## MODELLING AND SIMULATION OF MECHATRONIC SYSTEMS

$\angle(\mathbf{v},\mathbf{w}) = \arccos\frac{\mathbf{v}\cdot\mathbf{w}}{\|\mathbf{v}\|\,\|\mathbf{w}\|} \in [0,\pi]$

$\mathbf{R}(x,\theta)=\mathbf{R}(i,\theta)=\begin{bmatrix}1&0&0\\0&\cos\theta&-\sin\theta\\0&\sin\theta&\cos\theta\end{bmatrix}$ $\mathbf{R}(y,\theta)=\mathbf{R}(j,\theta)=\begin{bmatrix}\cos\theta&0&\sin\theta\\0&1&0\\-\sin\theta&0&\cos\theta\end{bmatrix}$ $\mathbf{R}(z,\theta)=\mathbf{R}(k,\theta)=\begin{bmatrix}\cos\theta&-\sin\theta&0\\\sin\theta&\cos\theta&0\\0&0&1\end{bmatrix}$

$(\mathbf{T}^a_b)^{-1}=\mathbf{T}^b_a=\begin{bmatrix}\mathbf{R}^b_a & \bar{t}^b_{ba}\\ \mathbf{0} & 1\end{bmatrix}, \quad \bar{t}^b_{ba}=-\mathbf{R}^b_a\bar{t}^a_{ab}.$

$\mathbf{i}_b=\alpha^1_1\mathbf{i}_a+\alpha^1_2\mathbf{j}_a+\alpha^1_3\mathbf{k}_a,$
$\mathbf{j}_b=\alpha^2_1\mathbf{i}_a+\alpha^2_2\mathbf{j}_a+\alpha^2_3\mathbf{k}_a,$
$\mathbf{k}_b=\alpha^3_1\mathbf{i}_a+\alpha^3_2\mathbf{j}_a+\alpha^3_3\mathbf{k}_a$

$\mathbf{R}^a_b=\begin{bmatrix}\alpha^1_1&\alpha^2_1&\alpha^3_1\\\alpha^1_2&\alpha^2_2&\alpha^3_2\\\alpha^1_3&\alpha^2_3&\alpha^3_3\end{bmatrix}.$

$\sin(\alpha+\beta)=\sin(\alpha)\cos(\beta)+\cos(\alpha)\sin(\beta)$

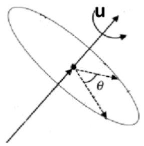
$\sin(\alpha-\beta)=\sin(\alpha)\cos(\beta)-\cos(\alpha)\sin(\beta)$

$\cos(\alpha+\beta)=\cos(\alpha)\cos(\beta)-\sin(\alpha)\sin(\beta)$

$\cos(\alpha-\beta)=\cos(\alpha)\cos(\beta)+\sin(\alpha)\sin(\beta)$

(Also rows and columns of R(ϑ) have norm 1)

**Angle-axis representation:** (And det(R)=1)




$$\mathbf{R}(\bar{u},\theta)=\begin{bmatrix}u_1^2(1-c_\theta)+c_\theta & u_1u_2(1-c_\theta)-u_3s_\theta & u_1u_3(1-c_\theta)+u_2s_\theta\\ u_1u_2(1-c_\theta)+u_3s_\theta & u_2^2(1-c_\theta)+c_\theta & u_2u_3(1-c_\theta)-u_1s_\theta\\ u_1u_3(1-c_\theta)-u_2s_\theta & u_2u_3(1-c_\theta)+u_1s_\theta & u_3^2(1-c_\theta)+c_\theta\end{bmatrix}$$

$c_\theta \equiv \cos\theta \quad s_\theta\equiv\sin\theta$

**Euler parameters:**

$\mathbf{R}=\mathbf{R}(\bar{u},\theta)\rightsquigarrow(v_1,v_2,v_3,v_4)$ $\quad v_1=u_1\sin\frac{\theta}{2},\quad v_2=u_2\sin\frac{\theta}{2},\quad v_3=u_3\sin\frac{\theta}{2},\quad v_4=\cos\frac{\theta}{2}$

$u_1^2+u_2^2+u_3^2=1\implies v_1^2+v_2^2+v_3^2+v_4^2=1$

$\mathbf{R}(\mathbf{v})=\begin{bmatrix}v_1^2-v_2^2-v_3^2+v_4^2 & 2(v_1v_2-v_3v_4) & 2(v_1v_3+v_2v_4)\\ 2(v_1v_2+v_3v_4) & -v_1^2+v_2^2-v_3^2+v_4^2 & 2(v_2v_3-v_1v_4)\\ 2(v_1v_3-v_2v_4) & 2(v_2v_3+v_1v_4) & -v_1^2-v_2^2+v_3^2+v_4^2\end{bmatrix}$

$v_4=\frac{1}{2}\sqrt{1+\mathrm{tr}\mathbf{R}}\quad v_1=\frac{1}{4v_4}(r_{32}-r_{23})$

$v_2=\frac{1}{4v_4}(r_{13}-r_{31})\quad v_3=\frac{1}{4v_4}(r_{21}-r_{12})\qquad \mathrm{tr}\,\mathbf{R}=r_{11}+r_{22}+r_{33}=1+2c_\theta\implies\theta=\pm\arccos\frac{\mathrm{tr}\,\mathbf{R}-1}{2}.$

