

## **String**

Transversal vibration of a tensioned string. Comparison with the beam.

## **Beam**

Elastic potential energy for an axial loaded beam.

Axial vibration of a beam (where is the maximum force, stationary solution)

Bending vibration of a beam

Moving loads

## **Modal Superimposition**

When we use the modal approach and what is the main advantage:

The main advantage of the modal approach is its ability to simplify the analysis of complex dynamic systems. By focusing on a reduced set of modes, engineers can gain a deeper understanding of the system's behavior, identify critical aspects, and efficiently evaluate the dynamic response. This approach saves computational resources and aids in making informed decisions during design, analysis, and optimization processes.

Modal superposition approach, stiffness matrix for a pinned-pinned beam

Damping matrix- how to deal with concentrated dampers. How to estimate  $\alpha$  and  $\beta$  in damping.

## **Modal parameter Identification**

Methods for the evaluation of the damping ratio and assumptions required to do these calculations

- Good spacing between resonance peaks
- Low damping

why we impact in different points with the hammer

## **FEM**

Shape function in FE

If we are considering the three internal actions in a node of the structure with the FEM, are they the same if we calculate them using the left element or right?

- Yes, there is continuity

If and why the N varies along the length of the finite element?

- In the context of finite element analysis, the axial force within an element can vary along its length depending on the loading and boundary conditions. The axial force refers to the internal force or stress acting parallel to the longitudinal axis of the element.
- Several factors can cause the axial force to vary:
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  - 1. Applied Loads: If there are non-uniformly distributed axial loads or external forces applied to the structure, the resulting axial force within the element will vary along its length. For example, if a beam is subjected to a concentrated load at a specific point, the axial force will be highest near that point and decrease as you move away from it.
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  - 2. Geometric Variations: Changes in the geometry or cross-section of the element can lead to variations in the axial force. For instance, if an element has a varying cross-section along its length (such as a tapered beam or a

stepped bar), the axial force will change accordingly.

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- 3. Boundary Conditions: The boundary conditions at the ends of the element can affect the distribution of axial forces. For example, if an element is fixed at one end and subjected to a load at the other, the axial force will be different at each end and vary gradually in between.
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- 4. Material Properties: Heterogeneous or anisotropic materials can result in non-uniform distributions of axial forces. Different material properties, such as modulus of elasticity or Poisson's ratio, can influence how the axial force is transmitted within the element.
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- It's important to consider these factors when analyzing structures using finite element methods. The varying axial forces along the length of an element can have implications for design, stress analysis, and determining the overall structural behavior. Proper consideration of these variations is essential for accurate modeling and simulation of the system.

Are the damping ratios the same if computed using different FRFs (different points of applying the force and measuring)

- Yes, the calculated damping values can vary when using different FRFs obtained from different excitations or measurements at different points. The reasons for these variations include:
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  - 1. Excitation Differences: The excitation signal used to measure the FRF can affect the resulting response. Different excitations may lead to variations in the amplitudes and phases of the FRF, which can impact the estimation of damping.

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- 2. Measurement Locations: The location at which the FRF is measured can also influence the calculated damping. Damping is influenced by the dynamic characteristics of the specific point being measured. Different points may have varying levels of damping due to local effects, coupling with other components, or resonances specific to that location.
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- 3. System Nonlinearity: Nonlinear behavior in the system can lead to variations in the FRF and affect the damping estimation. Different excitations or measurement points may exhibit different nonlinear responses, resulting in variations in the calculated damping values.
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- 4. Measurement Noise: Measurement noise and uncertainties can impact the accuracy of the FRF and, consequently, the calculated damping values. Variations in the quality and noise levels of the measurements can introduce discrepancies in the damping estimation.
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- Given these factors, it's important to carefully consider the consistency and reliability of the FRF data when calculating damping. If possible, it is recommended to conduct measurements under similar conditions, using consistent excitation signals and measurement locations, to reduce variations and ensure more reliable comparisons of damping values. Additionally, it can be beneficial to assess the repeatability and validity of the measurements through multiple trials or averaging techniques to mitigate the impact of noise and measurement uncertainties.
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Are there any differences between using half power points or the

slope method?

When calculating damping from the frequency response function (FRF) using different methods, such as the half-power points method or the slope method, it is expected that you may not obtain the exact same result. Here's why:

1. Assumptions: The different methods make different assumptions about the shape of the FRF or the behavior of the system. For example, the half-power points method assumes a symmetric response around the resonance peak and uses the -3dB points to estimate the bandwidth. On the other hand, the slope method assumes a linear slope on either side of the peak and calculates the damping based on that slope. These assumptions can introduce slight variations in the calculated damping values.

2. Sensitivity to Noise: The methods may have different sensitivities to noise or measurement uncertainties. The accuracy of the measured FRF can be influenced by various factors, such as noise, measurement errors, or signal processing techniques. Different methods may handle these factors differently, leading to variations in the calculated damping values.

3. Data Processing Techniques: The methods may employ different data processing techniques. For example, the half-power points method involves interpolating the FRF data to find the -3dB points, while the slope method may involve fitting a straight line to the linear region of the response. The choice of interpolation or fitting techniques can

introduce differences in the calculated damping values.

Why finite elements have an  $L_{max}$ ?

