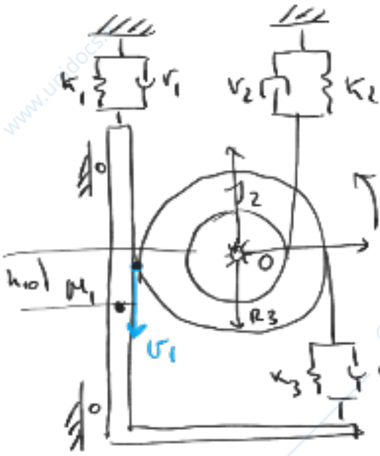
Ese21



$$E_c = \frac{1}{2} M_1 v_1^2 + \frac{1}{2} J_2 \omega_2^2$$

$$\omega_2 = \dot{\vartheta}$$

$$v_1 = -R_3 \dot{\vartheta}$$

$$E_c = \frac{1}{2} \underbrace{\left(M_1 R_3^2 + J_2\right)}_{J^*} \dot{\vartheta}^2$$

$$\frac{d}{dt}\frac{\partial E_c}{\partial \dot{\vartheta}} - \frac{\partial E_c}{\partial \vartheta} = J^* \ddot{\vartheta}$$

$$V = V_g + V_K$$

$$V_g = M_1 g h_1 + M_2 g h_2 \qquad h_2 = y_0 = \text{costante}$$

$$\frac{\partial^2 h_2}{\partial \vartheta^2} = 0$$

$$\frac{\partial V_g}{\partial \vartheta} = M_1 g \frac{\partial h_1}{\partial \vartheta} + \cancel{M_2 g \frac{\partial h_2}{\partial \vartheta}}^{\,0}$$

$$h_1 = h_{10} - R_3 \vartheta$$

$$= -M_1 g R_3 \quad \text{Coppia costante}$$

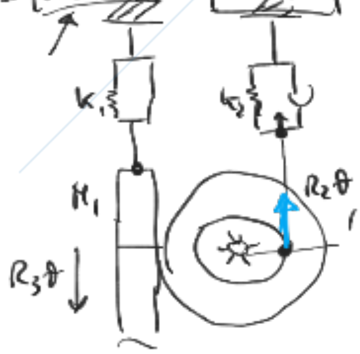
$$\frac{\partial h_1}{\partial \vartheta} = -R_3 \qquad \frac{\partial^2 h_1}{\partial \vartheta^2} = 0$$

$$K_g = M g \left.\frac{\partial^2 h_1}{\partial \vartheta^2}\right|_{\vartheta_0} = 0$$

$$V_K = \frac{1}{2} K_1 \Delta l_1^2 + \frac{1}{2} K_2 \Delta l_2^2 + \frac{1}{2} K_3 \Delta l_3^2$$

$$\frac{\partial V_{K_1}}{\partial \vartheta} = K_1 (\Delta l_{01} + R_3 \vartheta) R_3 = \boxed{\cancel{K \Delta l_{01} R_3}} + \boxed{K_1 R_3^2 \vartheta}$$

$$\boxed{V_{K_1} = \frac{1}{2} K (\Delta l_{01} + R_3 \vartheta)^2}$$



$$\Delta l_1 = \Delta l_{01} + \Delta l_{1d}$$

$$\Delta l_{1d} = R_3 \vartheta \qquad \frac{\partial \Delta l_{1d}}{\partial \vartheta} = R_3 \qquad \frac{\partial^2 \Delta l_{1d}}{\partial \vartheta^2} = 0$$

$$K_{K0}^{(1)} = 0 \qquad K_{K_1}^{(1)} = R_3^2 K_1$$

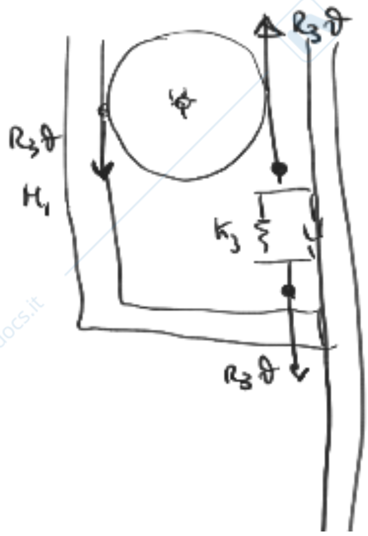
$$\Delta l_2 = \Delta l_{02} + \Delta l_{2d} \qquad \Delta l_{2d} = -R_2 \vartheta$$

