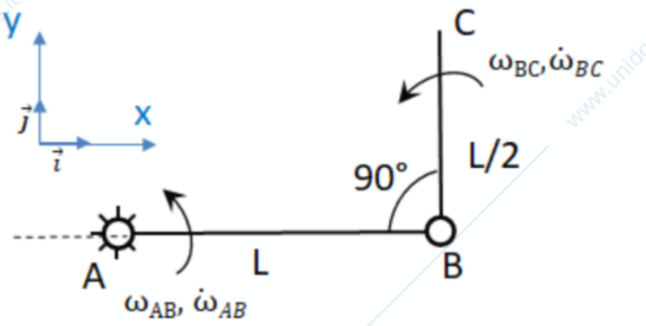
Mechanical Design - Exam simulation

10-06-2020

1

The system in the figure is made by two bars; the first one is hinged to the ground in A and hinged to the second bar in B.
Given the length of the two bars as in the picture, and knowing their angular velocities, at the time instant shown in the figure:

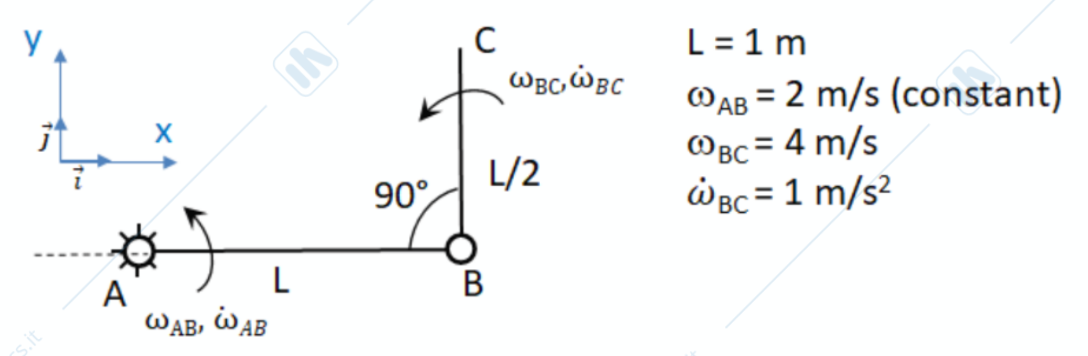


$L = 1$ m
$\omega_{AB} = 2$ m/s (constant)
$\omega_{BC} = 4$ m/s
$\dot{\omega}_{BC} = 1$ m/s²

- [ ] 1) The module of the velocity of point C is equal to 4 m/s
- [ ] 2) The module of the velocity of point B is equal to 2 m/s
- [ ] 3) The velocity of point B is purely vertical
- [ ] 4) The velocity of point C is purely horizontal
- [ ] 5) The system has one degree of freedom

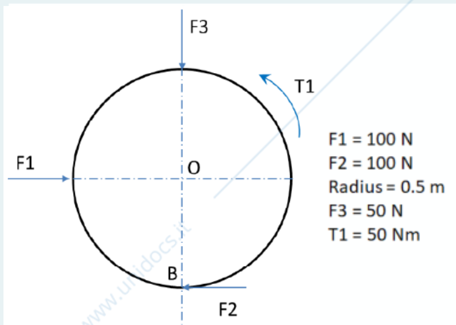
2

Considering the same system as the previous exercise, and knowing now also the angular acceleration of the two bars, as represented in the picture, at the time instant shown in the figure:



- [ ] 1) The acceleration of point C is equal to -4.5i-8j
- [ ] 2) The acceleration of point B is purely vertical
- [ ] 3) The absolute value of the centripetal component of the acceleration of point B is equal to 4 m/s2
- [ ] 4) The acceleration of point C is purely horizontal
- [ ] 5) The direction of the vector corresponding to the angular acceleration is orthogonal to the plane

