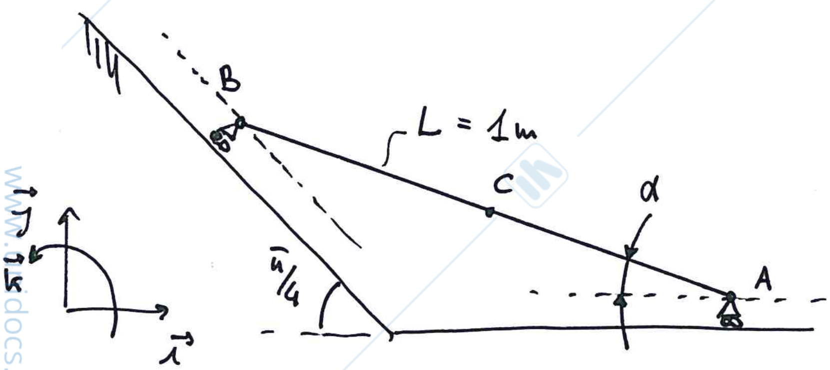
Cinematica del corpo rigido



Studio della cinematica di un'asta vincolata a due guide rettilinee

Gradi di libertà

1 corpo rigido nel piano = 3 pdl

2 carrelli = 2 × 1 pdv

1 pdl (e.g. α)

Noto $\alpha, \dot{\alpha}, \ddot{\alpha}$ determinare $\vec{v}_A, \vec{v}_B, \vec{v}_C, \vec{a}_A, \vec{a}_B, \vec{a}_C$ nell'atto di moto rappresentato ($\alpha = \pi/6$ rad, $\dot{\alpha} = 1$ rad/s, $\ddot{\alpha} = 0.1$ rad/s²)

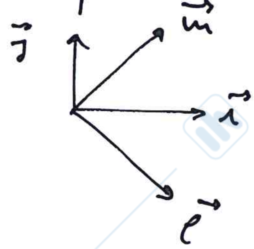
Th di Rivals

$$\vec{v}_B = \vec{v}_A + \vec{\omega} \wedge (B - A)$$

$$\vec{v}_A = v_A \vec{i}$$

$$\vec{v}_B = v_B \vec{e}$$

$\vec{e}$ versore della guida inclinata



$$\begin{cases} \vec{e} = \vec{i}\cos\frac{\pi}{4} - \vec{j}\sin\frac{\pi}{4} \\ \vec{n} = \vec{i}\sin\frac{\pi}{4} + \vec{j}\cos\frac{\pi}{4} \end{cases}$$

$$\begin{cases} \vec{e} = \frac{\sqrt{2}}{2}\vec{i} - \frac{\sqrt{2}}{2}\vec{j} \\ \vec{n} = \frac{\sqrt{2}}{2}\vec{i} + \frac{\sqrt{2}}{2}\vec{j} \end{cases}$$

$$v_B \vec{e} = v_A \vec{i} + (-\dot{\alpha}\vec{k}) \wedge L(-\cos\alpha\, \vec{i} + \sin\alpha\, \vec{j})$$

$$v_B \frac{\sqrt{2}}{2}\vec{i} - v_B \frac{\sqrt{2}}{2}\vec{j} = v_A \vec{i} + \dot{\alpha} L \cos\alpha \,\vec{j} + \dot{\alpha} L \sin\alpha \,\vec{i}$$

