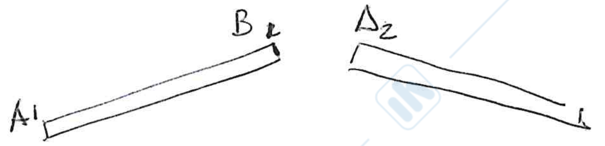
CINEMATICA DEI CORPI RIGIDI

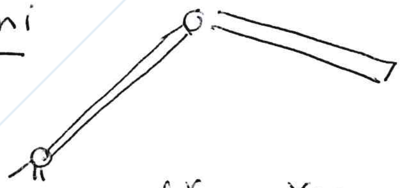
C.R mutuamente connessi

1) C.R 3 gdL

2) 

$3 \times n_{C.R} = 6$ gdL

3) Meccanismi

•) 

$3 \times 2 = 6$ gdL

$2 \times 2 = 4$ gdV

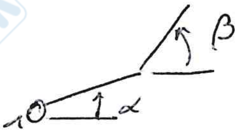
2 gdL residui

$$\begin{cases} x_{A_1} = 0 \\ y_{A_1} = 0 \end{cases} \qquad \begin{cases} x_{B_1} = x_{A2} \\ y_{B_1} = y_{A2} \end{cases}$$

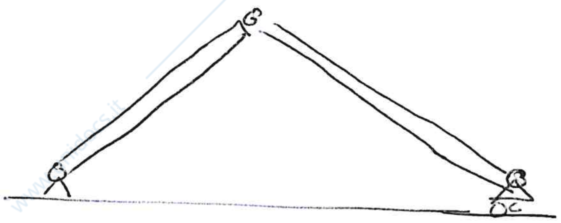
1 corpo è collegato solo al precedente, corpo

il telaio si parla di catena cinematica aperta

└ ROBOT



$\boxed{\alpha \text{ e } \beta \text{ indipendenti}}$

:) 

$3 \times 2 = 6$ gdL

$2 \times 2 = 4$ gdV

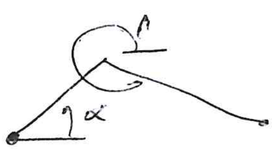
$1 \times 1 = 1$ gdV

1 gdL resta

$$\begin{cases} x_{A_1} = 0 \\ y_{A_1} = 0 \end{cases} \qquad \begin{cases} x_{B1} = x_{A2} \\ y_{B_1} = y_{A2} \end{cases} \qquad \big| \, y_{B_2} = 0$$

Se il corpo finale si richiude su telaio o su altri corpi ⇒ catena cinematica chiusa

Meccanismi x trasformare movimento



β e α dipendenti

chiusura devo legare tra α e β

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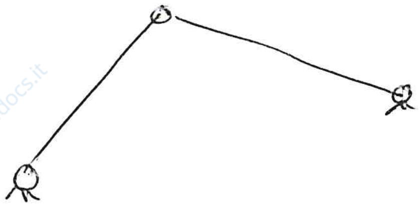
4) 

$\underline{2\times 3} = 6$ gdl

$\underline{3\times 2} = 6$ gdV

0 gdl

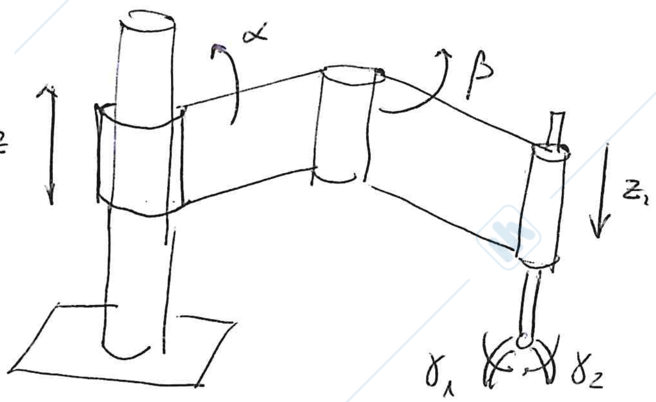
Strutture – isostatiche se gdl residui $= 0$

– iperstatiche se gdl residui < 0

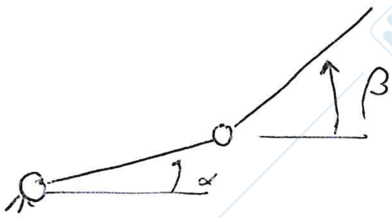
5) REGOLA DI GRÜBLER

$$\boxed{3\times n_{c.r} - 1\times n_{V_1} - 2\, n_{V_2} = n_{gdl\ residuo}}$$

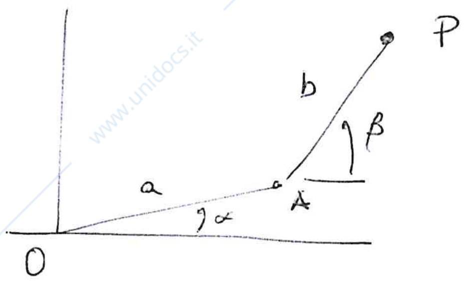
Due esempi



Vista dall'alto



MANIPOLATORE (R – R)



$$(P-O) = (A-O) + (P-A)$$

$$(P-O) = a\alpha^{i\alpha} + b e^{i\beta}$$

$$(P-O) = a e^{i\alpha} + b e^{i\beta}$$

$$\begin{cases} P_x = a\cos\alpha + b\cos\beta \\ P_y = a\sin\alpha + b\sin\beta \end{cases}$$