3

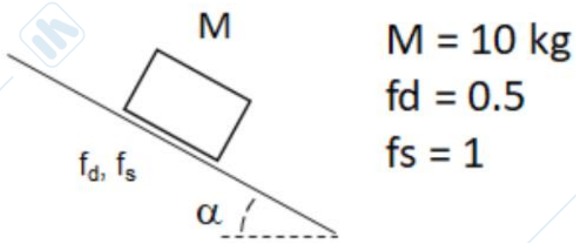


Considering the following body, subjected to the forces and torques indicated in the figure (assume that the weight of the body is negligible):

- [ ] 1. The body is in static equilibrium condition
- [ ] 2. The body would be in static equilibrium condition if F3=0
- [ ] 3. The equilibrium of the moments with respect to the centre of the disc is verified
- [ ] 4. The moment generated by the force F3 with respect to the pole B is equal to 50 Nm
- [ ] 5. The moment generated by the force F1 around the pole O is zero

4

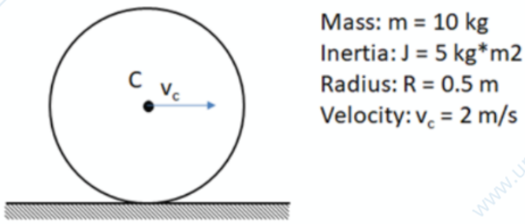
A mass is placed on a slope as represented in the picture, the static and dynamic friction coefficient at the contacting surface are fs and fd, respectively. Considering to be in the vertical plane (gravity to be considered):



- [ ] 1) The maximum slope to have adherence is alfa = 45°
- [ ] 2) The minimum slope to have adherence is alfa = 45°
- [ ] 3) If alfa = 0 (horizontal plane) the tangent reaction force is equal to 98.1 N
- [ ] 4) If alfa = 0 the normal reaction force is equal to 98.1 N
- [ ] 5) If alfa = 30° the body accelerates downside

5

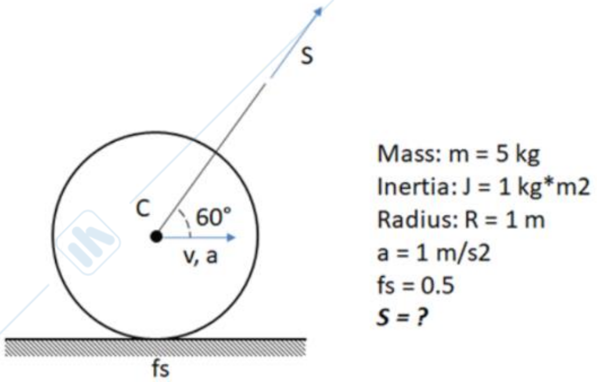
The disc in the picture is rolling without sliding on a horizontal plane. Knowing the velocity of the centre of the disc, its kinetic energy is equal to



- [ ] 5 J
- [ ] 10 J
- [ ] 20 J
- [ ] 40 J
- [ ] 60 J

6

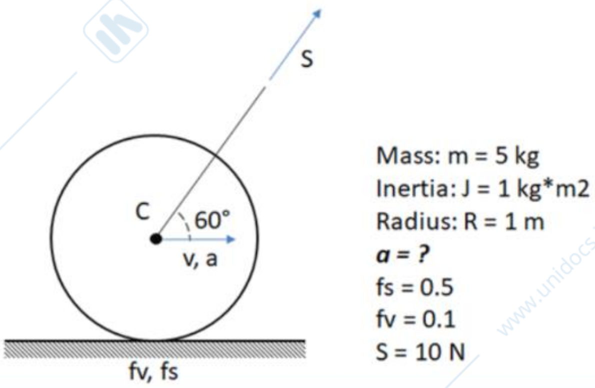
Consider the system in the figure, where a disc rolling without sliding on a horizontal plane is pulled through a rope connected in its centre as represented in the picture.
Knowing the acceleration of the centre of the disk:



- [ ] 1) The rope tension S to maintain this motion condition is equal to 20 N
- [ ] 2) The absolute value of the normal reaction force at the ground is equal to 43 N
- [ ] 3) The absolute value of the tangent reaction force at the ground is equal to 1 N
- [ ] 4) The rolling without sliding assumption is verified
- [ ] 5) The system has one degree of freedom

