}{2}\frac{dTAS^2}{dh}$$

$$\frac{1}{2}\frac{dTAS^2}{dh}\sin\gamma = \frac{T-D}{m} - g\sin\gamma$$

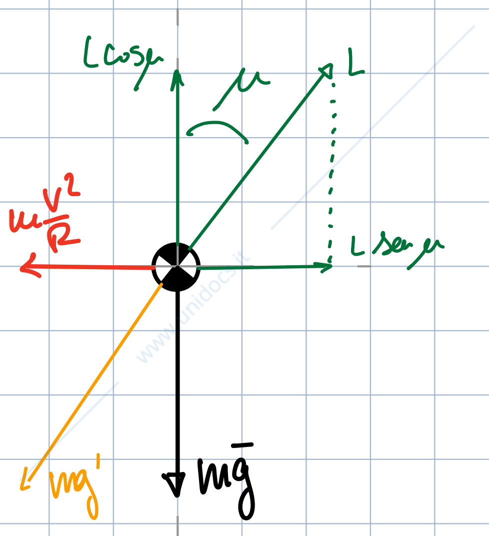
$$\frac{1}{2}\frac{dTAS^2}{dh}\sin\gamma + g\sin\gamma = \frac{T-D}{m}$$

$$\sin\gamma\left(1+\frac{1}{2g}\frac{dTAS^2}{dh}\right) = \frac{T-D}{W}$$

$$\sin\gamma = \frac{T-D}{W}\left(1+\frac{1}{2g}\frac{dTAS^2}{dh}\right)^{-1}$$

$$\sin\gamma = \frac{T-D}{W}K_E \qquad K_E = \left(1+\frac{1}{2g}\frac{dTAS^2}{dh}\right)^{-1}$$

VIRATA



VIRATA COORDINATA

$$\begin{cases} V = cost \\ h = cost \\ R = cost \\ \beta = 0° \end{cases} \qquad \begin{array}{l} T = D \\ L\cos\mu = mg \\ L\sin\mu = m\frac{V^2}{R} \end{array}$$

$$\tan\mu = \frac{V^2}{gR} \qquad V = \Omega R \qquad n_{zB} = \frac{mg}{mg\cos\mu} = \frac{1}{\cos\mu}$$

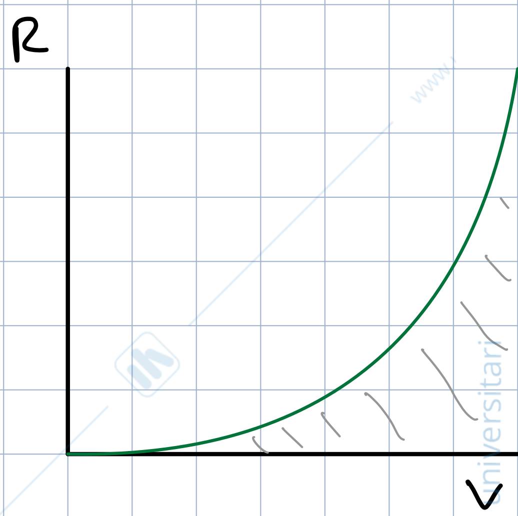
raggio minimo n_z

$$R = \frac{V^2}{g\tan\mu} = \frac{V^2}{g}\frac{\cos\mu}{\sin\mu} = \frac{V^2}{g n_z}\frac{1}{\sin\mu} \qquad \sin\mu = \sqrt{1-\cos^2\mu}$$

$$= \frac{V^2}{\ }\ \frac{1}{\ } = \frac{V^2}{\ }\ \frac{1}{\ }$$

$$g u_z \sqrt{1-\cos^2\mu} \qquad g u_z \sqrt{1-(1/u_z)^2}$$

$$= \frac{v^2}{\sqrt{g^2 u_z^2 \left(1-\frac{1}{u^2}\right)}} = \frac{v^2}{g\sqrt{u_z^2-1}}$$



