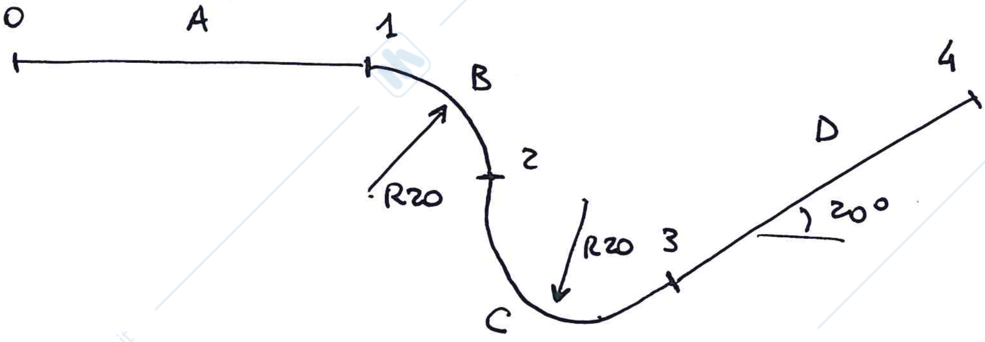
Cinematica del punto

Formula 1 – 1ª variante di Monza



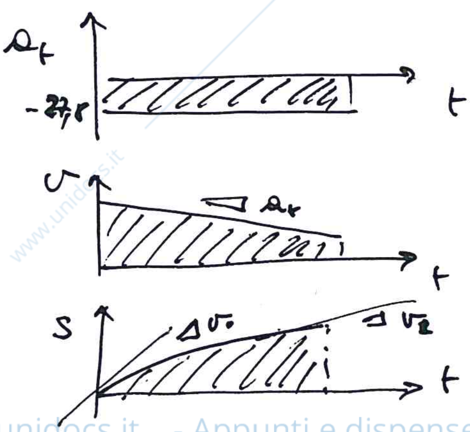
	t	a_t	a_n	v	s
0	0	?	?	340 km/h	0
1	2.6	?	?	80 km/h	?
2	?	0 m/s²	?	80 km/h	?
3	?	?	?	90 km/h	?
4	?	10 m/s²	?	?	s_3 + 100 m

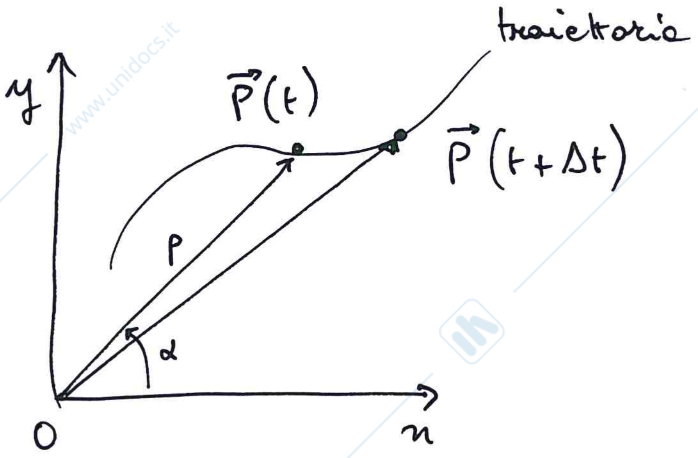
Tratto A – moto uniformemente decelerato rettilineo

$$a_{tA} = \frac{\Delta v_A}{\Delta t_A} = \frac{(80 - 340)/3.6 \text{ m/s}}{2.6 \text{ s}} = -27{,}8 \text{ m/s}^2$$

$$a_{nA} = 0 \text{ m/s}^2$$

$$s_1 = s_0 + v_0 \Delta t_A + \frac{1}{2} a_{tA} \Delta t_A^2 = 151{,}6 \text{ m}$$



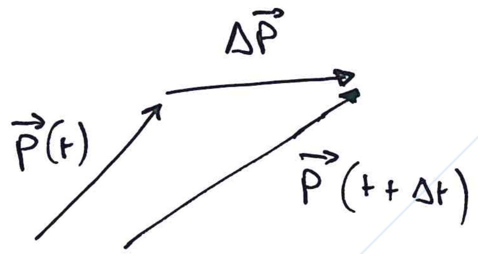


vettore posizione

$$\vec{P}(t) = (P-O) = x\vec{i} + y\vec{j}$$

$$= p e^{i\alpha}$$

velocità $\quad \vec{v} = \dfrac{d\vec{P}}{dt} = \lim\limits_{\Delta t \to 0} \dfrac{\vec{P}(t+\Delta t) - \vec{P}(t)}{\Delta t}$