7

Consider the same example as before, but now including the presence of rolling friction, with rolling friction coefficient fv. Considering to impose a rope tension as indicated in the figure:



- [ ] 1) The acceleration of the centre of the disc is equal to 0.16
- [ ] 2) The effect of the rolling friction is a variation of the tangent ground reaction force
- [ ] 3) The effect of the rolling friction is a variation of the point of application of the normal ground reaction force, resulting in a torque opposing to the motion
- [ ] 4) The normal ground reaction force is equal to 45 N
- [ ] 5) The rolling without sliding assumption is verified

- [ ] Send me an email receipt of my responses

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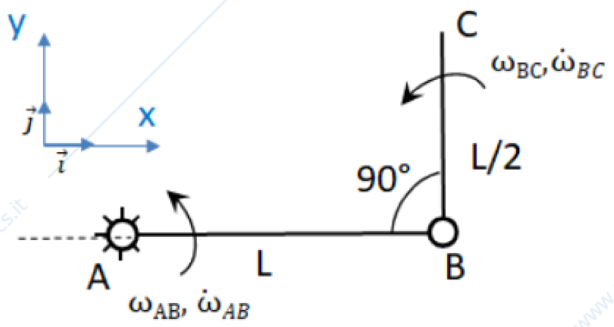
Solutions with explanations

1

The system in the figure is made by two bars; the first one is hinged to the ground in A and hinged to the second bar in B.
Given the length of the two bars as in the picture, and knowing their angular velocities, at the time instant shown in the figure:



$L = 1$ m
$\omega_{AB} = 2$ m/s (constant)
$\omega_{BC} = 4$ m/s
$\dot\omega_{BC} = 1$ m/s^2

- [ ] 1) The module of the velocity of point C is equal to 4 m/s
- [ ] 2) The module of the velocity of point B is equal to 2 m/s ✓
- [ ] 3) The velocity of point B is purely vertical ✓
- [ ] 4) The velocity of point C is purely horizontal
- [ ] 5) The system has one degree of freedom

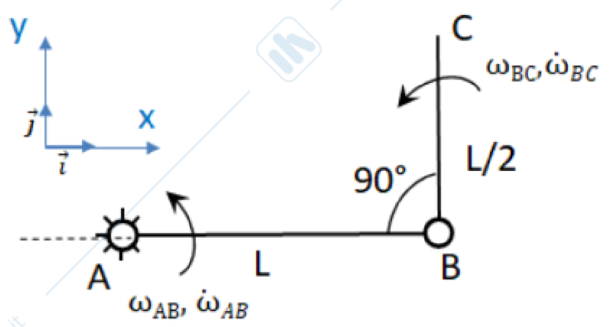
1.
$$\vec v_C = \vec v_B + \vec\omega_{BC} \wedge \frac{L}{2}\vec j$$
$$= \vec\omega_{AB} \wedge L\vec i + \omega_{BC}\frac{L}{2}\vec k \wedge \vec j$$
$$= \omega_{AB} L\, \vec k \wedge \vec i + \omega_{BC}\frac{L}{2}\,\vec k \wedge \vec j$$
$$= \omega_{AB} L\, \vec j - \omega_{BC}\frac{L}{2}\,\vec i$$
$$= 2\vec j - 2\vec i$$
$$|\vec v_C| = \sqrt{2^2 + 2^2} = 2\sqrt{2}$$

$$\text{DoF} = \underbrace{2}_{\text{Body number}} \times 3 - \overbrace{2 - 2}^{\text{Pin}} = 2$$

$$\vec v_B = \vec v_A + \vec\omega_{AB} \wedge L\vec i$$
$$= 0 + \omega_{AB} L\, \vec k \wedge \vec i$$
$$= 2\vec j \Rightarrow \text{vertical upwards}$$
$$|\vec v_B| = 2$$