**Quaternions:** (q and (-q) [UNIT NORM!!] give same R(ϑ)! ) $\quad R(\bar{\boldsymbol{u}},\theta)\sim q=\cos\left(\frac{\theta}{2}\right)+\sin\left(\frac{\theta}{2}\right)\bar{\boldsymbol{u}}$

$u_0=\pm\frac{1}{2}\sqrt{(1+r_{11}+r_{22}+r_{33})}$ $\qquad \mathbf{R}(\mathrm{u})=(u_0^2-\mathbf{u}^\top\mathbf{u})\mathbf{I}+2\mathbf{u}\mathbf{u}^\top-2u_0\mathbf{S}(\mathbf{u})=\qquad\theta\in[0,\pi]=[0,180°]$

$u_1=\frac{1}{4u_0}(r_{32}-r_{23})\quad u_1=\frac{1}{2}\mathrm{sgn}(r_{32}-r_{23})\sqrt{(1+r_{11}-r_{22}-r_{33})}$

$u_2=\frac{1}{4u_0}(r_{13}-r_{31})\quad u_2=\frac{1}{2}\mathrm{sgn}(r_{13}-r_{31})\sqrt{(1-r_{11}+r_{22}-r_{33})}$

$u_3=\frac{1}{4u_0}(r_{21}-r_{12})\quad u_3=\frac{1}{2}\mathrm{sgn}(r_{21}-r_{12})\sqrt{(1-r_{11}-r_{22}+r_{33})}$

$$\begin{bmatrix}u_0^2+u_1^2-u_2^2-u_3^2 & 2(u_1u_2-u_3u_0) & 2(u_1u_3+u_2u_0)\\ 2(u_1u_2+u_3u_0) & u_0^2-u_1^2+u_2^2-u_3^2 & 2(u_2u_3-u_1u_0)\\ 2(u_1u_3-u_2u_0) & 2(u_2u_3+u_1u_0) & u_0^2-u_1^2-u_2^2+u_3^2\end{bmatrix}$$

R(ϕ, ϑ, ψ) = R(ϕ+π, −ϑ, ψ+π) $\qquad \mathbf{R} \Leftrightarrow \mathrm{u}\qquad \mathbf{R}^\mathrm{T}\Leftrightarrow \mathrm{u}^*$

**Euler ZXZ:** (O-current-current)

$\phi=\mathrm{atan2}(r_{13},\ -r_{23})\pm2k\pi\qquad\theta=\mathrm{atan2}(s_\phi r_{13}-c_\phi r_{23},\ r_{33})\pm2k\pi$

$\psi=\mathrm{atan2}(-c_\phi r_{12}-s_\phi r_{22},\ c_\phi r_{11}+s_\phi r_{21})\pm2k\pi$

**singularity if** $\theta=0,\pi$

$\theta=0,\pi\iff r_{13}=r_{23}=0$

$\mathbf{R}_{\phi,\theta,\psi}=\mathbf{R}(\phi,\theta,\psi)=\mathbf{R}_{z,\phi}\mathbf{R}_{x,\theta}\mathbf{R}_{z,\psi}=\mathbf{R}(\mathbf{k},\phi)\mathbf{R}(\mathrm{i},\theta)\mathbf{R}(\mathbf{k},\psi)$

$\alpha=\mathrm{atan2}(y,x)=\tan^{-1}\left(\frac{y}{x}\right)=$

$$=\begin{bmatrix}c_\phi c_\psi-s_\phi c_\theta s_\psi & -c_\phi s_\psi-s_\phi c_\theta c_\psi & s_\phi s_\theta\\ s_\phi c_\psi+c_\phi c_\theta s_\psi & -s_\phi s_\psi+c_\phi c_\theta c_\psi & -c_\phi s_\theta\\ s_\theta s_\psi & s_\theta c_\psi & c_\theta\end{bmatrix}=\begin{cases}0°\le\alpha\le90° & x\ge0;\ y\ge0\\ 90°\le\alpha\le180° & x\le0;\ y\ge0\\ -180°\le\alpha\le-90° & x\le0;\ y\le0\\ -90°\le\alpha\le0° & x\ge0;\ y\le0\end{cases}$$

[ atan (-1) = -45 atan(1) = 45 atan( 1/0 ) = 90 atan( -1/0 ) = -90 atan( 0/1 ) = 0 atan( 0/-1 ) = 180 ]

**RPY (ZYX):** (O-current-current)

(XYZ O-fixed-fixed)

$\mathbf{R}=\mathbf{R}(\mathrm{k},\theta_z)\mathbf{R}(\mathrm{j},\theta_y)\mathbf{R}(\mathrm{i},\theta_x)$

$$R(\theta_x,\theta_y,\theta_z)=\begin{bmatrix}c_{\theta_y}c_{\theta_z} & s_{\theta_x}s_{\theta_y}c_{\theta_z}-c_{\theta_x}s_{\theta_z} & c_{\theta_x}s_{\theta_y}c_{\theta_z}+s_{\theta_x}s_{\theta_z}\\ c_{\theta_y}s_{\theta_z} & s_{\theta_x}s_{\theta_y}s_{\theta_z}+c_{\theta_x}c_{\theta_z} & c_{\theta_x}s_{\theta_y}s_{\theta_z}-s_{\theta_x}c_{\theta_z}\\ -s_{\theta_y} & s_{\theta_x}c_{\theta_y} & c_{\theta_x}c_{\theta_y}\end{bmatrix}$$