- We want to be in the quasi static region- below the first resonance frequency i.e mode 1  $\omega$  for pin-pinned beam. SO we don't have to worry about resonance effects.

### **Rigid Bodies**

Kinetic energy of a rigid body derivation (Rival's theorem) -> integral over vol -> cancel middle terms -> eqn with  $m$  and  $J_g$  matrices)

Rigid body in the space, when the mass matrix is completely diagonal

- always?

when matrix  $JG$  is diagonal?

- When  $x_l, y_l, z_l$  are aligned with principle axis of inertia

Components of inertia tensor and their meaning-

- Products and moments of inertia.
- POI - Moment of inertia (MOI) describes the amount of force, or torque, required to change the rate of rotation of an object. Product of inertia (POI) reveals how an object might be imbalanced - integral of  $xy$  over  $dA$ .

Cardano's angles.

- are a set of three angles that describe the orientation of a rigid body in three-dimensional space
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Define the angular speed as function of cardan angles

$$-\omega_L = [A_L] \dot{\theta}$$

why is convenient to pass into local coordinates instead of global and how to do that

Local to global - use rotation matrix

if the formula works for both local and global and what we choose

- Better to choose the local so that J does not change with time.

### **Rotors**

Balancing technique for a rigid rotor. : Jeffcott's model, how to balance a rotor (both equilibrium and influence coefficients methods).

(how we get the equation with  $\Omega/\omega_0$ )

When measure the force instead of the vibration for the balancing

- Depends on the "softness" of the bearings- if they are soft then better to use the vibrations. If they are stiff then better to use the force (too much noise in vibration measurements)

How many balancing planes are necessary for balancing a rigid rotor

- At least 2. 4 equations to be satisfied (no eccentricity- x,y and no Moments of inertia wrt to axis x,y)

balancing rigid and flexible rotors, difference, f

- Rigid rotors are easier to balance. Only 2 planes needed. For flexible rotors the number of planes depends on the steady state regime speed of the rotor (number of measuring sections depends = all critical speeds below, plus one above, plus the regime speed.)
- (so number of balancing planes = ns?)

Coherence meaning

### **Assignment questions**

What is the first small peak in the FRF of DH1 (only log-log scale) (1 assignment)

Methods for estimating the damping ratio (Half power points/Slope of the phase diagram) (1 assignment)

Computation of the mode shape (1 assignment)

Hypothesis of the single-mode identification technique (1 assignment)

Differences between modal and FEM FRFs (2 assignment)

Alternatives for point 5 (2 assignment)

Effect of the concentrated mass on the damping (2 assignment)

why for some of the chosen points there where less peaks than modes, internal action in FE, discussion on galloping modelling

Will there be difference (Assignment 2-3c) if I choose different elements in the calculation of bending moment, shear force in section G?

How did you calculate h with half power peaks on Matlab? Is there a better way of doing it? (es. interpolating (?)). Number of lobes of wheel mode shapes: why do you expect that?



Transversal vibration refers to the motion of a string or a beam when it is displaced perpendicular to its longitudinal axis. Both tensioned strings and beams exhibit transverse vibration behavior, but there are some key differences in their characteristics and mathematical descriptions.

### 1. **Tensioned String:**

A tensioned string is a one-dimensional element that is typically long and slender. When it is plucked, struck, or otherwise excited, it vibrates in a transverse manner, forming standing waves. The motion of the string is governed by the wave equation, and its vibration modes are quantized and harmonic. The fundamental frequency (first mode) is the lowest possible frequency of vibration, and higher modes are integer multiples of the fundamental frequency.

Key points about the transversal vibration of a tensioned string:

- The equation governing the transverse vibration of a tensioned string is a partial differential wave equation known as the wave equation.
- The vibration modes of a tensioned string are harmonic and discrete.
- The fundamental frequency of vibration depends on the tension in the string, its length, and its linear mass density (mass per unit length).
- The frequencies of the higher modes are integer multiples of the fundamental frequency.
- The motion of a tensioned string is one-dimensional (transverse to the length of the string).

### 2. **Beam:**

A beam, on the other hand, is a three-dimensional element that is typically shorter and thicker compared to a string. Beams can be of various shapes (e.g., rectangular, circular, I-shaped) and are often used to support loads. When subjected to transverse vibrations, beams exhibit more complex behavior than strings due to their bending and torsional stiffness.

Key points about the transversal vibration of a beam:

- The equation governing the transverse vibration of a beam is a partial differential equation known as the Euler-Bernoulli beam equation or the Timoshenko beam equation (which considers shear effects as well).
- The vibration modes of a beam are continuous and can be approximated as a continuous spectrum of frequencies.
- Beams have multiple vibration modes, including bending, torsional, and coupled modes.
- The fundamental frequency and higher frequencies of a beam depend on its material properties, shape, dimensions, and boundary conditions.
- The motion of a beam is two-dimensional, involving both bending and transverse displacement.

In summary, both tensioned strings and beams can exhibit transverse vibration behavior, but the mathematical models governing their vibrations are different. Strings have discrete and harmonic vibration modes, while beams have continuous and more complex vibration modes due to their bending and torsional characteristics. Understanding these differences is crucial when analyzing and designing systems involving tensioned strings or beams.