$$\frac{\partial \Delta l_{2d}}{\partial \vartheta} = -R_2 \qquad \frac{\partial^2 \Delta l_{2d}}{\partial \vartheta^2} = 0$$

$$K_{K0}^{(2)} = 0 \qquad K_{K_1}^{(2)} = R_2^2 K_2$$

$$\Delta l_3 = \Delta l_{03} + \Delta l_{3d} \qquad \Delta l_3 = R_3 \vartheta + R_3 \vartheta$$

$$\Delta l_{3d} = 2 R_3 \vartheta$$



$$K_{K0}^{(3)} = 0 \qquad K_{K_1}^{(3)} = 4 R_3^2 K_3$$

$$K^* = R_3^2 K_1 + R_2^2 K_2 + 4 R_3^2 K_3$$

$$\frac{\partial V}{\partial \vartheta} = K^* \vartheta$$

$$D = \frac{1}{2} r_1 \Delta l_1^2 + \frac{1}{2} r_2 \Delta l_2^2 + \frac{1}{2} r_3 \Delta l_3$$

$$\Delta l_1 = \Delta l_{10} + R_3 \vartheta \qquad \Delta \dot{l}_1 = R_3 \dot{\vartheta}$$

$$r^* = r_1 R_3^2 + r_2 R_2^2 + r_3 4 R_3^2$$

$$\frac{\partial D}{\partial \dot{\vartheta}} = r^* \dot{\vartheta}$$

$$Q = \frac{\delta L_c}{\delta \vartheta} = -C(t) \qquad \delta L_c = \vec{C} \cdot \vec{\delta \vartheta_c} = -C \delta \vartheta$$

$$\delta \vartheta_c = \delta \vartheta$$

$$J^* \ddot{\vartheta} + r^* \dot{\vartheta} + K^* \vartheta = -C(t)$$

$\omega_0 = \sqrt{\frac{K^*}{J^*}}$ $\left[\frac{rad}{s}\right]$ pulsazione propria del sistema non smorzato

$f_0 = \frac{\omega_0}{2\pi}$ $\left[\frac{1}{s}\right]$ = $[Hz]$ frequenza propria del sistema non smorzato

$$\vartheta(t) = \vartheta_{OA}(t) + \vartheta_p(t)$$

$\vartheta_{OA}(t)$: risposta libera

$\vartheta_p(t)$: risposta alla forzante $C(t)$ ← risposta a regime

$$\vartheta_{OA} \xrightarrow[t \to \infty]{} 0$$

$$C(t) = C_0 e^{i\Omega t} \qquad \vartheta_p(t) = \bar{\vartheta}_p e^{i\Omega t}$$ ← Ipotesi sol a regime

$$\dot{\vartheta}_p(t) = i\Omega \bar{\vartheta}_p e^{i\Omega t}$$

$$\ddot{\vartheta}_p(t) = -\Omega^2 \bar{\vartheta}_p e^{i\Omega t}$$

$$J^*\left(-\Omega^2 \bar{\vartheta}_p e^{i\Omega t}\right) + r^*\left(i\Omega \bar{\vartheta}_p e^{i\Omega t}\right) + k^* \bar{\vartheta}_p e^{i\Omega t} = -C_0 e^{i\Omega t}$$

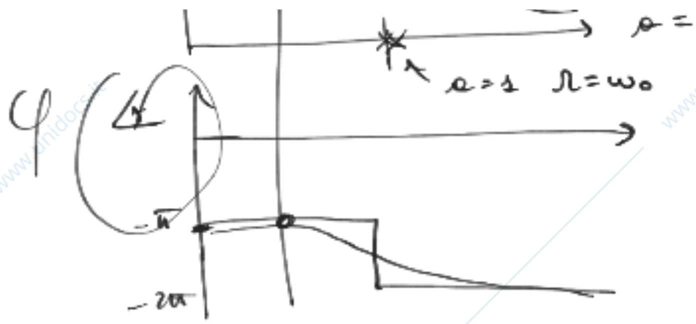
$$\left(-J^*\Omega^2 + i\Omega r^* + k^*\right) \bar{\vartheta}_p \cancel{e^{i\Omega t}} = -C_0 \cancel{e^{i\Omega t}}$$

