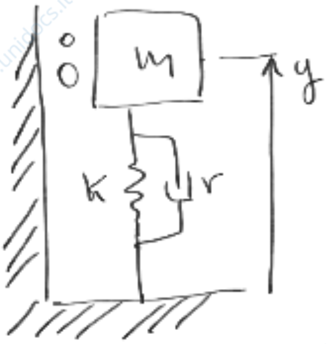
Ese18_VibrazioniLibere1gdl



- Equilibri dinamici (D'Alembert)
- Teorema E_c (Bilancio Potenze)
- Eq Lagrange

$$\underbrace{\frac{d}{dt}\left(\frac{\partial E_c}{\partial \dot{y}}\right) - \frac{\partial E_c}{\partial y}}_{-F_{in}} + \underbrace{\frac{\partial D}{\partial \dot{y}}}_{-F_{smorz}} + \underbrace{\frac{\partial V}{\partial y}}_{-F_{cons}} = \underbrace{Q_y}_{\frac{\delta \mathcal{L}}{\delta y}}$$

$$L_{F_{conserv}} = \Delta V$$

$$E_c = \cancel{\frac{1}{2} m v_x^2} + \frac{1}{2} m v_y^2 + \cancel{\frac{1}{2} J \omega^2} \qquad v_x = 0,\quad v_y = \dot{y},\quad \omega = 0$$

$$E_c = \frac{1}{2} m \dot{y}^2$$

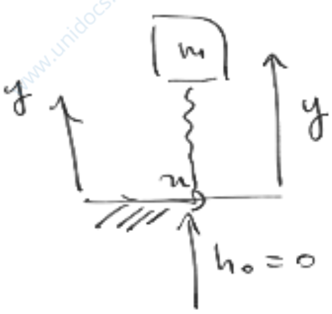
$$\frac{\partial E_c}{\partial \dot{y}} = m\dot{y} \qquad \frac{d}{dt}\left(\frac{\partial E_c}{\partial \dot{y}}\right) = \frac{d}{dt}(m\dot{y}) = m\ddot{y} \qquad \frac{\partial E_c}{\partial y} = 0$$

$$\frac{d}{dt}\left(\frac{\partial E_c}{\partial \dot{y}}\right) - \frac{\partial E_c}{\partial y} = m\ddot{y}$$

$$F_{cons} = -\operatorname{grad} V$$

$$\frac{\partial D}{\partial \dot{y}} \qquad D = \frac{1}{2} r \dot{\Delta l}_r^2$$

$$\frac{\partial V}{\partial y} \qquad V = V_g + V_k \qquad \underline{V_g = mgh} \qquad h = h_0 + y$$



$$V_g = mgh = mgy$$

$$V_k = \frac{1}{2} k \Delta l^2 \qquad F_k = -\frac{\partial V_k}{\partial \Delta l} = \underline{\underline{-k\Delta l}}$$

$$\Delta l \qquad \overset{+}{\longleftrightarrow}$$

$$\Delta l = l(t) - l_0$$

$$l(t) = y(t) \qquad V_k = \frac{1}{2} k (y - l_0)^2$$

$$V = \frac{1}{2} k (y - l_0)^2 + mgy \qquad \frac{\partial V}{\partial y} = k(y - l_0) + mg$$

$$D = \frac{1}{2} r \dot{\Delta l}^2 \qquad \Delta l = y - l_0 \qquad \underline{\underline{\dot{\Delta l} = \dot{y}}}$$

$$\frac{\partial D}{\partial \dot{y}} = \frac{\partial}{\partial \dot{y}}\left(\frac{1}{2} r \dot{y}^2\right) = r\dot{y}$$

$$Q_y = 0$$

$$\underbrace{\frac{d}{dt}\left(\frac{\partial E_c}{\partial \dot{y}}\right) - \frac{\partial E_c}{\partial y}} + \frac{\partial D}{\partial \dot{y}} + \frac{\partial V}{\partial y} = Q_y$$

tutte le forze funzione del tempo, non conservative, tutte le forze che non sono considerate

