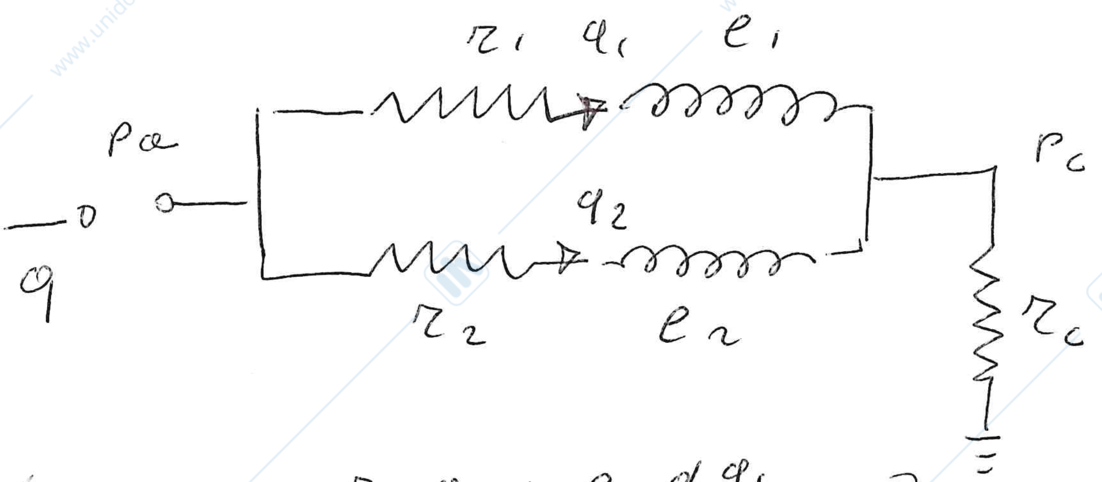
Esercizio



$$\begin{cases} p_a - p_c = r_1 q_1 + e_1 \dfrac{dq_1}{dt} \\ p_a - p_c = r_2 q_2 + e_2 \dfrac{dq_2}{dt} \end{cases} \quad \text{eq. di stato}$$

$$p_c = r_c (q_1 + q_2) \quad \text{eq. ausiliaria}$$

$$\begin{cases} e_1 \dfrac{dq_1}{dt} = -r_1 q_1 - r_c (q_1 + q_2) + p_a \\ e_2 \dfrac{dq_2}{dt} = -r_c (q_1 + q_2) - r_2 q_2 + p_a \end{cases}$$

$$\begin{cases} \dfrac{dq_1}{dt} = -\dfrac{(r_1 + r_c)}{e_1} q_1 - \dfrac{r_c}{e_1} q_2 + \dfrac{p_a}{e_1} \\ \dfrac{dq_2}{dt} = -\dfrac{r_c}{e_2} q_1 - \dfrac{(r_2 + r_c)}{e_2} q_2 + \dfrac{p_a}{e_2} \end{cases}$$

$$A = \begin{bmatrix} -\dfrac{(r_1 + r_c)}{e_1} & -\dfrac{r_c}{e_1} \\ -\dfrac{r_c}{e_2} & -\dfrac{(r_2 + r_c)}{e_2} \end{bmatrix} \qquad B = \begin{bmatrix} \dfrac{1}{e_1} \\ \dfrac{1}{e_2} \end{bmatrix}$$

Eq. uscita: $\quad q = q_1 + q_2$

$$c = [\,1 \quad 1\,] \qquad d = [\,\phi \quad \phi\,]$$

Funzione di trasferimento:

$$h(s) = C(sI-A)^{-1}B =$$

$$= \begin{bmatrix} 1 & 1 \end{bmatrix} \frac{\mathrm{adj}(sI-A)}{(s-\lambda_1)(s-\lambda_2)} \begin{bmatrix} \frac{1}{e_1} \\ \frac{1}{e_2} \end{bmatrix} =$$

$$= \begin{bmatrix} 1 & 1 \end{bmatrix} \frac{\begin{bmatrix} s + \frac{r_2+r_c}{e_2} & -\frac{r_c}{e_1} \\ -\frac{r_c}{e_2} & s + \frac{r_1+r_c}{e_1} \end{bmatrix}}{(s-\lambda_1)(s-\lambda_2)} \begin{bmatrix} \frac{1}{e_1} \\ \frac{1}{e_2} \end{bmatrix}$$