R(ϑx, ϑy, ϑz) = R(ϑx+π, π−ϑy, ϑz+π)

**singularity if** $\theta_y=\pm90°$

$\theta_x=\mathrm{atan2}(r_{32},\ r_{33})\pm2k\pi$

$\theta_z=\mathrm{atan2}(-c_{\theta_x}r_{12}+s_{\theta_x}r_{13},\ c_{\theta_x}r_{22}-s_{\theta_x}r_{23})\pm2k\pi$

$\theta_y=\mathrm{atan2}(-r_{31},\ s_{\theta_x}r_{32}+c_{\theta_x}r_{33})\pm2k\pi$

$$R(\theta_x,\mp90°,\theta_z)=\begin{bmatrix}0&\sin(\theta_x\pm\theta_z)&\cos(\theta_x\pm\theta_z)\\0&\cos(\theta_x\pm\theta_z)&-\sin(\theta_x\pm\theta_z)\\\pm1&0&0\end{bmatrix}.$$

**KINEMATICS:**

$\mathbf{v}(t)=\dot{\mathbf{r}}(t)=\frac{d\mathbf{r}(t)}{dt}=\lim_{\Delta t\to0}\frac{\mathbf{r}(t+\Delta t)-\mathbf{r}(t)}{\Delta t}$

$\mathbf{a}(t)=\ddot{\mathbf{r}}(t)=\frac{d\mathbf{v}(t)}{dt}=\lim_{\Delta t\to0}\frac{\mathbf{v}(t+\Delta t)-\mathbf{v}(t)}{\Delta t}.$

$\mathbf{j}(t)=\dddot{\mathbf{r}}(t)=\frac{d\mathbf{a}(t)}{dt}=\lim_{\Delta t\to0}\frac{\mathbf{a}(t+\Delta t)-\mathbf{a}(t)}{\Delta t}.$

$\mathbf{r}=r_x\mathbf{i}+r_y\mathbf{j}+r_z\mathbf{k}\implies\dot{\mathbf{r}}=\dot{r}_x\mathbf{i}+\dot{r}_y\mathbf{j}+\dot{r}_z\mathbf{k}+r_x\frac{d\mathbf{i}}{dt}+r_y\frac{d\mathbf{j}}{dt}+r_z\frac{d\mathbf{k}}{dt}$

$\mathbf{v}(t)=(\dot\theta\mathbf{k})\times\mathbf{r}(t)=(\dot\theta\mathbf{n})\times\mathbf{r}(t)\qquad\boldsymbol{\omega}=\omega_x\mathbf{i}+\omega_y\mathbf{j}+\omega_z\mathbf{k}$

$\underbrace{\frac{d\mathbf{i}}{dt}\cdot\mathbf{j}}_{=:\omega_z}+\mathbf{i}\cdot\frac{d\mathbf{j}}{dt}=0,\quad\underbrace{\frac{d\mathbf{j}}{dt}\cdot\mathbf{k}}_{=:\omega_x}+\mathbf{j}\cdot\frac{d\mathbf{k}}{dt}=0,\quad\underbrace{\frac{d\mathbf{k}}{dt}\cdot\mathbf{i}}_{=:\omega_y}+\mathbf{k}\cdot\frac{d\mathbf{i}}{dt}=0$

$\overline{O_bP}=x_P\mathbf{i}_b+y_P\mathbf{j}_b+z_P\mathbf{k}_b\qquad\mathbf{v}_P-\mathbf{v}_{O_b}=\boldsymbol{\omega}_b\times\overline{O_bP}\qquad\mathbf{v}_P(t)=\mathbf{v}_{O_b}+\boldsymbol{\omega}_b(t)\times\overrightarrow{O_b(t)P(t)}$

$\bar{a}\times\bar{b}=\begin{pmatrix}a_x\\a_y\\a_z\end{pmatrix}\times\begin{pmatrix}b_x\\b_y\\b_z\end{pmatrix}=\begin{bmatrix}0&-a_z&a_y\\a_z&0&-a_x\\-a_y&a_x&0\end{bmatrix}\begin{pmatrix}b_x\\b_y\\b_z\end{pmatrix}=\mathbf{S}(\bar{a})\bar{b}=-\mathbf{S}(\bar{b})\bar{a}=-\bar{b}\times\bar{a}.$

$\mathbf{S}(\bar\omega(t))=\dot{\mathbf{R}}(t)(\mathbf{R}(t))^\top$

$\bar{v}_P=\bar{v}_O+\begin{pmatrix}\dot{x}_P\\\dot{y}_P\\\dot{z}_P\end{pmatrix}+\mathbf{S}(\bar\omega)\begin{pmatrix}x_P\\y_P\\z_P\end{pmatrix}$

$\alpha_E(t)=\begin{bmatrix}\phi(t)\\\theta(t)\\\psi(t)\end{bmatrix},\quad\alpha_{RPY}(t)=\begin{bmatrix}\theta_x(t)\\\theta_y(t)\\\theta_z(t)\end{bmatrix}$