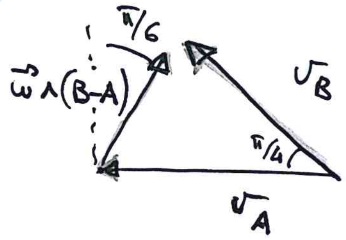
$$\begin{cases} v_B \frac{\sqrt{2}}{2} = v_A + \dot{\alpha} L \sin\alpha \\ -v_B \frac{\sqrt{2}}{2} = +\dot{\alpha} L \cos\alpha \end{cases} \longrightarrow \quad v_B = -\frac{\cancel{2}\sqrt{2}}{\cancel{2}}\dot{\alpha} L \cos\alpha$$

$$v_A = \dot{\alpha} L \left(-\cos\alpha - \sin\alpha\right)$$

@ $\alpha = \pi/6$, $\dot{\alpha} = 1$ rad/s

$$v_A = -\frac{\cancel{2}\sqrt{2}}{\cancel{2}}\cdot\frac{\sqrt{3}}{2}\ m/s$$

$$v_B = -\left(\frac{\sqrt{3}}{2} + \frac{1}{2}\right) m/s$$



$$\vec{v}_c = \vec{v}_A + \vec{\omega} \wedge (c - A) = v_A \vec{i} + \dot{\alpha} L \cos\alpha \,\vec{j} + \dot{\alpha} L \sin\alpha \,\vec{i}$$

$$= \left(v_A + \dot{\alpha} L \sin\alpha\right)\vec{i} + \dot{\alpha} L \cos\alpha \,\vec{j}$$

$$= v_{cx}\,\vec{i} + v_{cy}\,\vec{j}$$

$$\vec{a}_B = \vec{a}_A + \overbrace{\dot{\vec{\omega}} \wedge (B - A)}^{a_t} + \overbrace{\vec{\omega} \wedge (\vec{\omega} \wedge (B - A))}^{a_n}$$

← t e n del moto rotatorio di B intorno ad A

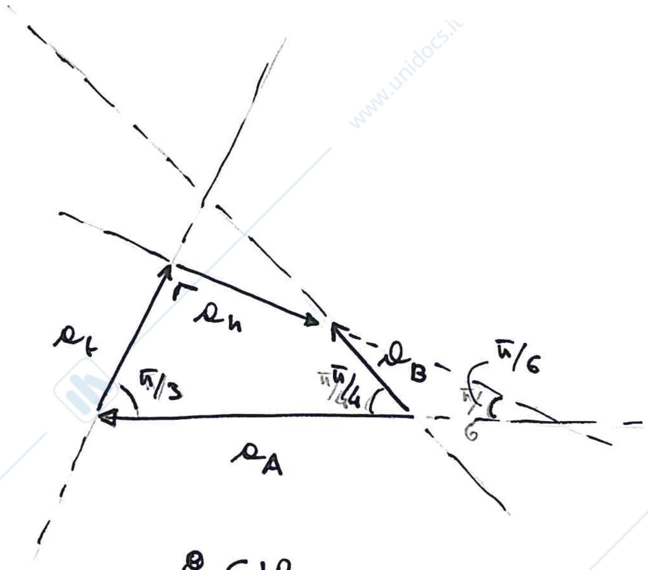
$$\vec{a}_A = a_A \vec{i}$$

$$\vec{a}_B = a_B \vec{e} = a_B \left(\frac{\sqrt{2}}{2}\vec{i} - \frac{\sqrt{2}}{2}\vec{j}\right)$$

$$\dot{\vec{\omega}} = -\ddot{\alpha}\vec{k}$$

$$a_B \frac{\sqrt{2}}{2}\vec{i} - a_B \frac{\sqrt{2}}{2}\vec{j} = a_A \vec{i} + \ddot{\alpha} L \cos\alpha \,\vec{j} + \ddot{\alpha} L \sin\alpha \,\vec{i} - \dot{\alpha}^2 L \left(-\cos\alpha \,\vec{i} + \sin\alpha \,\vec{j}\right)$$