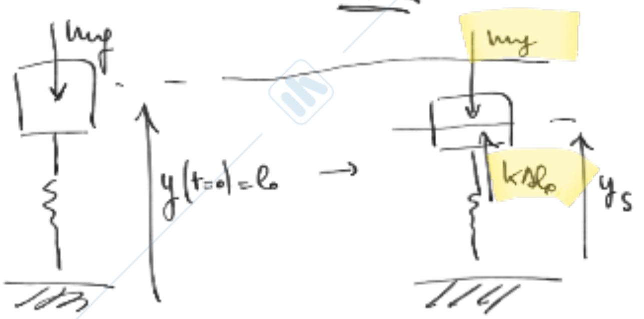
$$m\ddot{y} + r\dot{y} + \underline{k(y-l_0) + mg} = 0$$

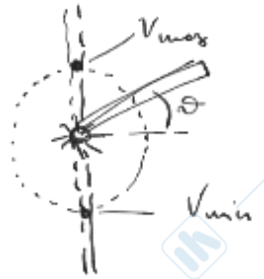
Posizione di equilibrio statico $(\ddot{y} = \dot{y} = 0)$

$$\frac{\partial V}{\partial y} = Q_{y0} \qquad \frac{\partial V}{\partial y} = 0 \leftarrow \text{trovare massimi e minimi di } V$$

$$\underline{k(y_s - l_0)} + mg = 0$$

$$y_s = l_0 - \frac{mg}{k}$$





$$V = mgh = mg\frac{L}{2}\sin\vartheta$$

$$\frac{\partial V}{\partial \vartheta} = mg\frac{L}{2}\cos\vartheta = 0$$

$$\cos\vartheta = 0 \qquad \vartheta = \frac{\pi}{2} + n\pi$$

All'equilibrio

$$k(y_s - l_0) = -mg$$

$$k(l_0 - y_s) = mg$$

$$k\,\Delta l_0 = mg \qquad \Delta l_0 \text{ precarico}$$

$$y = y_s + y_d \rightarrow \dot{y} = \dot{y}_d, \quad \ddot{y} = \ddot{y}_d$$

$$m\ddot{y} + r\dot{y} + k(y-l_0) + mg = 0$$

$$m\ddot{y}_d + r\dot{y}_d + k(y_s + y_d - l_0) + mg = 0$$

$$m\ddot{y}_d + r\dot{y}_d + ky_d + \underbrace{\cancel{k(y_s - l_0) + mg}}_{\frac{\partial V}{\partial y}=0} = 0$$

eq statico, costante

$$m\ddot{y}_d + r\dot{y}_d + ky_d = 0 \qquad u(t) = u_0 + u_1(t)$$

$y_d \quad y_d \quad y_d$ $\qquad$ y_d

↑ spostamento dinamico

$$\begin{cases} y_d(t) = Y_0 e^{\lambda t} \\ \dot{y}_d = \lambda Y_0 e^{\lambda t} \\ \ddot{y}_d = \lambda^2 Y_0 e^{\lambda t} \end{cases}$$

Ip soluzione

$$m\lambda^2 Y_0 e^{\lambda t} + r\lambda Y_0 e^{\lambda t} + K Y_0 e^{\lambda t} = 0$$

$$(m\lambda^2 + r\lambda + k) Y_0 e^{\lambda t} = 0 \qquad e^{\lambda t} \neq 0 \quad \forall t$$

$Y_0 = 0 \rightarrow y_d(t) = 0 \quad \forall t \rightarrow$ sol banale

$$m\lambda^2 + r\lambda + k = 0$$

$$\lambda_{1/2} = -\frac{r}{2m} \mp \sqrt{\left(\frac{r}{2m}\right)^2 - \frac{k}{m}}$$

$$\omega_0 = \sqrt{\frac{k}{m}}$$

$$r_c = 2m\omega_0$$

$$h = \frac{r}{r_c}$$

$$\frac{r}{2m} \cdot \frac{\omega_0}{\omega_0} = \frac{r\omega_0}{2m\omega_0} = \frac{r}{r_c}\omega_0 = h\omega_0$$