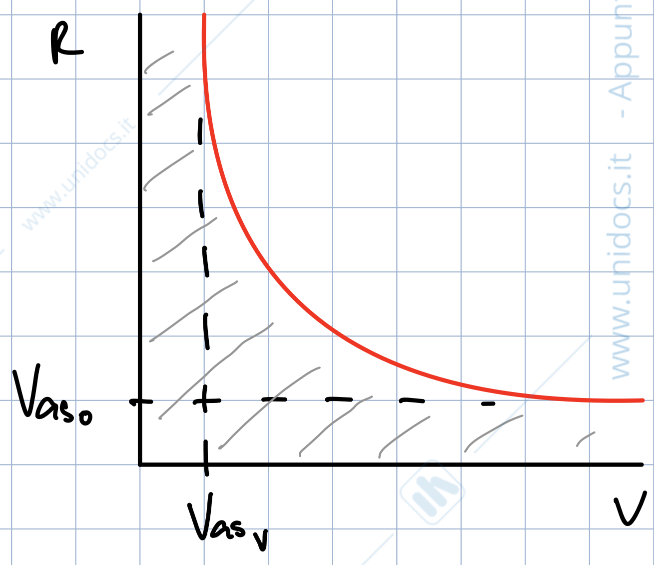
$$R^{min}\Big|_{u_z} = \frac{v^2}{g\sqrt{u_z^{max\,2}-1}}$$

raggio minimo C_L

$$R = \frac{v^2}{g\sqrt{u_z^2-1}} : \left\{ u_z = \frac{L}{W} \right\} : \frac{v^2}{g\sqrt{\frac{1}{w^2}L^2-1}} =$$

$$= \frac{v^2}{g\sqrt{\frac{1}{w^2}\left(\frac{1}{2}\rho S v^2 C_L\right)^2-1}} = \frac{v^2}{g\sqrt{\left(\frac{\rho S v^2}{2w}\right)^2 C_L^2-1}}$$

$$R^{min}\Big|_{C_L} = \frac{v^2}{g\sqrt{\left(\frac{\rho S v^2}{2w}\right)^2 C_L^{max\,2}-1}}$$



AS VERT) $\left(\frac{\rho S v^2}{2w}\right)^2 C_L^{max\,2}-1=0$

$$\frac{\rho^2 S^2 v^4}{4w^2} C_L^{max\,2} = 1$$

$$V = \sqrt[4]{\frac{4w^2}{\rho^2 S^2 C_L^{m\,2}}} = \sqrt{\frac{2w}{\rho S C_L^{m\,2}}}$$

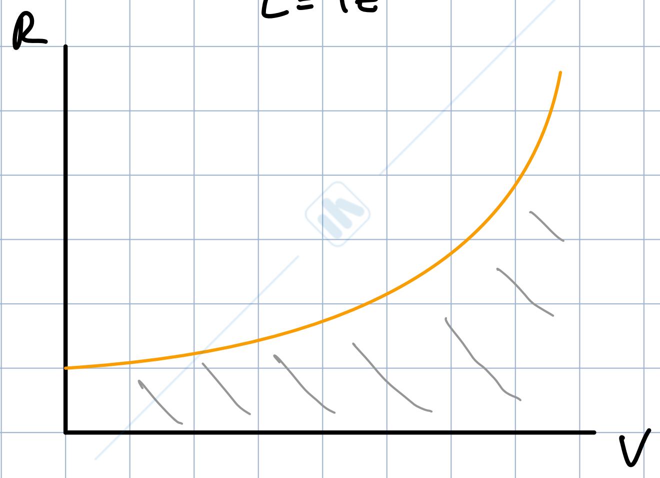
AS OR) $\lim_{V\to 0} R = \frac{1}{g\sqrt{\rho^2 S^2 C_L^{m\,2}}} = \frac{1}{g\,\rho S C_L^{m}} = \frac{V_{as_V}^2}{g}$