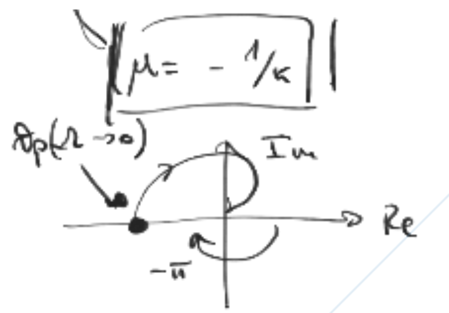
$$\boxed{\bar{\vartheta}_p = \frac{-C_0}{-J^*\Omega^2 + i\Omega r^* + k^*}}$$ ← numero complesso $||$ e $\angle$

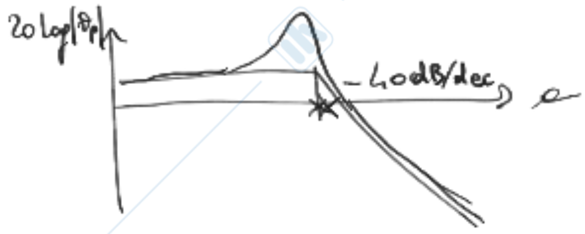
$$|\bar{\vartheta}_p| = \frac{C_0}{\sqrt{(k^* - J^*\Omega^2)^2 + \Omega^2 r^{*2}}} \qquad \varphi = \operatorname{atan}\left(\frac{+\Omega r^*}{k^* - J^*\Omega^2}\right)$$

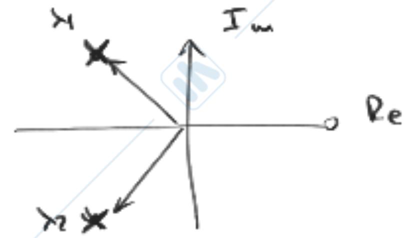
$$\boxed{\bar{\vartheta}_p \xrightarrow[\Omega \to 0]{} -\frac{C_0}{k}}$$

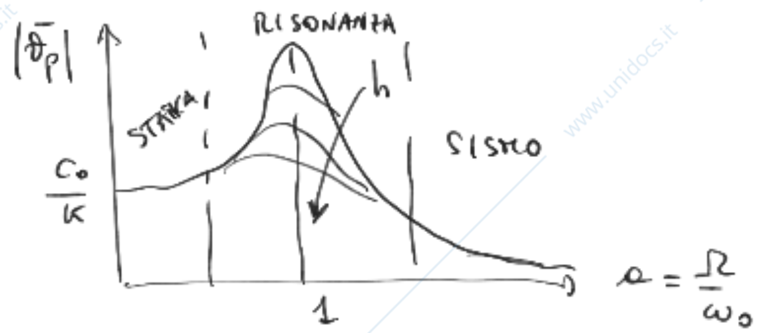
$$TF = \frac{\vartheta_p}{C_0} = \frac{-1}{Js^2 + rs + K}$$











$a < 1$ quasi statica

$a \simeq 1$ risonanza

$a \gg 1$ sismografica

$$\bar\vartheta_p = \frac{-C_0}{\underbrace{K - J\Omega^2}_{a} + i\underbrace{\Omega r}_{b}} = -C_0\left(\frac{1}{a+ib}\right)\cdot\frac{a-ib}{a-ib}$$

$$= -C_0\left(\frac{a-ib}{a^2+b^2}\right) = -\frac{C_0}{a^2+b^2}(a-ib)$$

$$= -\frac{C_0 a}{a^2+b^2} + \frac{C_0 b}{a^2+b^2}\, i$$

$$\tan\varphi = \frac{\frac{\cancel{C_0} b}{\cancel{a^2+b^2}}}{-\frac{\cancel{C_0} a}{\cancel{a^2+b^2}}} = -\frac{b}{a} = \left(-\frac{\Omega r}{k - J\Omega^2}\right)$$

$$J^*\ddot\vartheta + r\dot\vartheta + k\vartheta = -C_0$$

$$\theta_p = \frac{-\omega}{-J\Omega^2 + K + i\Omega r}$$

poli $\Rightarrow$ $\quad -J\Omega^2 + K + i\Omega r = 0$

$$\boxed{\lambda_{1/2} = -\frac{r}{2J} \mp \sqrt{\left(\frac{r}{2J}\right)^2 - \frac{K}{J}}}$$