$\bar\omega=\dot\phi\begin{bmatrix}0\\0\\1\end{bmatrix}+\dot\theta\begin{bmatrix}\cos\phi\\\sin\phi\\0\end{bmatrix}+\dot\psi\begin{bmatrix}\sin\phi\sin\theta\\-\cos\phi\sin\theta\\\cos\theta\end{bmatrix}\neq\dot\alpha_E.$

$\bar\omega=\dot\theta_z\begin{bmatrix}0\\0\\1\end{bmatrix}+\dot\theta_y\begin{bmatrix}-\sin\theta_z\\\cos\theta_z\\0\end{bmatrix}+\dot\theta_x\begin{bmatrix}\cos\theta_z\cos\theta_y\\\sin\theta_z\cos\theta_y\\-\sin\theta_y\end{bmatrix}\neq\dot\alpha_{RPY}.$

**DVKF:**

$\mathbf{r}(q)=\overline{OP(q)}=x_P(q)\mathbf{i}+y_P(q)\mathbf{j}+z_P(q)\mathbf{k}.$

$\mathbf{v}(q,\dot q)=\frac{d}{dt}\mathbf{r}(q(t))=\frac{\partial x_P(q)}{\partial q}\dot q\,\mathbf{i}+\frac{\partial y_P(q)}{\partial q}\dot q\,\mathbf{j}+\frac{\partial z_P(q)}{\partial q}\dot q\,\mathbf{k}.$

$\delta\bar r\approx\frac{\partial\bar r}{\partial q}\delta q=\sum_{i=1}^m\frac{\partial\bar r}{\partial q_i}\delta q_i$

$\boldsymbol\omega=\boldsymbol\omega(\phi,\theta,\psi,\dot\phi,\dot\theta,\dot\psi),\quad\dot\phi=\frac{\partial\phi}{\partial q}\dot q,\ \dot\theta=\frac{\partial\theta}{\partial q}\dot q,\ \dot\psi=\frac{\partial\psi}{\partial q}\dot q.$

**DYNAMICS AND LAGRANGE:**

$\mathbf{v}_C=\frac{1}{M}\sum_{i=1}^N m_i\mathbf{v}_i,\quad\mathbf{a}_C=\frac{1}{M}\sum_{i=1}^N m_i\mathbf{a}_i.\qquad\mathbf{p}=\mathbf{h}_L=\sum_i\underbrace{m_i\mathbf{v}_i}_{=\mathbf{p}_i=\mathbf{h}_{L,i}}=M\mathbf{v}_C.$

$\mathbf{r}_C=\frac{1}{M}\sum_{i=1}^N m_i\mathbf{r}_i,\quad M=\sum_i m_i.$

$\mathbf{L}^{O_b}=\underbrace{\sum_i(m_i\mathbf{r}_i)}_{=M\mathbf{r}_C}\times\mathbf{v}_{O_b}+\underbrace{\sum_i(m_i\mathbf{r}_i\times(\boldsymbol\omega\times\mathbf{r}_i))}_{=:\hat{\boldsymbol\Gamma}^{O_b}(\boldsymbol\omega)}.$

$\mathbf{L}^O=\mathbf{h}_A^O=\sum_i\underbrace{\mathbf{r}_i\times m_i\mathbf{v}_i}_{=\mathbf{L}_i^O=\mathbf{h}_{A,i}^O}=\sum_i\mathbf{r}_i\times\mathbf{p}_i.$

$\hat{\boldsymbol\Gamma}^{O'}(\boldsymbol\omega)=\hat{\boldsymbol\Gamma}^O(\boldsymbol\omega)-M\mathbf{t}_C^O\times(\boldsymbol\omega\times\mathbf{t}_C^O)+M\mathbf{t}_C^{O'}\times(\boldsymbol\omega\times\mathbf{t}_C^{O'}).\qquad\Gamma\bar\omega=\sum_i\Gamma_i\bar\omega.$

$\Gamma_i=m_i\left(\|\bar r_i\|^2\mathbf{I}_3-\bar r_i\bar r_i^\top\right)=m_i\begin{bmatrix}y_i^2+z_i^2&-x_iy_i&-x_iz_i\\-y_ix_i&x_i^2+z_i^2&-y_iz_i\\-z_ix_i&-z_iy_i&x_i^2+y_i^2\end{bmatrix}\qquad D\,tgx=\frac{1}{cos^2x}$

**The diagonal entries are moments of inertia with respect to the axes Oi, Oj, Ok** (basic physics):

$\Gamma_{11}=\sum_i m_i(y_i^2+z_i^2)=I_\mathbf{i},\ \Gamma_{22}=\sum_i m_i(x_i^2+z_i^2)=I_\mathbf{j},\ \Gamma_{33}=\sum_i m_i(x_i^2+y_i^2)=I_\mathbf{k}.$

$\mathbf{t}_C^O\mapsto(t_x,t_y,t_z)^\top,\quad\Gamma_O=\Gamma_C+M\begin{bmatrix}t_y^2+t_z^2&-t_xt_y&-t_xt_z\\-t_xt_y&t_x^2+t_z^2&-t_yt_z\\-t_xt_z&-t_yt_z&t_x^2+t_y^2\end{bmatrix}\qquad\frac{d}{dt}\left(\frac{\partial\mathcal{K}^*}{\partial\dot q_i}\right)-\frac{\partial\mathcal{K}}{\partial q_i}+\frac{\partial\mathcal{P}}{\partial q_i}+\frac{\partial\mathcal{D}_i}{\partial\dot q_i}=\mathcal{F}_i$

