

# MIDA 1

## Prediction

### 1. Introduction

- Prediction problem, prediction error
- Transfer function, zeros and poles
- Random Variable  $v$ :  $\mathbb{E}[v]$ ,  $Var(v)$
- Random Vector  $\underline{v}$ :  $\mathbb{E}[\underline{v}]$ ,  $Var(\underline{v})$ ,  $\rho$
- Random/Stochastic Process  $v(t)$ :  $\mathbb{E}[v(t, s)]$ ,  $Var(v(t))$ ,  $\gamma(t_1, t_2)$
- Stationary Process
- White Noise (i.e. Unpredictable Signal)

### 2. MA, AR and ARMA Processes

- MA(n): Moving Average Process
  - $\mathbb{E}[v(t)]$ ,  $Var(v(t))$ ,  $\gamma(\tau)$
  - Dynamic representation
  - MA(n) vs. White Noise
  - MA( $\infty$ )
- AR(n): Autoregressive Process
  - AR(1) as MA( $\infty$ )
  - AR(n) vs. MA(n)
  - $\gamma(\tau)$ : Yule-Walker equations
- ARMA( $n_a, n_c$ )

### 3. Frequency Domain

- Spectral representation: real spectrum  $\Gamma(\omega)$ , complex spectrum  $\phi(z)$
- Anti-transform formula
- Fundamental theorem of spectral analysis (*magic formula in real and complex form*)
- ARMA process: dynamical representation (long division and interpretation of coefficients  $w_i$ )
- ARMA process: multiplicity of representations
- Canonical representation

### 4. Prediction

- Fake problem (knowing the past of  $\eta$ )
- Real problem (knowing the past of  $v$ )
- Prediction for MA(1), AR(1), ARMA( $n_a, n_c$ )
- Exogenous variables: ARX, ARMAX

## Identification

### 5. PEM (Prediction Error Minimization) methods

- LS (Least Squares) identification method
- Identifiability: invertibility of  $S(N)$  and  $R(N)$
- Persistent excitation
- Uniqueness of estimation
- ML (Maximum Likelihood) method
- Asymptotic analysis of PEM methods: performance of identification models
- Asymptotic behaviour of PEM
- LS estimate procedure

### 6. Model Complexity Selection

- Naive approach
- FPE (Final Prediction Error)
- AIC (Akaike Information Criteria)
- AIC vs. FPE
- MDL (Minimum Description Length)

### 7. Durbin Levinson Algorithm

- Recursion from AR(n) to AR(n+1)

### 8. Time Series Analysis

### 9. Recursive identification methods

# MIDA 2

## Chapter 1

*Non-parametric (direct/constructive) black box identification of I/O systems using state-space models.*

- Representations:
  1. State space
  2. Transfer function (I/O)
  3. Convolution of input and input response (IR)
- Change of representation (\*)
- Observability, controllability of a dynamical system
- (\*) 4SID method (without/with noisy meas. of IR)

## Chapter 2

*Parametric identification of black box I/O systems with a frequency domain approach.*

- Parametric system identification:
  - Step 1: Frequency domain dataset
  - Step 2: Model class selection  $M(\theta)$
  - Step 3: Performance index  $J(\theta)$
  - Step 4: Optimization  $\hat{\theta}$

## Chapter 3

*Kalman filter for software sensing using feedback on white box models.*

- Motivations and goals of Kalman filter theory
- Software sensing
- Basic system
- KF for the basic solution of the basic system
- Block scheme representation of KF
- Extensions of basic problem of basic systems:
  1. Exogenous input
  2. Multi-step prediction
  3. Filter 11
  4. Time-varying system
  5. Non-linear systems (\*)
- Asymptotic solution of KF:
  - 1<sup>st</sup> and 2<sup>nd</sup> asymptotic theorems
- (\*) Extension of KF to non-linear systems

## Chapter 4

*Black box methods for softw. sensing without feedback.*

- Black box estimate of  $\hat{x}(t)$  from  $y(t)$ ,  $u(t)$
- Black box software sensing vs. KF software sensing
- Architectures: 4 types of

## Chapter 5

*Gray-box system identification <sup>(1)</sup> using Kalman filter, <sup>(2)</sup> using simulation error methods (SEM).*

- Gray-box system identification using KF
- Parametric system identification based on SEM
- SEM in black box methods:
  - PEM (prediction error methods) vs. SEM

## Chapter 6

*Minimum variance control (MVC). Design of optimal feedback controllers using the theoretical background of the MIDA 1 course.*

- Setup of the problem
- Simplified problem 1: noise free
- Simplified problem 2: with noise
- General solution
- Stability and performance analysis

## Appendix

- Discretization of an analog system