$\frac{r}{2J} = h\omega_0 \qquad h = r/r_c = \frac{r}{2J\omega_0}$

$0 < h < 1$ sottosmorzato $\lambda_{1/2}$ $\mathbb{C}$

$h = 1$ smorz critico $\lambda_{1/2}$ $\mathbb{R}$ coinc <0

$h > 1$ ipersmorzato $\lambda_{1/2}$ $\mathbb{R}$ <0

$x_0 = 0$

$\dot{x}_0 = V_0 = 1\ m/s$

$m = 5\ kg \qquad k = 500\ N/m$

$E_c = \frac{1}{2} m \dot{x}^2$

$V = \frac{1}{2} k \Delta l^2 \qquad \Delta l = \Delta l_0 + x$

$m\ddot{x} + kx = 0 \quad \leftarrow \quad \begin{cases} x(t) = X_0 e^{\lambda t} \\ \dot{x} = \lambda X_0 e^{\lambda t} \\ \ddot{x} = \lambda^2 X_0 e^{\lambda t} \end{cases}$

$(\lambda^2 m + k) X_0 \cancel{e^{\lambda t}} = 0$

$\lambda^2 = -k/m \qquad \lambda_{1/2} = \mp i\sqrt{\frac{k}{m}} \qquad \omega_0 = 10\ rad/s$

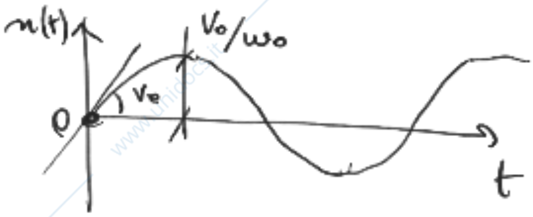
$x(t) = A \cos\omega_0 t + B \sin\omega_0 t$

$x(t=0) = 0 \quad \rightarrow \quad A = 0$

$\dot{x}(t) = -\omega_0 A \sin\omega_0 t + \omega_0 B \cos\omega_0 t$

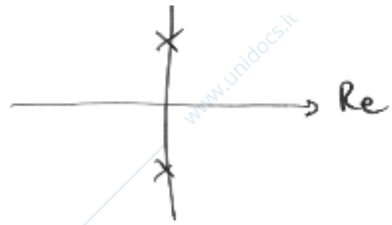
$\dot{x}(t=0) = V_0 \qquad \omega_0 B = V_0 \qquad B = \frac{V_0}{\omega_0}$

$x(t) = \frac{V_0}{\omega_0} \sin\omega_0 t$



Ampiezza $= B = \frac{V_0}{\omega_0} = 0,1\ m$

Im



$v_c = -\dot{\vartheta} R$

$\omega = \dot{\vartheta}$

$m = 0{,}7\ kg$

$R = 1{,}3\ m$

$k = 150\ N/m$

f_0?

$$E_c = \frac{1}{2} m v_c^2 + \frac{1}{2} J \omega^2$$

$$E_c = \frac{1}{2} \left(mR^2 + J \right) \dot{\vartheta}^2$$

$$J = \frac{1}{2} m R^2$$

$$J^* = mR^2 + J = \frac{3}{2} m R^2 = 1{,}77\ kg\, m^2$$

$$V = V_K = \frac{1}{2} k \Delta\ell^2$$

$$\Delta\ell_d = 2R\vartheta \qquad \frac{\partial \Delta\ell_d}{\partial \vartheta} = 2R \qquad \frac{\partial^2 \Delta\ell_d}{\partial \vartheta^2} = 0$$

$$K^* = K_{K1} = k\, 4R^2 = 1014\ \frac{Nm}{rad} \qquad k_{K1} = k \left(\left. \frac{\partial \Delta\ell_d}{\partial \vartheta} \right|_{\vartheta=0} \right)^2$$

$$\omega_0 = \sqrt{\frac{K^*}{J^*}} = \sqrt{572{,}9} = 23{,}9\ rad/s \qquad f_0 = \frac{\omega_0}{2\pi} = 3{,}8\ Hz$$