$\mathcal{K}^*=\frac{1}{2}M\|\mathbf{v}_{O_b}\|^2+M\mathbf{v}_{O_b}\cdot(\boldsymbol\omega\times\mathbf{r}_C)+\frac{1}{2}\boldsymbol\omega\cdot\hat{\boldsymbol\Gamma}^{O_b}(\boldsymbol\omega)\qquad\mathcal{D}_i(\dot{\boldsymbol q})=\frac{1}{2}\beta_i\|\dot{\boldsymbol q}\|^2\qquad\mathcal{P}_e=\frac{1}{2}k_e|e|^2$

| | Angolo | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Op.** | $0$ | $\frac{\pi}{6}$ | $\frac{\pi}{4}$ | $\frac{\pi}{3}$ | $\frac{\pi}{2}$ | $\frac{2\pi}{3}$ | $\frac{3\pi}{4}$ | $\frac{5\pi}{6}$ | $\pi$ | $\frac{7\pi}{6}$ | $\frac{5\pi}{4}$ | $\frac{4\pi}{3}$ | $\frac{3\pi}{2}$ | $\frac{5\pi}{3}$ | $\frac{7\pi}{4}$ | $\frac{11\pi}{6}$ |
| sin | $0$ | $\frac{1}{2}$ | $\frac{\sqrt2}{2}$ | $\frac{\sqrt3}{2}$ | $1$ | $\frac{\sqrt3}{2}$ | $\frac{\sqrt2}{2}$ | $\frac{1}{2}$ | $0$ | $-\frac{1}{2}$ | $-\frac{\sqrt2}{2}$ | $-\frac{\sqrt3}{2}$ | $-1$ | $-\frac{\sqrt3}{2}$ | $-\frac{\sqrt2}{2}$ | $-\frac{1}{2}$ |
| cos | $1$ | $\frac{\sqrt3}{2}$ | $\frac{\sqrt2}{2}$ | $\frac{1}{2}$ | $0$ | $-\frac{1}{2}$ | $-\frac{\sqrt2}{2}$ | $-\frac{\sqrt3}{2}$ | $-1$ | $-\frac{\sqrt3}{2}$ | $-\frac{\sqrt2}{2}$ | $-\frac{1}{2}$ | $0$ | $\frac{1}{2}$ | $\frac{\sqrt2}{2}$ | $\frac{\sqrt3}{2}$ |
| tan | $0$ | $\frac{\sqrt3}{3}$ | $1$ | $\sqrt3$ | $\nexists$ | $-\sqrt3$ | $-1$ | $-\frac{\sqrt3}{3}$ | $0$ | $\frac{\sqrt3}{3}$ | $1$ | $\sqrt3$ | $\nexists$ | $-\sqrt3$ | $-1$ | $-\frac{\sqrt3}{3}$ |

$dW_N=\mathbf{N}\cdot\omega\,dt$

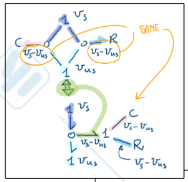
$\mathcal{P}_g=-m\mathbf{g}\cdot\mathbf{x}_C^O$

## BOND GRAPHS

| Dominio | P | e | f |
|---|---|---|---|
| Elettrico | $e\cdot i$ | $e = d\lambda/dt$ | $i = dQ/dt$ |
| Meccanico (Rotazione) | $T\cdot\omega$ | $T = dp_\theta/dt$ | $\omega = d\theta/dt$ |
| Meccanico (Traslazione) | $F\cdot v$ | $F = dp/dt$ | $v = dq/dt$ |
| Idraulico | $p\cdot Q$ | $p = dp_Q/dt$ | $Q = dV/dt$ |
| Generalizzazione | $e\cdot f$ | $e = dp/dt$ | $f = dq/dt$ |

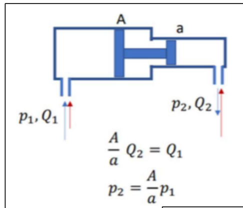
$\rightarrow p_Q = \frac{\rho\ell}{A}\cdot Q$

| Dominio | Relazione Lineare | Relazione Lineare | Relazione Lineare |
|---|---|---|---|
| Variabili generalizzate | $e = R\cdot f$, $f = Ge = e/R$ | $p = I\cdot f$, $f = p/I$ | $q = C\cdot e$, $e = q/C$ |
| Traslazione meccanica | $F = \beta\cdot v$ | $p = m\cdot v$, $v = p/m$ | $x = C\cdot f$, $f = k\cdot x$ |
| Rotazione meccanica | $\tau = c\omega$ | $p_\tau = J\omega$, $\omega = p_\tau/J$ | $\theta = C\tau$, $\tau = k\theta$ |
| Sistemi idraulici | $P = RQ$ | $p_p = IQ$, $Q = p_p/I$ | $V = CP$, $P = V/C$ |
| Sistemi elettrici | $e = R\cdot i$, $i = Ge = e/R$ | $\lambda = L\cdot i$, $i = \lambda/L$ | $q = C\cdot e$, $e = q/C$ |



