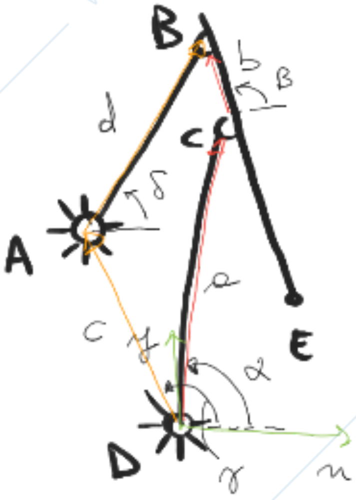
Cinematica gru portuale



3×3 gdl

4×2 gdv

1 gdl residuo

$$a e^{i\alpha} + b e^{i\beta} = c e^{i\gamma} + d e^{i\delta}$$

a, b, c, d sono fissi e noti

$\alpha = \alpha(t)$ var indipendente

β, δ variabili, incognite

γ fisso e noto

$\rightarrow \beta, \delta$

$$\begin{cases} a\cos\alpha + b\cos\beta = c\cos\gamma + d\cos\delta \\ a\sin\alpha + b\sin\beta = c\sin\gamma + d\sin\delta \end{cases} \longrightarrow \beta(\alpha), \delta(\alpha)$$

$$\begin{cases} -a\dot\alpha\sin\alpha - b\dot\beta\sin\beta = -d\dot\delta\sin\delta \\ a\dot\alpha\cos\alpha + b\dot\beta\cos\beta = d\dot\delta\cos\delta \end{cases}$$

$$\underbrace{\begin{bmatrix} -b\sin\beta & d\sin\delta \\ b\cos\beta & -d\cos\delta \end{bmatrix}}_{[A]_{2\times 2}} \begin{Bmatrix} \dot\beta \\ \dot\delta \end{Bmatrix} = \begin{Bmatrix} a\sin\alpha \\ a\cos\alpha \end{Bmatrix}_{2\times 1} \dot\alpha$$

$$\begin{Bmatrix} \dot\beta \\ \dot\delta \end{Bmatrix} = \underbrace{[A]^{-1} \begin{Bmatrix} a\sin\alpha \\ a\cos\alpha \end{Bmatrix}}_{[\Lambda(\alpha)]_{2\times 1}} \dot\alpha$$

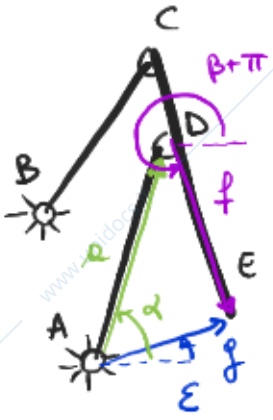
Matrice Jacobiana

$$\begin{cases} -a\ddot\alpha\sin\alpha - a\dot\alpha^2\cos\alpha - b\ddot\beta\sin\beta - b\dot\beta^2\cos\beta = -d\ddot\delta\sin\delta - d\dot\delta^2\cos\delta \\ a\ddot\alpha\cos\alpha - a\dot\alpha^2\sin\alpha + b\ddot\beta\cos\beta - b\dot\beta^2\sin\beta = d\ddot\delta\cos\delta - d\dot\delta^2\sin\delta \end{cases}$$

$$\begin{vmatrix} -b\sin\beta & d\sin\delta \end{vmatrix} \begin{Bmatrix} \ddot\beta \end{Bmatrix} \quad \begin{Bmatrix} a\sin\alpha \end{Bmatrix} \ddot\alpha \quad \begin{Bmatrix} a\cos\alpha \end{Bmatrix} \dot\alpha^2 \quad \begin{Bmatrix} b\cos\beta \end{Bmatrix} \dot\beta^2 \quad \begin{Bmatrix} -d\cos\delta \end{Bmatrix} \dot\delta^2$$

$$\begin{vmatrix} b\cos\beta' & -d\cos\delta \end{vmatrix} \begin{Bmatrix} \ddot{\delta} \end{Bmatrix} = \begin{Bmatrix} a\cos\alpha \end{Bmatrix} \dot{\alpha} + \underbrace{\begin{Bmatrix} a\sin\alpha \end{Bmatrix} \dot{\alpha} + \begin{Bmatrix} b\sin\beta' \end{Bmatrix} \dot{\beta} + \begin{Bmatrix} -d\sin\delta \end{Bmatrix} \dot{\delta}}$$

$$\begin{Bmatrix} \ddot{\beta} \\ \ddot{\delta} \end{Bmatrix} = \underbrace{[A]^{-1} \begin{bmatrix} a\sin\alpha \\ a\cos\alpha \end{bmatrix}}_{[\Lambda(\alpha)]} \ddot{\alpha} + [A]^{-1} \cdot \begin{Bmatrix} \quad \end{Bmatrix}$$