2

Considering the same system as the previous exercise, and knowing now also the angular acceleration of the two bars, as represented in the picture, at the time instant shown in the figure:



L = 1 m
ω_{AB} = 2 m/s (constant)
ω_{BC} = 4 m/s
$\dot{\omega}_{BC}$ = 1 m/s²

- 1) The acceleration of point C is equal to -4.5i-8j ✓
- 2) The acceleration of point B is purely vertical
- 3) The absolute value of the centripetal component of the acceleration of point B is equal to 4 m/s2 ✓
- 4) The acceleration of point C is purely horizontal
- 5) The direction of the vector corresponding to the angular acceleration is orthogonal to the plane ✓

2. $\vec{a}_B = \vec{a}_A + \dot{\vec{\omega}}_{AB} \wedge L\vec{i} - \omega_{AB}^2 L\vec{i}$

$= 0 + 0 - 4\vec{i}$

$= -4\vec{i}$ ← Horizontal to the left

centripetal/normal component

$\omega_{AB} = 2$ m/s (constant)

$\Downarrow$

$\dot{\omega}_{AB} = 0$

$\vec{a}_C = \vec{a}_B + \dot{\vec{\omega}}_{BC} \wedge \frac{L}{2}\vec{j} - \omega_{BC}^2 \frac{L}{2}\vec{j}$

$= -4\vec{i} + \dot{\omega}_{BC}\frac{L}{2}\vec{k}\wedge\vec{j} - \omega_{BC}^2\frac{L}{2}\vec{j}$

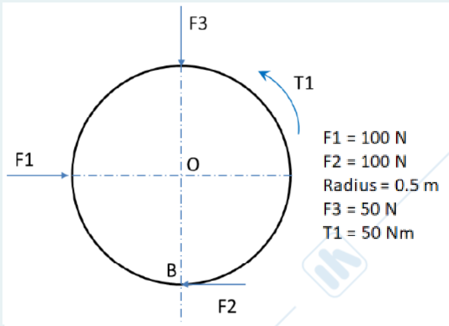
$= -4\vec{i} - \dot{\omega}_{BC}\frac{L}{2}\vec{i} - 8\vec{j}$

$= -4\vec{i} - 0.5\vec{i} - 8\vec{j}$

$= -4.5\vec{i} - 8\vec{j}$

The direction of the angular acceleration shown in the figure is $\vec{k}$ which is orthogonal to the plane

3



Considering the following body, subjected to the forces and torques indicated in the figure (assume that the weight of the body is negligible):

- 1. The body is in static equilibrium condition
- 2. The body would be in static equilibrium condition if F3=0 ✓
- 3. The equilibrium of the moments with respect to the centre of the disc is verified ✓
- 4. The moment generated by the force F3 with respect to the pole B is equal to 50 Nm
- 5. The moment generated by the force F1 around the pole O is zero ✓

3. The equations of static equilibrium of the system.

$\Sigma F_x : F_1 - F_2 = 0$ $\quad F_1 = F_2 = 100\ N$ → This condition is satisfied ✓

$\Sigma F_y : -F_3 = 0$ $\quad F_3 = 50\ N$ → not satisfied ✗ Satisfied Only If $F_3 = 0$

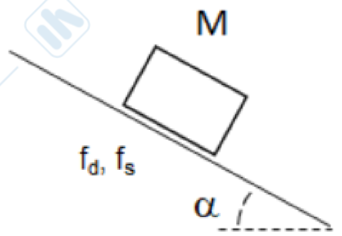
$\Sigma M_o : T_1 - F_2 R = 0$ $\quad F_2 R = 100 \times 0.5 = 50\ Nm = T_1$ Satisfied. ✓

Moment generated by F_3 wrt B is $F_3 \cdot 0 = 0$ (F_3 passes through pole B, no moment)