$\overset{e_1}{\underset{f_1}{\rightharpoonup}}$ TF $\overset{e_2}{\underset{f_2}{\rightharpoonup}}$ $\qquad f_2 = \tau f_1$, $\quad e_1 = \tau e_2$

$\overset{e_1}{\underset{f_1}{\rightharpoonup}}$ TF $\overset{e_2}{\underset{f_2}{\rightharpoonup}}$ $\qquad f_1 = \frac{1}{\tau} f_2$, $\quad e_2 = \frac{1}{\tau} e_1$

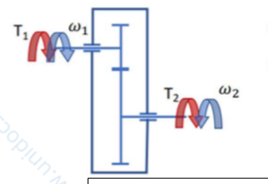


$\frac{A}{a} Q_2 = Q_1$

$p_2 = \frac{A}{a} p_1$

$Q_1 = \frac{A_1}{A_2} Q_2$

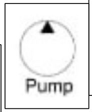
$P_2 = \frac{A_1}{A_2} P_1$



$T_2\tau = T_1$

$\omega_2 = \tau\,\omega_1$

$\omega_1 > \omega_2$

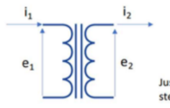


$Q = D\cdot\omega$

$Q = D\cdot\Delta\omega$ $\qquad [D] = cm^3/giro$

$T = D\cdot\Delta p$

MECHANICS $\frac{T}{\omega}$ $\overset{D}{\text{TF}}$ $\frac{\Delta p}{Q}$ HYDRAULICS



$e_2\tau = e_1$

$i_2 = \tau\, i_1$

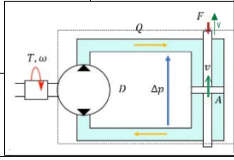
Just valid in AC and steady state conditions

**Power screw**

- Transformer ratio $\tau = \frac{p}{2\pi}$
- Flow relation: $\underset{NUT}{v} = \tau\underset{SCREW}{\omega}$
- Effort equilibrium: $T = \tau F$

Rotary mech. $\overset{e_1}{\underset{f_1}{\rightharpoonup}}$ TF $\overset{e_3}{\underset{f_3}{\rightharpoonup}}$ Linear mech.

$\tau = D/A$

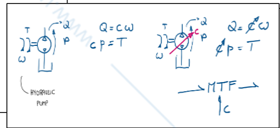


$f_3 = \frac{D}{A} f_1 \leftrightarrow v = \frac{D}{A}\omega$

$v = \tau\omega$

$e_1 = \frac{D}{A} e_3 \leftrightarrow T = \frac{D}{A} F$

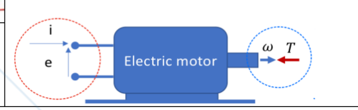
$T = \tau F$



$Q = c\omega$, $c\,p = T$

$Q = \cancel{c}\,\omega$, $\cancel{c}\,p = T$

$\rightarrow$ MTF $\rightarrow$ $1c$



$T = K_m\cdot i$

$e = K_m\cdot\omega$

### VOICE COIL:

$F = 2\pi\cdot B\cdot r\cdot N\cdot i = K_m\cdot i$

$e = K_m\cdot v$

$\overset{e_1}{\underset{f_1}{\rightharpoonup}}$ GY $\overset{e_2}{\underset{f_2}{\rightharpoonup}}$

$e_2 = K_i f_1$

$e_1 = K_i f_2$

$\overset{e_1}{\underset{f_1}{\rightharpoonup}}$ GY $\overset{e_2}{\underset{f_2}{\rightharpoonup}}$ I $\Leftrightarrow$ $\overset{e_1}{\underset{f_1}{\rightharpoonup}}$ C $\qquad$ $\overset{e_1}{\underset{f_1}{\rightharpoonup}}$ GY $\overset{e_2}{\underset{f_2}{\rightharpoonup}}$ C $\Leftrightarrow$ $\overset{e_1}{\underset{f_1}{\rightharpoonup}}$ I

$e_1 = r\cdot f_2 = \frac{r}{I} p_2 = \frac{r}{I}\int e_2 dt = \frac{r}{I}\int r f_1 dt = \frac{r^2}{I} q_1$

$f_1 = \frac{e_2}{r} = \frac{1}{Cr} q_2 = \frac{1}{Cr}\int f_2 dt = \frac{1}{Cr}\int \frac{e_1}{r} dt = \frac{1}{Cr^2} p_1$

### GALVANOMETER:

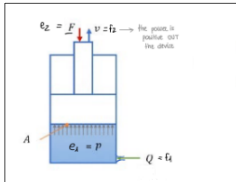
$\overset{e}{\underset{i}{\rightharpoonup}}$ MGY $\overset{T}{\underset{\omega}{\rightharpoonup}}$ (modulated by $\theta$)

$F_{B2C} = BNLi$

$T = BNLD\cos\theta\; i$

$T = \Phi_0\cos\theta\; i$

$BNLD = \Phi_0$ $\qquad e = \Phi_0\cos\theta\;\omega$



Hydraulics $\overset{p}{\underset{Q}{\rightharpoonup}}$ $\overset{e_1}{\underset{f_1}{}}$ TF $\overset{F}{\underset{v}{\rightharpoonup}}$ $\overset{e_2}{\underset{f_2}{}}$ Mechanics