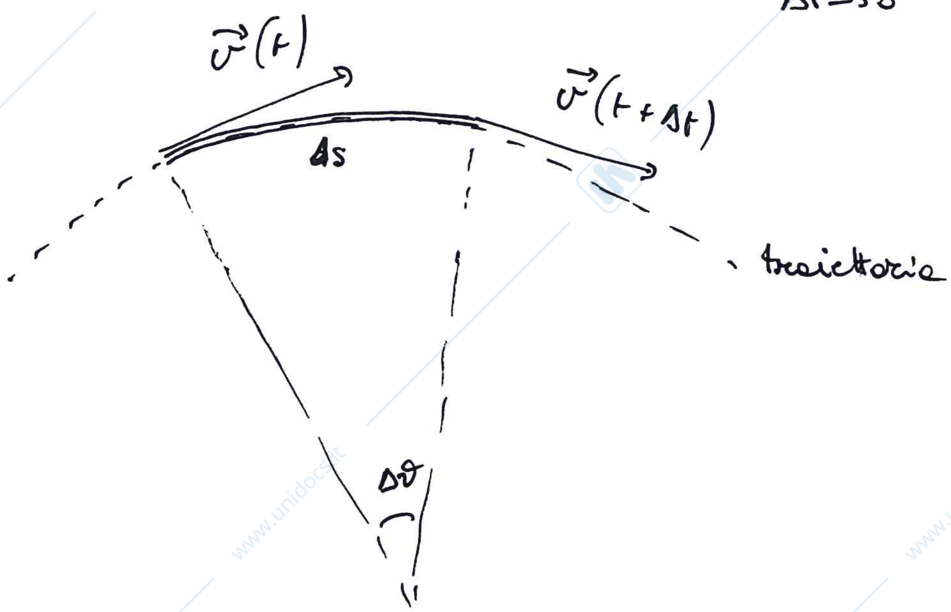


$$\vec{v} = \frac{d\vec{P}}{dt} = \dot{s}\,\vec{\tau}$$

$S = \int_0^t |\vec{v}|\,dt$ ascissa curvilinea è uno scalare

$\vec{v} = \dot{s}\,\vec{\tau}$ $\quad \vec{\tau}$ è il versore tangente alla traiettoria

Accelerazione $\quad \vec{a} = \dfrac{d\vec{v}}{dt} = \lim\limits_{\Delta t \to 0} \dfrac{\vec{v}(t+\Delta t) - \vec{v}(t)}{\Delta t}$

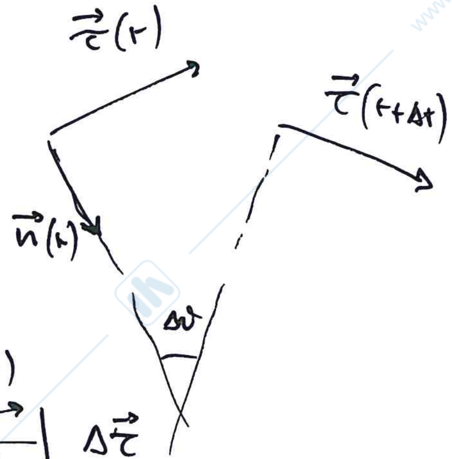


$$\vec{v} = \dot{s}\,\vec{\tau}$$

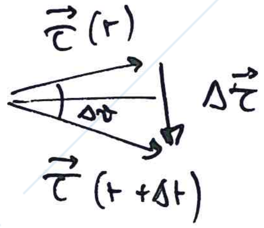
$$\frac{d\vec{v}}{dt} = \ddot{s}\,\vec{\tau} + \dot{s}\,\frac{d\vec{\tau}}{dt}$$

derivata di un versore (Formula di Poisson)

$$\frac{d\vec{\tau}}{dt} = \lim_{\Delta t \to 0} \frac{\vec{\tau}(t+\Delta t) - \vec{\tau}(t)}{\Delta t}$$