Raggio minimo T

$$R = \frac{v^2}{g\sqrt{n_z^2-1}} = \frac{v^2}{g\sqrt{\frac{1}{W^2}T^2E^2-1}}$$

$n = \frac{L}{W}$ $\quad E = \frac{L}{D}$ $\quad D = T$ $\quad T = \frac{L}{E}$

$L = TE$



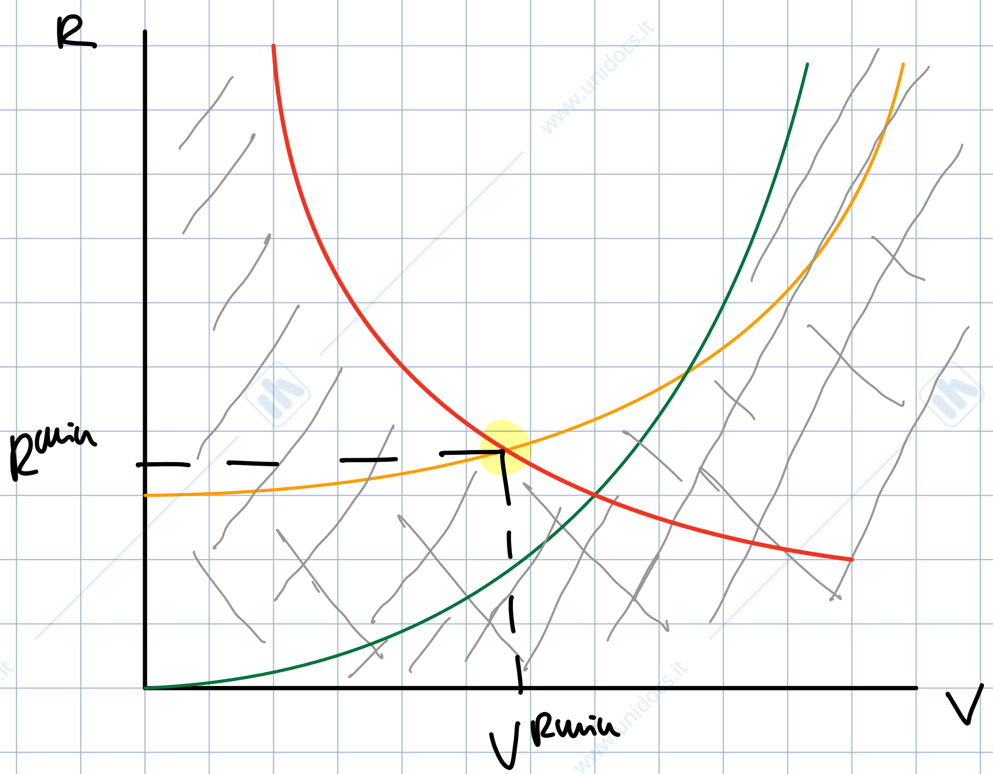
$$R^{min}\Big|_T = \frac{v^2}{g\sqrt{\frac{1}{W^2}E^2T_{max}^2-1}}$$

E) $T = D = \frac{1}{2}\rho S v^2 (C_{D0} + kC_L^2)$

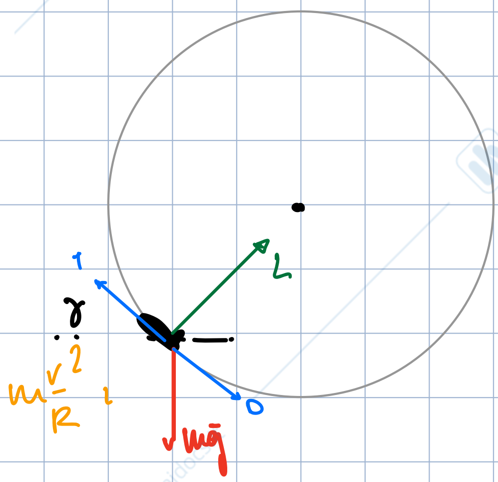
$$C_{D0} + kC_L^2 = \frac{2T}{\rho S v^2} \qquad C_L = \sqrt{\frac{2T}{\rho S v^2 k} - \frac{C_{D0}}{k}}$$

$$E = \frac{C_L}{C_D} = \frac{\sqrt{\frac{2T}{\rho S v^2 k} - \frac{C_{D0}}{k}}}{\cancel{C_{D0}} + \cancel{k}\left(\frac{2T}{\rho S v^2 \cancel{k}} - \frac{\cancel{C_{D0}}}{\cancel{k}}\right)} = \frac{\sqrt{\frac{2T}{\rho S v^2 k} - \frac{C_{D0}}{k}}}{\frac{2T}{\rho S v^2}} = (T, V)$$

R minimo



LOOP PERFETTO



$T = D$

$L = m\frac{v^2}{R} + mg\cos\gamma$

$\stackrel{!}{=} mg\left(\frac{v^2}{gR} + \cos\gamma\right)$

$n_z = \left(\frac{v^2}{gR} + \cos\gamma\right)$ $\quad n_z^{max} = \left(\frac{v^2}{gR} + 1\right)$

DECOLLO