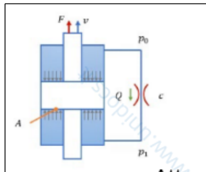
$f_2 = f_1/A$

$e_1 = e_2/A$

$V = \frac{Q}{A}$ $\qquad P = \frac{F}{A}$

$Q = A\cdot\Delta v$ $\qquad F = A\cdot\Delta p$

MECHANICS $\rightharpoonup$ $\overset{A}{\text{TF}}$ $\rightharpoonup$ R:C HYDRAULICS



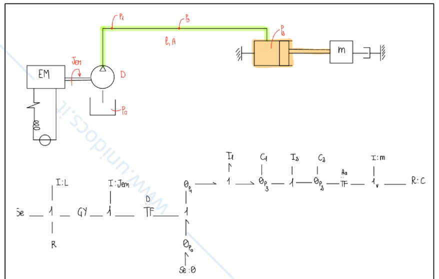
HYDRAULIC DAMPER

$f_2 = A f_1$

$e_1 = A e_2$

- FLOW RELATION $\quad Q = A\cdot v$
- EFFORT EQUILIBRIUM $\quad F = A\cdot\Delta p$
- RESTRICTOR CONSTITUTIVE RELATION $\quad \Delta p = c\cdot Q$

Attention to damper → R: c !!



## CAPACITIVE TRANSDUCER:

$F(q,Q) = \frac{\partial E(q,Q)}{\partial q} = -\frac{1}{2}\frac{Q^2}{C^2}\frac{\partial C}{\partial q} = -\frac{1}{2}e^2\frac{\partial C}{\partial q} = \frac{1}{2}\frac{Q^2}{\varepsilon_0\varepsilon_r A}$

$\overset{F}{\underset{v}{\rightharpoonup}}$ $\underset{\text{POTENTIAL STATES}}{\underset{q\;Q}{CC}}$ $\overset{e}{\underset{i}{\leftharpoondown}}$

$e_1 = \frac{q_1^2}{2\varepsilon_0\varepsilon_r A}$ $\qquad F = \frac{Q^2}{2\varepsilon_0\varepsilon_r A}$ CONTROLLA CHE SIANO SEMPRE QUESTE EQU.

$e_2 = \frac{q_1\cdot q_2}{\varepsilon_0\cdot\varepsilon_r\cdot A}$ $\qquad e = \frac{q\cdot Q}{\varepsilon_0\varepsilon_r A}$

$C = \frac{\varepsilon A}{d}$ where typ. or A(q) or d(q)

So $F - F_0 = -K_q\,\delta q + K_e\,\delta e$

$Q - Q_0 = -K_e\,\delta q + C_0\,\delta e$

$\frac{d}{dt}(Q - Q_0) = i = -\underbrace{K_e\,\delta\dot q}_{\text{FROM TF}} + \underbrace{C_0\frac{d\delta e}{dt}}_{\text{FROM C}}$

C: $-K_q$ $\qquad$ $C_0$

$\overset{F-F_0}{\rightharpoonup}$ 1 $\overset{T}{\underset{\dot q}{\rightharpoonup}}$ TF $\overset{e}{\underset{i}{\rightharpoonup}}$ 0 $\leftharpoondown e-e_0$

$E(q,Q) = \frac{1}{2}\frac{Q^2}{C(q)}$

$e(q,Q) = \frac{\partial E(q,Q)}{\partial Q} = \frac{Q}{C(q)}$

## ELECTROMAGNET:

- $E(q,\lambda) = \int_{\lambda=0}^{\lambda_f} i\,d\lambda = \int \frac{\lambda\cdot R(q)}{N^2}\,d\lambda = \frac{R(q)}{2N^2}\cdot\lambda^2 \simeq \frac{q\,\lambda^2}{\mu_0 S_{GAP}\cdot N^2} = \frac{\lambda^2}{2\,L(q)}$

VELOCITÀ DELL'ELECTROM.

$\overset{F}{\underset{v}{\rightharpoonup}}$ $\underset{\text{POTENTIAL STATES}}{\underset{q\;\lambda}{CI}}$ $\overset{e}{\underset{i}{\leftharpoondown}}$

$F(q,\lambda) = \left.\frac{\partial E(q,\lambda)}{\partial q}\right|_{\lambda=CONST} = \frac{\lambda^2}{\mu_0 S_{GAP}\cdot N^2} = -\frac{1}{2}i^2\frac{\partial L}{\partial q}$

$i(q,\lambda) = \frac{\partial E(q,\lambda)}{\partial\lambda} = \frac{2q\lambda}{\mu_0 S_{GAP} N^2} = \frac{\lambda}{L(q)}$

C $-K_q$; I $L_0$; $S_f: i_0$

$\overset{F}{\rightharpoonup}$ 1 $\overset{}{\rightharpoonup}$ $\overset{K_i}{\text{GY}}$ $\rightharpoonup$ 1 $\leftharpoondown$ 0 $\leftharpoondown$ $i - i_0$

$v = \frac{d(q-q_0)}{dt}$, $\frac{d(\lambda-\lambda_0)}{dt} = e$

Se $F_0$

$F = \frac{\lambda^2}{\alpha}$

$i = \frac{2\lambda q}{\alpha}$

DALLE EQUAZIONI DEL CAPACITIVE TRANSDUCER, QUINDI CONTROLLA CHE VENGANO QUESTE EQU.