$$\Delta\vec{\tau} = \vec{\tau}(t+\Delta t) - \vec{\tau}(t)$$



$$= 2|\vec{\tau}| \sin\left(\frac{\Delta\vartheta}{2}\right) \vec{n}$$

$$\simeq |\vec{\tau}| \Delta\vartheta \, \vec{n} \qquad |\vec{\tau}| = 1 \text{ versore}$$

$$= \frac{\Delta s}{R} \vec{n} \qquad \Delta\vartheta = \frac{\Delta s}{R}$$

$$\frac{d\vec{\tau}}{dt} = \lim_{\Delta t \to 0} \frac{\Delta\vec{\tau}}{\Delta t} = \lim_{\Delta t \to 0} \frac{\Delta\vartheta}{\Delta t} \vec{n} = \omega \vec{n} = \omega \vec{b} \wedge \vec{\tau}$$

$$= \lim_{\Delta t \to 0} \frac{\Delta s}{R \Delta t} \vec{n} = \frac{\dot{s}}{R} \vec{n}$$

Accelerazione

$$\vec{a} = \frac{d\vec{v}}{dt} = \ddot{s}\,\vec{\tau} + \frac{\dot{s}^2}{R} \vec{n} = \vec{a}_t + \vec{a}_n$$

cambia il modulo di v
$\parallel$
$\dot{v}\,\vec{\tau}$

cambia la direzione di v
$\parallel$
$\frac{v^2}{R} \vec{n}$

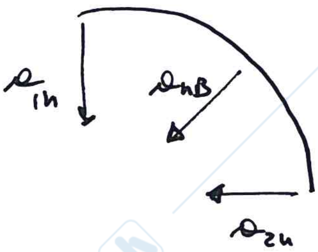
tratto B – moto circolare uniforme

$$a_{t_B} = 0\ m/s^2 = \frac{v_2 - v_1}{\Delta t_B}$$

$$\Delta S_B = S_2 - S_1 = \Delta\theta_B \cdot R_B = \frac{\pi}{2} \cdot 22\,m = \cancel{34{,}6\ m} \rightarrow 31{,}4\ m$$

$$\Delta t_B = \frac{\Delta S_B}{v_B} = \cancel{1.55\ s} \rightarrow 1{,}41\ s$$

$$a_{nB} = \frac{v_B^2}{R_B} = 24{,}7\ m/s^2 = a_{1n} = a_{2n}$$



Tratto C – moto circolare uniformemente accelerato

$$\begin{cases} a_{tc} = \dfrac{v_3 - v_2}{\Delta t_c} \\ \Delta S_B = \Delta\theta_c R_c = v_2 \Delta t_c + \dfrac{1}{2} a_{tc} \Delta t_c^2 \end{cases} \qquad \Delta S_c = 38{,}4\ m$$

$$\Delta S_c = v_2 \Delta t_c + \frac{1}{2} \frac{v_3 - v_2}{\cancel{\Delta t_c}} \Delta t_c^{\cancel{2}}$$

$$\Delta S_c = \Delta t_c \left(v_2 + \frac{1}{2}(v_3 - v_2) \right)$$

$$\Delta t_c = \cancel{1.46\ s} \rightarrow 1{,}63\ s$$

$$a_{tc} = \cancel{1.90\ m/s^2} \rightarrow 1{,}71\ m/s^2$$

$$a_{n2} = 24{,}7\ m/s^2 \qquad a_{n3} = \frac{v_3^2}{R_c} = 31{,}25\ m/s^2$$

Tratto D - moto uniformemente accelerato

$\Delta s_D = 100$ m (dato)

$\Delta s_D = v_3 \Delta t_D + \frac{1}{2} a_D \Delta t_D^2$

$\frac{1}{2} a_D \Delta t_D^2 + v_3 \Delta t_D - \Delta s_D = 0$

$\Delta t_D = -\frac{v_3}{a_D} \mp \sqrt{\left(\frac{v_3}{a_D}\right)^2 + \frac{\Delta s_D}{a_D/2}}$

$= -2.5 \mp 5{,}12$ — $-7{,}62$ s non accettabile non siamo in Star Trek; $2{,}62$ s

$v_4 = v_3 + a_D \Delta t_D = 51{,}2$ m/s $= 184{,}3$ km/h