$$\lambda_{1/2} = -h\omega_0 \mp \sqrt{(h\omega_0)^2 - \omega_0^2}$$

$$= -h\omega_0 \mp \omega_0\sqrt{h^2 - 1}$$

$h = 0 \rightarrow \lambda_{1/2} = \mp \omega_0\sqrt{-1} = \mp i\omega_0$

$$y(t) = Ae^{\lambda_1 t} + Be^{\lambda_2 t}$$

$$y(t) = Ae^{i\omega_0 t} + Be^{-i\omega_0 t}$$

$$= A\cos\omega t + iA\sin\omega_0 t + $$

$$+ B\cos\omega_0 t - iB\sin\omega_0 t$$

$$= (A+B)\cos\omega_0 t + i(A-B)\sin\omega_0 t$$

$y(t)$ numero reale

$A + B = a \qquad B = a - A$

$i(A-B) = b \qquad i \cdot i(A - a + A) = bi$

$-(2A - a) = bi$

$2A - a = -bi \qquad A = \frac{a - bi}{2}$

$$B = a - \frac{a-bi}{2}$$

$$B = \frac{a+bi}{2}$$

$(h=0)$

$$y(t) = \left(\frac{a-bi}{2} + \frac{a+bi}{2}\right)\cos\omega_0 t + i\left(\frac{a-bi}{2} - \frac{a+bi}{2}\right).$$

$- \sin \omega_0 t$

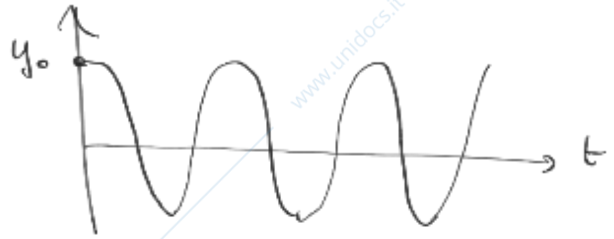
$$= a \cos \omega_0 t + i(-bi) \sin \omega_0 t$$

$$= a \cos \omega_0 t + b \sin \omega_0 t$$

$$\begin{cases} y(t=0) = y_0 \\ \dot{y}(t=0) = 0 \end{cases} \qquad \begin{cases} a = y_0 \\ \left(-\omega_0 a \sin \omega_0 t + b\omega_0 \cos \omega_0 t\right)\Big|_{t=0} = 0 \end{cases}$$

$$b = 0$$

$$y_d(t) = a \cos \omega_0 t = y_0 \cos \omega_0 t$$



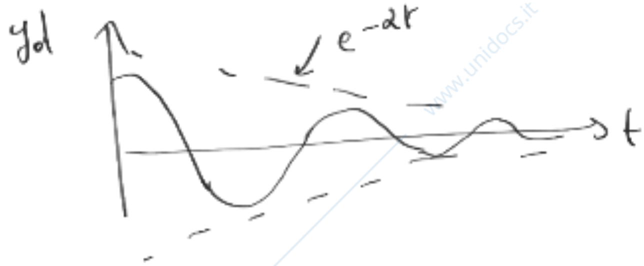
$0 < h < 1$

$$y_d(t) = e^{-\alpha t}\left(A \cos \omega t + B \sin \omega t\right)$$

$$\lambda_{1/2} = -h\omega_0 \mp i\,\omega_0 \sqrt{1-h^2}$$

$$\alpha = h\omega_0$$

$$\omega = \omega_0 \sqrt{1-h^2}$$



$h = 1$

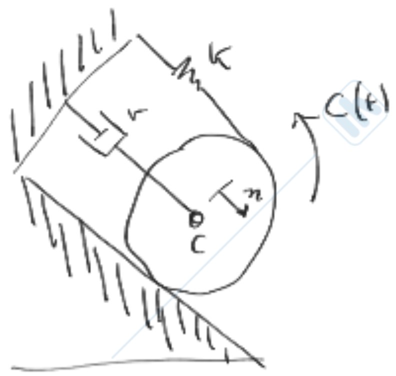
$$\lambda_{1/2} = -\omega_0$$

$$y_d(t) = A e^{-\omega_0 t} + B t e^{-\omega_0 t}$$

$h > 1$

$$\lambda_{1/2} = -h\omega_0 \mp \omega_0 \sqrt{h^2-1}$$

$$y_d(t) = A e^{-\lambda_1 t} + B e^{-\lambda_2 t}$$



$$\frac{d}{dt}\frac{\partial E_c}{\partial \dot{x}} - \frac{\partial E_c}{\partial x} + \frac{\partial D}{\partial \dot{x}} + \frac{\partial V}{\partial x} = Q_x$$

