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Microeconomics, Winter 2018
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Midterm Exam, February 7, 2018: SOLUTION

You have 60 minutes for this exam.
Each answer must contain a rigorous proof of all non trivial statements.

This is a closed book exam: no notes in any format are permitted. No electronic devices are allowed (cell phones must be turned off).

Problem 1 (50 %)

(I) (**a**) Describe the difference between as well as the format of the profit maximization problems for (i) a price taking firm in a competitive market, (ii) a monopoly and (iii) a price discriminating monopoly. (**b**) What is the formula for demand price elasticity? What is the distinction between inelastic demand, elastic demand and unit elastic demand? Illustrate for the first two.

Solution (5 minutes): See Lecture Notes.

(II) Suppose the utility function of a consumer with income M is $u(c_1, c_2) = c_1 + \frac{1}{e}c_2^e$ with $1 > e > 0$. (**a**) Assuming that c_1 can take any value in $\Re$ find, for $(p_1, p_2) \in \Re^2_{++}$, the solution to the utility maximization problem. Illustrate using indifference curves and budget constraint.
Solution (5 minutes): The budget constraint $p_1c_1 + p_2c_2 = M$ can be rearranged as $c_1 = \frac{M}{p_1} - \frac{p_2}{p_1}c_2$ which we insert in the maximization problem to get

$$\max_{c_2} \frac{M}{p_1} - \frac{p_2}{p_1}c_2 + \frac{1}{e}c_2^e \text{ with } FOC - \frac{p_2}{p_1} + c_2^{e-1} = 0 \Leftrightarrow c_2 = \left[\frac{p_1}{p_2}\right]^{\frac{1}{1-e}}$$

This then means that

$$D(p_1, p_2, M) = \left(\frac{M}{p_1} - \left[\frac{p_1}{p_2}\right]^{\frac{e}{1-e}}, \left[\frac{p_1}{p_2}\right]^{\frac{1}{1-e}}\right)$$

(**b**) In country A, which has N_A inhabitants, each inhabitant has the utility function $u_A(c_1, c_2) = c_1 + 2\sqrt{c_2}$ and income M_A. What is then the aggregate demand for commodity 2 in country A? What is the price elasticity of demand (for commodity 2) in this country? (**c**) Country B consumers have the utility function $u_B(c_1, c_2) = c_1 + \frac{3}{2}c_2^{2/3}$ and income M_B. With N_B inhabitants, what is the aggregate demand in country B? What is the price elasticity of demand (for commodity 2) in this country?

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Solution (max 5 minutes): $D_{A2}(p_1, p_2) = N_A \left[\frac{p_1}{p_2}\right]^2$. Then demand price elasticity in A is

$$\eta_{DA} = \frac{dD_{A2}}{dp_2} \frac{p_2}{D_{A2}(p_1, p_2)} = -2N_A p_1^2 p_2^{-3} \frac{p_2}{N_A p_1^2 p_2^{-2}} = -2$$

Similarly, $D_{B2}(p_1, p_2) = N_B \left[\frac{p_1}{p_2}\right]^3$ and demand price elasticity in B is

$$\eta_{DB} = \frac{dD_{B2}}{dp_2} \frac{p_2}{D_{B2}(p_1, p_2)} = -3N_B p_1^3 p_2^{-4} \frac{p_2}{N_B p_1^3 p_2^{-3}} = -3$$

(III) A monopoly produces commodity 2 with the production function $\phi : \Re_+^2 \to \Re$ where, for $z_1 \geq 0$

$$\phi(z_1, z_2) = az_1 \text{ for } z_2 \geq \overline{z}_2, \phi(z_1, z_2) = 0 \text{ for } z_2 < \overline{z}_2$$

and where $\overline{z}_2 > 0$. (**a**) Interpret this production function. With input prices $(w_1, w_2) \in \Re_{++}^2$, show that the monopoly's cost function has the format $C(q) = F + cq$ for $q > 0, C(0) = 0$. What are the expressions for F and for c in this equation? (NOTE: If you cannot answer this question proceed to the next question assuming $C(q) = F + cq$ with $c > 0$.)
Solution (3 minutes): For $q = 0$ the cost minimizing choice of inputs is $(z_1, z_2) = (0, 0)$. For $q > 0$ the cost minimizing choice of inputs is $(z_1, z_2) = (q/a, \overline{z}_2)$. Thus the cost function is

$$C(q) = 0 \text{ for } q = 0, C(q) = w_2\overline{z}_2 + \frac{w_1}{a}q \text{ for } q > 0$$

i.e. $F = w_2\overline{z}_2$ and $c = w_1/a$.