Moment generated by F_1 wrt O is $F_1 \cdot 0 = 0$

4

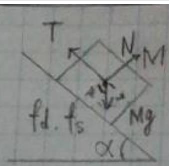
A mass is placed on a slope as represented in the picture, the static and dynamic friction coefficient at the contacting surface are fs and fd, respectively. Considering to be in the vertical plane (gravity to be considered):



M = 10 kg
fd = 0.5
fs = 1

- 1) The maximum slope to have adherence is alfa = 45° ✓
- 2) The minimum slope to have adherence is alfa = 45°
- 3) If alfa = 0 (horizontal plane) the tangent reaction force is equal to 98.1 N
- 4) If alfa = 0 the normal reaction force is equal to 98.1 N ✓
- 5) If alfa = 30° the body accelerates downside

4.



Assume the angle is α_m when the friction reaches its maximum T_m

$$\begin{cases} T_m - Mg\sin\alpha_m = 0 & \Rightarrow T_m = f_s N = Mg\sin\alpha_m \quad ① \\ N - Mg\cos\alpha_m = 0 & \Rightarrow N = Mg\cos\alpha_m \quad ② \end{cases}$$

$$\frac{①}{②} \Rightarrow f_s = \tan\alpha_m = 1 \Rightarrow \alpha_m = 45^\circ$$

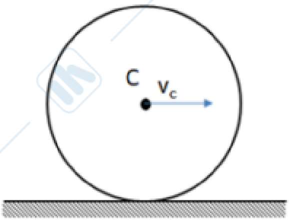
If $\alpha = 0$ Tangent reaction force $T = Mg\sin 0 = 0$

Normal reaction force $N = Mg\cos 0 = Mg = 98.1\ N$

If $\alpha = 30^\circ < \alpha_m = 45^\circ \Rightarrow$ the body remains static.

5

The disc in the picture is rolling without sliding on a horizontal plane. Knowing the velocity of the centre of the disc, its kinetic energy is equal to



Mass: m = 10 kg
Inertia: J = 5 kg*m2
Radius: R = 0.5 m
Velocity: v_c = 2 m/s

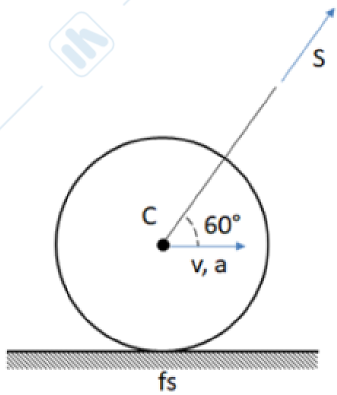
- [ ] 5 J
- [ ] 10 J
- [ ] 20 J
- [ ] 40 J
- [ ] 60 J ✓

5. kinetic energy

$$\left.\begin{aligned} E &= \tfrac{1}{2} m v_c^2 + \tfrac{1}{2} J \omega_c^2 \\ \omega &= v_c/R = 2/0.5 = 4 \end{aligned}\right\} \Rightarrow E = \tfrac{1}{2} \times 10 \times 2^2 + \tfrac{1}{2} \times 5 \times 4^2 = 60\ J$$

6

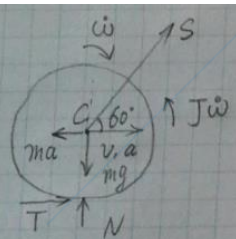
Consider the system in the figure, where a disc rolling without sliding on a horizontal plane is pulled through a rope connected in its centre as represented in the picture.
Knowing the acceleration of the centre of the disk:



Mass: m = 5 kg
Inertia: J = 1 kg*m2
Radius: R = 1 m
a = 1 m/s2
fs = 0.5
S = ?

- 1) The rope tension S to maintain this motion condition is equal to 20 N
- 2) The absolute value of the normal reaction force at the ground is equal to 43 N
- 3) The absolute value of the tangent reaction force at the ground is equal to 1 N ✓
- 4) The rolling without sliding assumption is verified ✓
- 5) The system has one degree of freedom ✓

5.



The direction α is known, the direction of $\dot\omega$ can be determined by right hand rule as shown in the FBD.