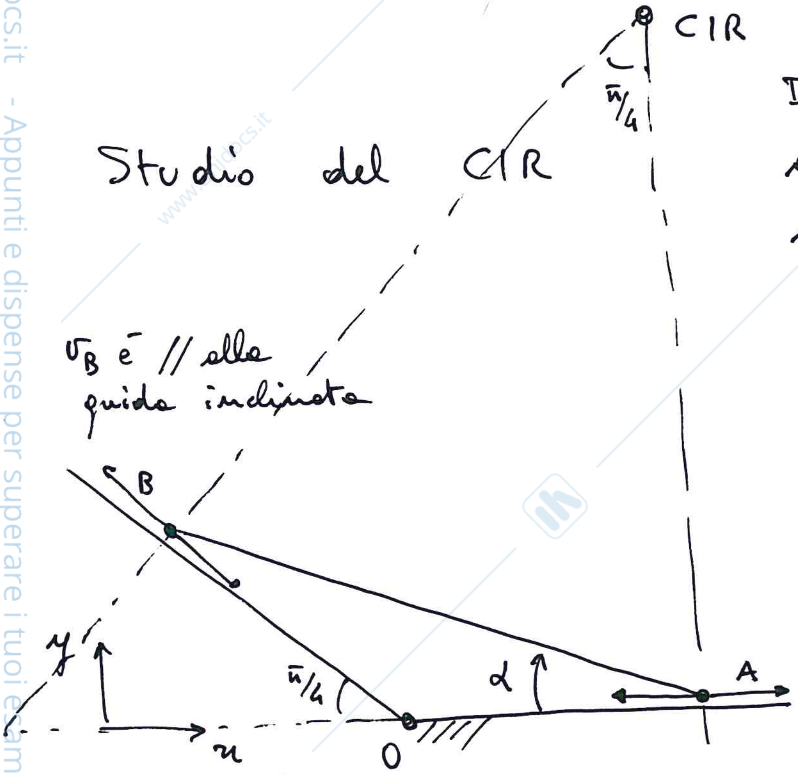
$$\begin{cases} a_B \frac{\sqrt{2}}{2} = a_A + \ddot{\alpha} L \sin\alpha + \dot{\alpha}^2 L \cos\alpha \\ -a_B \frac{\sqrt{2}}{2} = \ddot{\alpha} L \cos\alpha - \dot{\alpha}^2 L \sin\alpha \end{cases}$$



Studio del CIR

Il CIR dell'asta si trova all'incrocio delle rette ⊥ a due velocità del corpo

v_B è // alla guida inclinata

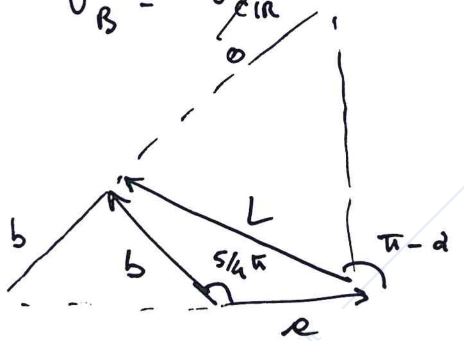


v_A è diretta come x, cioè parallela alla guida orizzontale

$$\vec{v}_A = \cancel{\vec{v}_{CIR}}^{\,0} + \vec{\omega} \wedge (A - CIR)$$

$$\vec{v}_B = \cancel{\vec{v}_{CIR}}^{\,0} + \vec{\omega} \wedge (B - CIR)$$

Bisogna determinare la posizione del CIR.
La posizione del CIR cambia nel tempo $v_{CIR} = 0$ ma $a_{CIR} \neq 0$



$$a e^{i0} + L e^{i(\pi-\alpha)} = b e^{i\frac{5}{4}\pi}$$

$$\begin{cases} a + L\cos(\pi-\alpha) = b\cos\frac{5}{4}\pi \\ L\sin(\pi-\alpha) = b\sin\frac{5}{4}\pi \end{cases}$$

$$\begin{cases} a = -L\sin(\pi-\alpha) - L\cos(\pi-\alpha) \\ b = \sqrt{2}\,L\sin(\pi-\alpha) \end{cases}$$

$$x_{CIR} = a$$

$$y_{CIR} = a + 2b\frac{\sqrt{2}}{2}$$