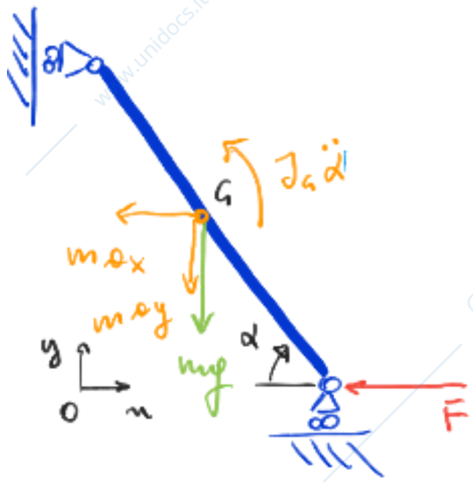
Dinamica del corpo rigido



Nota F calcolare $\alpha, \dot{\alpha}, \ddot{\alpha}$

Condizione iniziale $\alpha(t=0) = \pi/4$

$\dot{\alpha}(t=0) = 0$

1) Aggiungiamo le forze d'inerzia
2) Calcolo della cinematica tenendo α come variabile indipendente

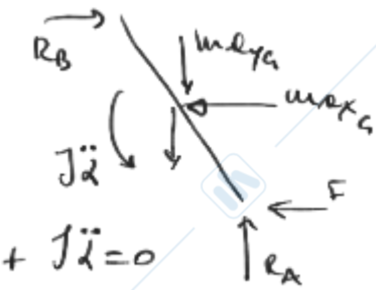
$$\begin{cases} x_A = L\cos\alpha \\ y_A = 0 \end{cases} \qquad \begin{cases} x_B = 0 \\ y_B = L\sin\alpha \end{cases} \qquad \begin{cases} x_G = \frac{L}{2}\cos\alpha \\ y_G = \frac{L}{2}\sin\alpha \end{cases}$$

$$\begin{cases} \dot{x}_A = -L\sin\alpha\,\dot{\alpha} \\ \dot{y}_A = 0 \end{cases} \qquad \begin{cases} \dot{x}_B = 0 \\ \dot{y}_B = L\cos\alpha\,\dot{\alpha} \end{cases} \qquad \begin{cases} \dot{x}_G = -\frac{L}{2}\sin\alpha\,\dot{\alpha} \\ \dot{y}_G = \frac{L}{2}\cos\alpha\,\dot{\alpha} \end{cases}$$

$$\begin{cases} \ddot{x}_A = -L\cos\alpha\,\dot{\alpha}^2 - L\sin\alpha\,\ddot{\alpha} \\ \ddot{y}_A = 0 \end{cases} \qquad \begin{cases} \ddot{x}_G = -\frac{L}{2}\cos\alpha\,\dot{\alpha}^2 - \frac{L}{2}\sin\alpha\,\ddot{\alpha} \\ \ddot{y}_G = -\frac{L}{2}\sin\alpha\,\dot{\alpha}^2 + \frac{L}{2}\cos\alpha\,\ddot{\alpha} \end{cases}$$

$$\begin{cases} \ddot{x}_B = 0 \\ \ddot{y}_B = -L\sin\alpha\,\dot{\alpha}^2 + L\cos\alpha\,\ddot{\alpha} \end{cases}$$

Soluzione con equilibri dinamici (alla D'Alembert)



$$\begin{cases} \sum F_x = 0 & -F - ma_{x_G} + R_B = 0 \\ \sum F_y = 0 & R_A - mg - ma_{y_G} = 0 \\ \sum M_z^{(G)} = 0 & R_A \frac{L}{2}\cos\alpha - R_B\frac{L}{2}\sin\alpha - F\frac{L}{2}\sin\alpha + J\ddot{\alpha} = 0 \end{cases}$$

3 equazioni in 3 incognite $R_A, R_B, \ddot{\alpha}$

Soluzione con il bilancio di potenze

$$\sum W = 0 \qquad -F\dot{x}_A - mg\dot{y}_G - m\ddot{x}_G\dot{x}_G - m\ddot{y}_G\dot{y}_G - J\ddot{\alpha}\dot{\alpha} = 0$$

$$-F\cdot\left(-L\sin\alpha\,\dot\alpha\right)-mg\left(\frac{L}{2}\cos\alpha\,\dot\alpha\right)-m\left(-\frac{L}{2}\cos\alpha\,\dot\alpha^2-\frac{L}{2}\sin\alpha\,\ddot\alpha\right)\left(-\frac{L}{2}\sin\alpha\,\dot\alpha\right)+$$

$$-m\left(-\frac{L}{2}\sin\alpha\,\dot\alpha^2+\frac{L}{2}\cos\alpha\,\ddot\alpha\right)\left(+\frac{L}{2}\cos\alpha\,\dot\alpha\right)-J\,\ddot\alpha\,\dot\alpha=0$$

$$FL\sin\alpha-mg\frac{L}{2}\cos\alpha-m\frac{L^2}{4}\cancel{\sin\alpha\cos\alpha\,\dot\alpha^2}-m\frac{L^2}{4}\sin^2\alpha\,\ddot\alpha+m\frac{L^2}{4}\cancel{\sin\alpha\cos\alpha\,\dot\alpha^2}+$$
$$-m\frac{L^2}{4}\cos^2\alpha\,\ddot\alpha-J_4\ddot\alpha=0$$

$$FL\sin\alpha-mg\frac{L}{2}\cos\alpha-\left(m\frac{L^2}{4}+J_4\right)\ddot\alpha=0$$

Partiamo da $\alpha=\alpha(t=0)\longrightarrow\ddot\alpha(t=0)=\dfrac{FL\sin\alpha-mg\frac{L}{2}\cos\alpha}{m\frac{L^2}{4}+J_4}$

$$\begin{cases}\dot\alpha(t+\Delta t)=\dot\alpha(t=0)+\ddot\alpha(t=0)\,\Delta t\\ \alpha(t+\Delta t)=\alpha(t=0)+\dot\alpha(t=0)\,\Delta t\end{cases}$$

$$\begin{cases}\dot\alpha_{k+1}=\dot\alpha_k+\ddot\alpha_k\,\Delta t\\ \alpha_{k+1}=\alpha_k+\dot\alpha_k\,\Delta t\end{cases}\longleftarrow$$

Integrazione numerica col metodo di Eulero avanti