$f_1 = \frac{2\cdot p_1 q_2}{\alpha}$ $\qquad \alpha = \mu_0 N^2 S$

$e_2 = \frac{p_1^2}{\alpha}$

$F(q,i) = F(q_0,i_0) + \underbrace{\left.\frac{\partial F}{\partial q}\right|_{q_0,i_0}}_{=-K_q}(q-q_0) + \underbrace{\left.\frac{\partial F}{\partial i}\right|_{q_0,i_0}}_{=K_i}(i-i_0)$

$\lambda(q,i) = \lambda(q_0,i_0) + \underbrace{\left.\frac{\partial\lambda}{\partial q}\right|_{q_0,i_0}}_{=-K_i}(q-q_0) + \underbrace{\left.\frac{\partial\lambda}{\partial i}\right|_{q_0,i_0}}_{=L_0}(i-i_0)$

**Rtot = Rgap + Rfe = R(q,θ) +** $\frac{l_{FE}}{\mu_{FE}S_{FE}}$

(No causal prop in IC & CC element!)

## PIEZOELECTRIC TRANSDUCER:

$F = K_p\,q + \Theta\,e$

$Q = -\Theta\,q + C_p\,e$

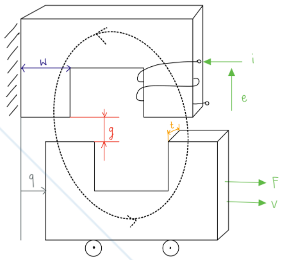
$\frac{d}{dt}$: $i = -\Theta\,v + C_p\frac{de}{dt}$

C: $K_p$ $\qquad$ C: $C_p$

$S_e: F_{in}$ $\overset{F}{\underset{V}{\rightharpoonup}}$ 1 $\rightharpoonup$ $\overset{\Theta}{\text{TF}}$ $\rightharpoonup$ 0 $\overset{e}{\underset{i}{\leftharpoondown}}$ $S_f: i_{in}$

## RELUCTANCE TRANSDUCER:

---> $L = \frac{N^2}{R}$ $\qquad E = \frac{1}{2}\frac{\lambda^2}{L(q)}$ $\qquad R = \frac{2g}{\mu_0(w-|q|)t}$ $\qquad F = -\frac{1}{2}\frac{\lambda^2}{L^2}\frac{\partial L}{\partial q}$



$T = \frac{\partial E}{\partial\vartheta} = -\frac{1}{2}\frac{\lambda^2}{L(\vartheta)^2}\frac{dL}{d\vartheta}$

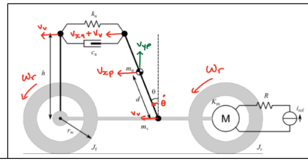
$T = T_\circ + \left.\frac{\partial T}{\partial\vartheta}\right|_\circ\vartheta + \left.\frac{\partial T}{\partial i}\right|_\circ(i - i_\circ)$

$\lambda = \lambda_\circ + \left.\frac{\partial\lambda}{\partial\vartheta}\right|_\circ\vartheta + \left.\frac{\partial\lambda}{\partial i}\right|_\circ(i - i_\circ)$ → (CIRCULAR)

**Gy + R/C/I → R/I/C**

**TF + R/C/I → R/C/I**

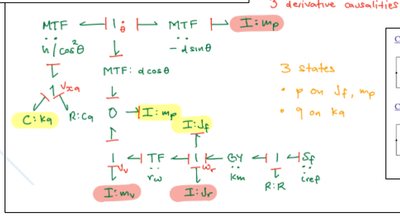
**TF + TF → TF** $\qquad$ **Se/Sf + GY/TF → Se/Sf**



$x_P = x_V + d\sin\theta$ $\qquad \frac{d}{dt}$ $\qquad v_{xP} = \dot x_P = v_V + \dot\theta\, d\cos\theta$

$y_P = d\cos\theta$ $\qquad v_{yP} = \dot y_P = -\dot\theta\, d\sin\theta$

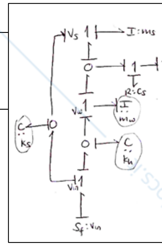
$x_a = h\tan\theta$ $\qquad \frac{d}{dt} \Rightarrow v_{xa} = \dot x_a = \frac{h\dot\theta}{\cos^2\theta}$

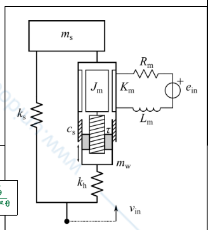


3 derivative causalities

3 states

- p on $J_f$, $m_p$
- q on $k_a$

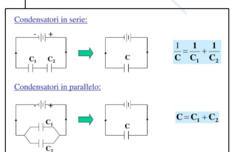




**HYDRAULIC:** ($C_h$ => wide, $I_h$ => small)

HYDRAULIC COMPLIANCE $\qquad C_h = A\cdot\ell\cdot K_h$

HYDRAULIC INERTIA $\qquad I_h = \frac{\rho\ell}{A}$



Condensatori in serie: $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$

Condensatori in parallelo: $C = C_1 + C_2$