$$= \frac{\begin{bmatrix} s + \frac{r_2+r_c}{e_2} - \frac{r_c}{e_2} & s + \frac{r_1+r_c}{e_1} - \frac{r_c}{e_1} \end{bmatrix}}{(s-\lambda_1)(s-\lambda_2)} \begin{bmatrix} \frac{1}{e_1} \\ \frac{1}{e_2} \end{bmatrix}$$

$$= \frac{e_2 s + r_2 + e_1 s + r_1}{e_1 e_2 (s-\lambda_1)(s-\lambda_2)} =$$

$$= \frac{(e_1+e_2)s + r_1 + r_2}{e_1 e_2 (s-\lambda_1)(s-\lambda_2)}$$

funzione di trasferimento del circuito:

$$\frac{p_a(s)}{q(s)} = r_c + \frac{(r_1 + l_1 s)(r_2 + l_2 s)}{r_1 + r_2 + (l_1 + l_2)s} =$$

$$= \frac{r_c[r_1 + r_2 + (l_1 + l_2)s] + r_1 r_2 + r_1 l_2 s + r_2 l_1 s + l_1 l_2 s^2}{(l_1 + l_2)s + r_1 + r_2}$$

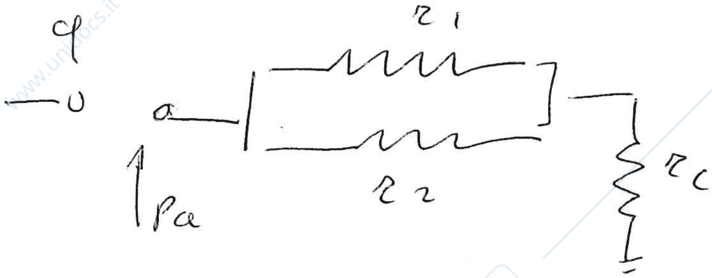
$$h(s) = \frac{q(s)}{p_a(s)} = \frac{(l_1 + l_2)s + r_1 + r_2}{l_1 l_2 s^2 + [r_1 l_2 + r_2 l_1 + r_c(l_1 + l_2)]s + r_1 r_2 + r_c(r_1 + r_2)}$$

facciamo una verifica per $s = \phi$

$$h(\phi) = \frac{r_1 + r_2}{r_1 r_2 + r_c(r_1 + r_2)} = \frac{1}{\frac{r_1 r_2}{r_1 + r_2} + r_c}$$

che è esattamente (l'impedenza)$^{-1}$ del circuito:

ammettenza



Calcolo degli autovalori:

$$p(s) = s^2 - Ts + \Delta =$$

$$= s^2 + \left[\frac{r_1 + r_c}{e_1} + \frac{r_2 + r_c}{e_2}\right] s +$$

$$+ \frac{r_1 + r_c}{e_1} \cdot \frac{r_2 + r_c}{e_2} - \frac{r_c^2}{e_1 e_2}$$

$$T = -\frac{r_1 + r_c}{e_1} - \frac{r_2 + r_c}{e_2} < \phi$$

$$\Delta = \frac{r_1 r_2 + r_1 r_c + r_c r_2}{e_1 e_2} > \phi$$

$$\lambda_{1,2} = \frac{T \pm \sqrt{T^2 - 4\Delta}}{2}$$

Si nota che $T^2 - 4\Delta =$

$$= \frac{r_1 + r_c}{e_1^2} + \frac{r_2 + r_c}{e_2^2} + 2\frac{(r_1 + r_c)(r_2 + r_c)}{e_1 e_2} -$$

$$- 4\left(\frac{r_1 + r_c}{e_1}\right)\left(\frac{r_2 + r_c}{e_2}\right) + \frac{r_c^2}{e_1 e_2}$$

$$= \left[\frac{r_1 + r_c}{e_1} - \frac{r_2 + r_c}{e_2}\right]^2 + \frac{r_c^2}{e_1 e_2} > \phi$$

quindi λ_1 e λ_2 sono reali

essendo $T < 0$
$\Delta > 0$

risultano due autovalori reali negativi

$\Downarrow$

stabilità asintotica