ESERCIZI SETTIMANA 12:

3) $T = \frac{3}{12}$

$S_0 = 52$

$K = 50$

$r = 0,12$

$\sigma = 0,30$

$c = S_0 \cdot N(d_1) - K e^{-rT} \cdot N(d_2)$

$d_1 = \frac{\ln\left(\frac{52}{50}\right) + \left(0,12 + \frac{0,30^2}{2}\right) \cdot \frac{3}{12}}{0,30\sqrt{\frac{3}{12}}} = 0,5365$

$d_2 = 0,5365 - 0,30\sqrt{\frac{3}{12}} = 0,3865$

$c = 52 \cdot N(0,5365) - 50e^{-0,12 \cdot \frac{3}{12}} \cdot N(0,3865) = 5,06$

4) $T = \frac{6}{12}$

$S_0 = 69$

$K = 70$

$r = 0,05$

$\sigma = 0,35$

$d_1 = \frac{\ln\left(\frac{69}{70}\right) + \left(0,05 + \frac{0,35^2}{2}\right) \cdot \frac{6}{12}}{0,35\sqrt{\frac{6}{12}}} = 0,1666$

$d_2 = 0,1666 - 0,35\sqrt{\frac{6}{12}} = -0,0809$

$p = 70e^{-0,05 \cdot \frac{6}{12}} \cdot N(0,0809) - 69 \cdot N(-0,1666) = 6,40$

5) $c = S_0 \cdot N(d_1) - K e^{-rT} \cdot N(d_2)$

$c + K e^{-rT} = S_0(N d_1) + K e^{-rT} \cdot N(-d_2)$

$p = K e^{-rT} \cdot N(-d_2) - S_0 \cdot N(-d_1)$

$p + S_0 = K e^{-rT} \cdot N(-d_2) + S_0 \cdot N(+d_1)$

PUT-CALL PARITY $\rightarrow$ $c + K e^{-rT} = p + S_0$

8) 2) $E[\ln S_T] = \ln S + \left(\mu - \frac{\sigma^2}{2}\right)(T-t)$

derivate $\rightarrow$ $f = e^{-r(T-t)}\left[\ln S + \left(r - \frac{\sigma^2}{2}\right)(T-t)\right]$

$\frac{\partial f}{\partial t} = e^{-r(T-t)}\left(r - \frac{\sigma^2}{2}\right) + r e^{-r(T-t)}\left[\ln S + \left(r - \frac{\sigma^2}{2}\right)(T-t)\right]$

$\frac{\partial f}{\partial S} = e^{-r(T-t)} \cdot \frac{1}{S}$

$\frac{\partial^2 f}{\partial S^2} = -e^{-r(T-t)} \cdot \frac{1}{S^2}$

a) $\frac{\partial f}{\partial t} + rS\frac{\partial f}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 f}{\partial S^2} =$

$= -e^{-r(T-t)}\left(r - \frac{\sigma^2}{2}\right) + re^{-r(T-t)}\left[\ln S + \left(r - \frac{\sigma^2}{2}\right)(T-t)\right] + rS \cdot e^{-r(T-t)} \cdot \frac{1}{S} + \frac{1}{2}\sigma^2 S^2 \cdot$

$\left(+e^{-r(T-t)} \cdot \frac{1}{S^2}\right) = e^{-r(T-t)}\left\{-r + \frac{\sigma^2}{2} + r\left[\ln S + \left(r + \frac{\sigma^2}{2}\right)(T-t)\right] + r - \frac{\sigma^2}{2}\right] =$

$= re^{-r(T-t)}\left[\ln S + \left(r - \frac{\sigma^2}{2}\right)(T-t)\right] = rf$

b) a) $T = \frac{1}{12}$

$S_0 = 30$

$K = 29$

$r = 0{,}05$

$\sigma = 0{,}25$

$$d_1 = \frac{\ln\left(\frac{30}{29}\right) + \left(0{,}05 + \frac{0{,}25^2}{2}\right)\cdot\frac{1}{12}}{0{,}25\sqrt{\frac{1}{12}}} = 0{,}4225$$

$d_2 = 0{,}4225 - 0{,}25\sqrt{\frac{1}{12}} = 0{,}2782$

$c = 30N(0{,}4228) - 29e^{-0{,}05\cdot\frac{1}{12}} \cdot N(0{,}2782) = 2{,}52.$

b) $p = 29e^{-0{,}05\cdot\frac{1}{12}} \cdot N(-0{,}2782) - 30 \cdot N(-0{,}4225) = 1{,}05$

c) $c + Ke^{-rt} = p + S_0$

$2{,}52 + 29 \cdot e^{-0{,}05\cdot\frac{1}{12}} = 1{,}05 + 30 \Rightarrow 31{,}05$