↑+↱

$$-ma + T + S\cos 60° = 0 \quad ①$$

$$N - mg + S\sin 60° = 0 \quad ②$$

$$J\dot\omega + TR = 0 \quad ③$$

Rolling without sliding ⇒ $a = \dot\omega R \Rightarrow \dot\omega = a/R = 1/1 = 1$ rad/s²

eq③ ⇒ $T = -J\dot\omega/R = -1\times 1/1 = -1$ N $\quad |T| = 1$ N

eq① ⇒ $S = (ma - T)\frac{1}{\cos 60°} = (5\times 1 - (-1))\times\frac{1}{\cos 60°} = 12$ N

eq② ⇒ $N = mg - S\sin 60° = 5\times 9.81 - 12\sin 60° = 38.66$ N

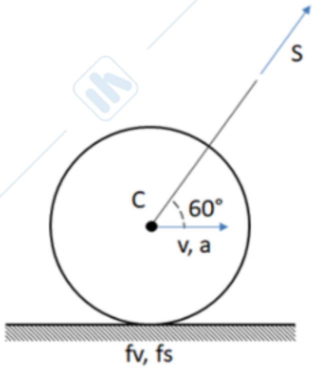
$T_{lim} = f_s|N| = 0.5\times 38.66 = 19.33$ N $> |T| = 1$ N Rolling w/o sliding OK

$DOF = 3 - \underset{\uparrow}{2} = 1$

contact (Rolling w/o sliding)

7

Consider the same example as before, but now including the presence of rolling friction, with rolling friction coefficient fv. Considering to impose a rope tension as indicated in the figure:



Mass: m = 5 kg
Inertia: J = 1 kg*m2
Radius: R = 1 m
***a* = ?**
fs = 0.5
fv = 0.1
S = 10 N

- 1) The acceleration of the centre of the disc is equal to 0.16 ✓
- 2) The effect of the rolling friction is a variation of the tangent ground reaction force
- 3) The effect of the rolling friction is a variation of the point of application of the normal ground reaction force, resulting in a torque opposing to the motion ✓
- 4) The normal ground reaction force is equal to 45 N
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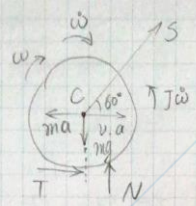

From the given directions of a & v, the directions of ω & $\dot{\omega}$ can be determined by right hand rule as shown in the diagram

$$-ma + T + S\cos 60° = 0 \quad (1)$$

$$N - mg + S\sin 60° = 0 \quad (2)$$

$$J\dot{\omega} + TR + Nf_vR = 0 \quad (3)$$

Rolling w/o sliding $\Rightarrow a = \dot{\omega}R$

eq. (2) $\Rightarrow N = mg - S\sin 60° = 5 \times 9.81 - 10 \times \sin 60° = 40.39\ N$

$$\left.\begin{aligned} &\text{eq. (3)} \Rightarrow J\frac{a}{R} + TR + Nf_vR = 0 \\ &\text{eq. (1)} \Rightarrow T = ma - S\cos 60° \end{aligned}\right\} \Rightarrow J\frac{a}{R} + (ma - S\cos 60°)R + Nf_vR = 0$$

$$J\frac{a}{R} + maR - S\cos 60° R + Nf_vR = 0$$

$$a\left(\frac{J}{R} + mR\right) = S\cos 60° R - Nf_vR$$

$$a = \frac{S\cos 60° R - Nf_vR}{J/R + mR} = \frac{10 \times \cos 60° \times 1 - 40.39 \times 0.1 \times 1}{1/1 + 5 \times 1} = 0.1602\ m/s^2 \approx 0.16$$

$T_{lim} = f_s|N| = 0.5 \times 40.39 = 20.1950\ N > |T| = 4.2\ N \Rightarrow$ Rolling w/o sliding OK

$T = ma - S\cos 60° = 5 \times 0.16 - 10 \times \cos 60° = -4.2\ N \Rightarrow |T| = 4.2\ N$