$$E_c = \frac{1}{2} m v_c^2 + \frac{1}{2} J\omega^2$$

$$v_c = \dot{x} \qquad \omega = -\frac{\dot{x}}{R}$$

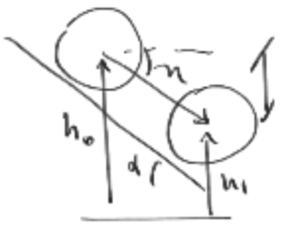
$$E_c = \frac{1}{2}\left(m + \frac{J}{R^2}\right)\dot{x}^2$$

$$\frac{d}{dt}\frac{\partial E_c}{\partial \dot{x}} - \frac{\partial E_c}{\partial x} = \underbrace{\left(m + \frac{J}{R^2}\right)}_{m^*}\ddot{x}$$

$$D = \frac{1}{2} r \dot{\Delta l}_r^2 \qquad \Delta l_r = \Delta l_{r0} + x \qquad \dot{\Delta l}_r = \dot{x}$$

$$\frac{\partial D}{\partial \dot{x}} = r\dot{x}$$

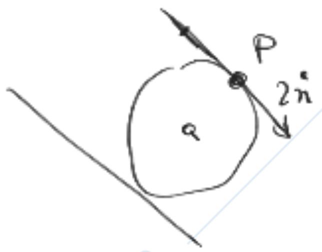
$$V = V_g + V_k$$



$$h = h_0 - x \sin\alpha$$

$$V_g = mgh = mg(h_0 - x\sin\alpha)$$

$$V_k = \frac{1}{2} K \Delta l^2 \qquad \Delta l = \Delta l_0 + \Delta l_d = \Delta l_0 + 2x$$

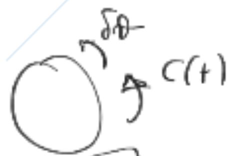


$$v_P = 2\dot{x}$$
$$\Delta l_d = 2x$$

$$V_k = \frac{1}{2} k(\Delta l_0 + 2x)^2$$

$$\frac{\partial V}{\partial x} = \frac{\partial V_g}{\partial x} + \frac{\partial V_k}{\partial x} = -mg\sin\alpha + k(\Delta l_0 + 2x)\cdot 2$$

$$Q_x = \frac{\delta L}{\delta x} = \frac{C\,\delta\vartheta}{\delta x} = \frac{-C/R\,\cancel{\delta x}}{\cancel{\delta x}}$$



$$\delta\vartheta = \underbrace{\boxed{\frac{\partial\vartheta}{\partial x}}}_{J}\delta x$$

$$Q_x = -\frac{1}{R} C(t)$$

$$- \tfrac{\cdot}{R}$$

$$\left(m + \frac{J}{R^2}\right)\ddot{x} + r\dot{x} + k(\Delta l_0 + 2x)2 - mg\sin\alpha = -\frac{C(t)}{R}$$

$$\underbrace{\left(m + \frac{J}{R^2}\right)}_{m^*}\ddot{x} + \underbrace{r}_{r^*}\dot{x} + \underbrace{4k}_{k^*}x = -\frac{C}{R} + mg\sin\alpha - 2k\Delta l_0$$

$$\omega_0 = \sqrt{\frac{k^*}{m^*}} = \sqrt{\frac{4k}{m + J/R^2}} \qquad r_c = 2m^*\omega_0$$

$$h = r/r_c$$

x_s ?

$$C(t) = C_0 e^{i\Omega t}$$

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