(**b**) The monopoly sells to the two countries, A and B described in (II), at different prices. Write the format of the monopoly's profit maximization problem and characterize the solution to it (assuming there is an interior solution), expressing the prices in terms of the marginal cost c. In which country is the price highest? Explain why.

Solution (max 5 minutes): With $P_A(q_A)$ and $P_B(q_B)$ being the inverse demand functions in country A and country B respectively, the general format is:

$$\max_{q_A \geq 0, q_B \geq 0} P_A(q_A)q_A + P_B(q_B)q_B - [F + c(q_A + q_B)]$$

and if the solution is interior we have

$$p_{2A}^* \left[1 + \frac{1}{\eta_{DA}}\right] = c = p_{2B}^* \left[1 + \frac{1}{\eta_{DB}}\right] \text{ that is } p_{2A}^* = 2c, p_{2B}^* = \frac{3}{2}c.$$

Thus since country A demand is less elastic than country B demand, the monopoly sets a higher pricer in the former country.
(**c**) Show that when either N_A or N_B (or both) is large, the monopoly will choose a positive output.

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Solution (max 5 minutes): Profit is

$$N_A\left[\frac{p_1}{2c}\right]^2 2c+N_B\left[\frac{p_1}{3c/2}\right]^3\frac{3c}{2}-F-[N_A\left[\frac{p_1}{2c}\right]^2+N_B\left[\frac{p_1}{3c/2}\right]^3]c=N_A\left[\frac{p_1}{2c}\right]^2 c+N_B\left[\frac{p_1}{3c/2}\right]^3\frac{c}{2}-F$$

which is >0 for sufficiently high N_A and/or N_B.

Problem 2 (50 %)

(I) A firm has the production function $\phi(z_1,z_2)=z_1+2A\sqrt{z_2}$. (**a**) Sketch some isoquants for this function and find the MRTS.

Solution (3 minutes):

$$\text{MRTS}_{12}(z_1,z_2)=\frac{\frac{\partial\phi(z_1,z_2)}{\partial z_1}}{\frac{\partial\phi(z_1,z_2)}{\partial z_2}}=\frac{1}{A/\sqrt{z_2}}=\frac{\sqrt{z_2}}{A}$$

For the isoquants, see the figure file.

(**b**) For this production function, find the conditional demand, the cost function (illustrate) and marginal costs (illustrate) as functions of the input price vector $(w_1,w_2)\in\Re^2_{++}$ and the output q.(NOTE: Corner solutions play an important role here).

Solution (10 minutes): For an interior solution we have

$$MRTS(z_1,z_2)=\frac{\sqrt{z_2}}{A}=\frac{w_1}{w_2}\Leftrightarrow z_2=\left(\frac{Aw_1}{w_2}\right)^2 \text{ and } z_1=q-2A\frac{Aw_1}{w_2}=q-2\frac{A^2w_1}{w_2}$$

For a corner solution we have $z_2=(\frac{q}{2A})^2$. With $\hat{q}=2A^2\frac{w_1}{w_2}$, this gives us the conditional demand

$$Z(w,q)=\left(0,\left(\frac{q}{2A}\right)^2\right)\text{ for } q<\hat{q},\ Z(w,q)=\left(q-2A^2\frac{w_1}{w_2},\left(\frac{Aw_1}{w_2}\right)^2\right)\text{ for } q\geq\hat{q}.$$

The cost function then is

$$C(w,q)=w_2\left(\frac{q}{2A}\right)^2\text{ for } q<\hat{q},\ C(w,q)=w_1\left(q-2A^2\frac{w_1}{w_2}\right)+w_2\left(\frac{Aw_1}{w_2}\right)^2\text{ for } q\geq\hat{q}.$$

See Figure file for an illustration. Marginal costs are

$$MC(w,q)=\frac{w_2q}{2A^2}\text{ for } q<\hat{q},\ MC(w,q)=w_1\text{ for } q\geq\hat{q}.$$

See the figure file for an illustration.