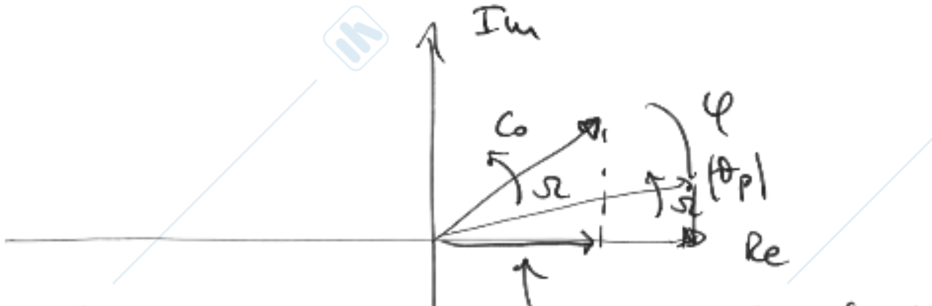
$$V_p = mgh \qquad h = -R\vartheta$$

$$K_{K0} = \cancel{mg \frac{\partial^2 h}{\partial \vartheta^2}}\ 0$$

$$V_g = -mgR\vartheta$$

$$\frac{\partial V_p}{\partial \vartheta} = -mgR$$

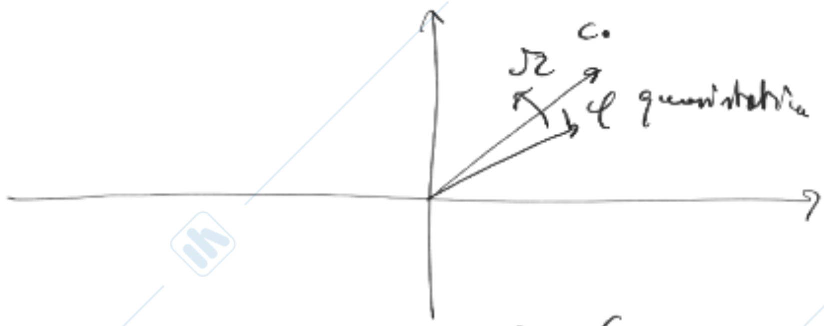
$$\vartheta = \vartheta_0 \cos(\omega_0 t)$$

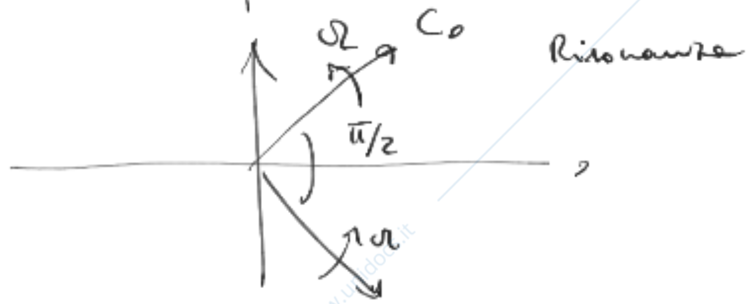


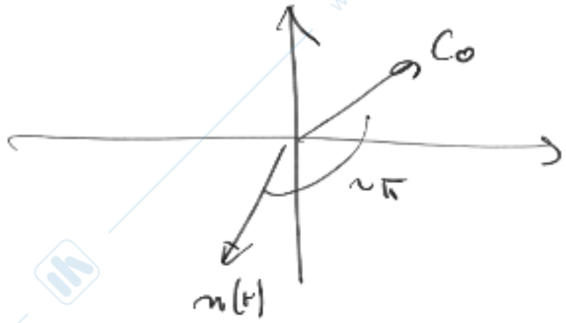
$$C(t) = C_0 \cos(\Omega t)$$

$$\bar{C}(t) = C_0 e^{i\Omega t}$$

$$x(t) = |\vartheta_p| \cos(\Omega t + \varphi)$$







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