Studiamo il moto di E

$$(E-O) = a e^{i\alpha} + f e^{i(\beta+\pi)}$$

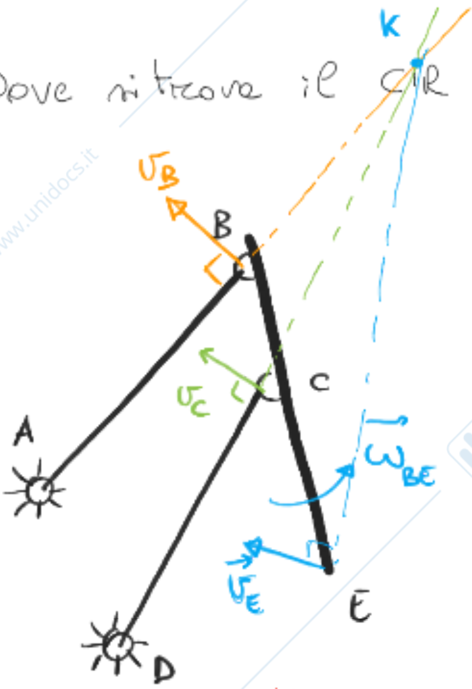
$$x_E + y_E i = a e^{i\alpha} + f e^{i(\beta+\pi)}$$

$$\begin{cases} x_E = a\cos\alpha + f\cos(\beta+\pi) \\ y_E = a\sin\alpha + f\sin(\beta+\pi) \end{cases}$$

$$\vec{v}_E = a\dot{\alpha} e^{i(\alpha+\pi/2)} + f\dot{\beta} e^{i(\beta+\pi)}$$

$$\vec{a}_E = a\ddot{\alpha} e^{i(\alpha+\pi/2)} - a\dot{\alpha}^2 e^{i\alpha} + f\ddot{\beta} e^{i(\beta+\pi)} - f\dot{\beta}^2 e^{i(\beta+\pi)}$$

Dove si trova il CIR dell'asta CE



K è il CIR dell'asta CE

Nota la geometria è nota anche la posizione di K

Possiamo quindi scrivere

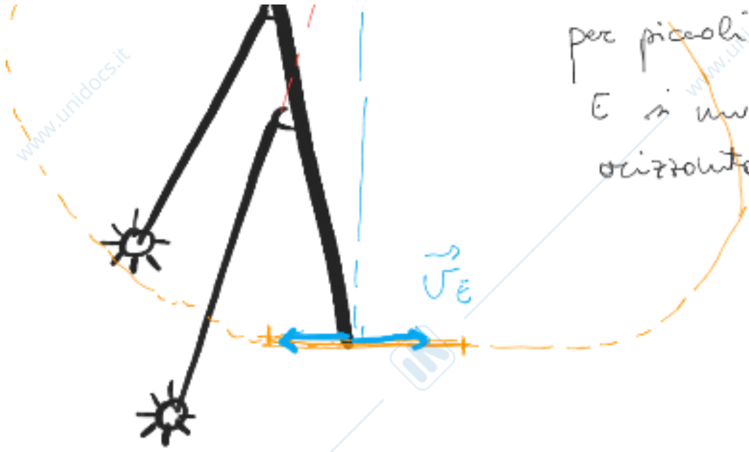
$$\vec{v}_C = \vec{\omega}_{BE} \wedge (C-K) = \dot{\alpha}\vec{k} \wedge (C-D)$$

$$\rightarrow \vec{\omega}_{BE}$$

$$\vec{v}_B = \vec{\omega}_{BE} \wedge (B-K) \rightarrow \vec{v}_B$$

$$\vec{v}_E = \vec{\omega}_{BE} \wedge (E-K) \rightarrow \vec{v}_E$$

traiettoria di E nel moto in grande



per piccoli spostamenti
E si muove circa in orizzontale