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(II) Suppose now input 1 is fixed at $z_1 = \overline{z}_1 > 0$ in the short run. (**a**) Sketch this situation in a figure with isoquants.

Solution (2 minutes): See Figure file.

(**b**) Find the short run conditional demand and cost function. Illustrate the short run cost function in the same figure where you have shown the long run cost function (from (I)) and comment.
Solution (3 minutes): Just using the fixed input $z_1 = \overline{z}_1$ a quantity of output $\overline{q} = \overline{z}_1$ can be produced, so for $q \leq \overline{q}$, $\tilde{Z}(q, w) = (\overline{z}_1, 0)$. For $q > \overline{q}$, solely input 2 is used for additional production, hence

$$\tilde{Z}(q, w; \overline{z}_1) = \left(\overline{z}_1, \left(\frac{\max\{0, q - \overline{q}\}}{2A}\right)^2\right)$$

Hence short run costs are

$$\tilde{C}(w, q; \overline{z}_1) = w_1\overline{z}_1 + w_2\left(\frac{\max\{0, q - \overline{q}\}}{2A}\right)^2$$

See Figure file for an illustration.

(**c**) Find for given (w_1, w_2) and $\overline{z}_1$ the value q^* s.t. $C(w, q^*) = \tilde{C}(w, q^*; \overline{z}_1)$. (NOTE: You might find this last question difficult).

Solution (3 minutes): If in the long run $z_1 > 0$, also $z_2 > 0$. To use the same quantity of input 2 in the short and long run means:

$$\tilde{Z}_2(q^*, w; , \overline{z}_1) = Z_2(q^*, w) \Leftrightarrow \left(\frac{q^* - \overline{z}_1}{2A}\right)^2 = \left(\frac{Aw_1}{w_2}\right)^2 \Leftrightarrow q^* = \frac{2A^2w_1}{w_2} + \overline{z}_1$$

Note that then $Z_1(w, q^*) = q^* - \frac{2A^2w_1}{w_2} = \overline{z}_1$, as should be the case.

(III) (**a**) Returning to the long run considered in (I) and assuming price taking behavior, set up the profit maximization problem and find long run supply as a function of the input price vector $w = (w_1, w_2)$ and output price p. Illustrate.

Solution (max 5 minutes): First note that for $p > w_1$ there is no solution to the profit maximization problem, since the firm can obtain unbounded profits by using input 1 to produce. Next note, that as q tends to 0 so does $MC(w, q)$. If a solution exists, the profit maximization problem, $\max_q p - C(w, q)$ then has the FOC $p = MC$. For $p < w_1$ the solution is found from $p = \frac{w_2 q}{2A^2} \Leftrightarrow q = \frac{2A^2 p}{w_2}$. For $p = w_1$ any $q \geq \hat{q}$ is a solution. We conclude:

$$S(p, (w_1, w_2)) = \frac{2A^2p}{w_2} \text{ for } p < w_1, S(p, (w_1, w_2)) = [A^2\frac{w_1}{w_2}, \infty) \text{ for } p = w_1$$

See figure file for an illustration.

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(b) Suppose the demand function is $D(p) = B - Cp$ Find for given (w_1, w_2) conditions on A, B and C s.t. the equilibrium price is w_1.

Solution (max 3 minutes): The requirement is that $D(w_1) > \hat{q}$, that is $B - Cw_1 \geq 2A^2 \frac{w_1}{w_2}$ which may be written in many formats, f.i. $B > \left(\frac{2A^2}{w_2} - C\right) w_1$.

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