α, β spazio dei giunti → p_x e p_y posizione del end effector

$$V_p = i a \dot\alpha e^{i\alpha} + i b \dot\beta e^{i\beta}$$

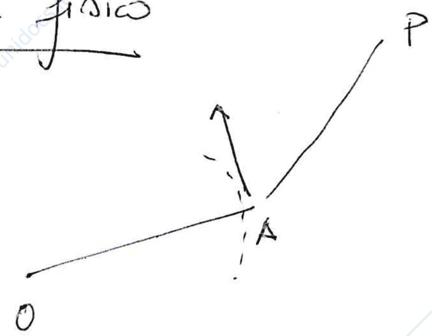
$$V_p = a \dot\alpha e^{i(\alpha+\frac{\pi}{2})} + b \dot\beta e^{i(\beta+\frac{\pi}{2})}$$

$$\begin{cases} V_x = -a\dot\alpha \operatorname{sen}\alpha - b\dot\beta \operatorname{sen}\beta \\ V_y = a\dot\alpha \cos\alpha + b\dot\beta\cos\beta \end{cases}$$

noto $\alpha(t) + \beta(t) \Rightarrow$ determinare V_p (in modulo e direzione oppure V_x e V_y

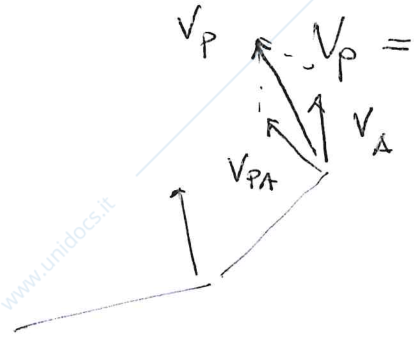
Osservazione

1) Signif. fisico



Vel A $\Rightarrow a\dot\alpha \perp$ a OA perchè appart a C.R. di rot

$V_p \Rightarrow$ Th di Rivals



$$\bar V_p = \bar V_A + \underbrace{\bar\omega_{PA} \wedge (P-A)}_{V_{PA}}$$

2)
$$\begin{vmatrix} V_x \\ V_y \end{vmatrix} = \underbrace{\begin{bmatrix} -a \operatorname{sen}\alpha & -b \operatorname{sen}\beta \\ a\cos\alpha & b\cos\beta \end{bmatrix}}_{} \begin{vmatrix} \dot\alpha \\ \dot\beta \end{vmatrix}$$

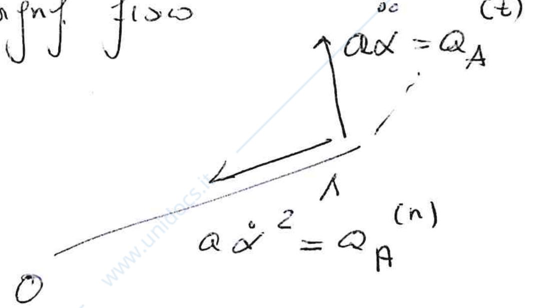
Matrice Jacobiana che lega le generiche coordinate V_x V_y alle coordin. libere $\dot\alpha$ e $\dot\beta$

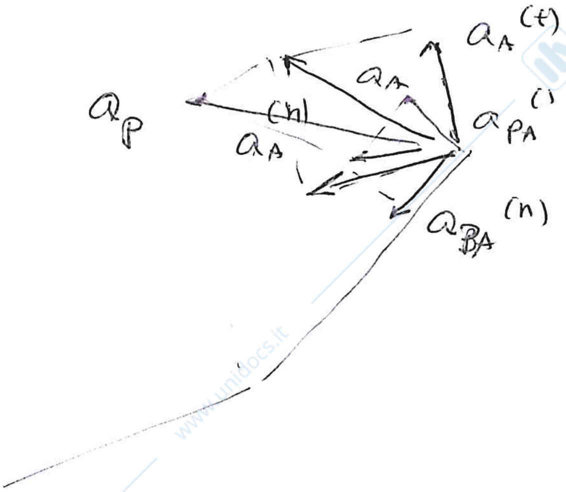
$$a_P = i a \ddot{\alpha} e^{i\alpha} - a \dot{\alpha}^2 + i b \ddot{\beta} e^{i\beta} - b \dot{\beta}^2 e^{i\beta}$$

$$\begin{cases} a_x = -a\ddot{\alpha}\,\mathrm{sen}\,\alpha - a\dot{\alpha}^2\cos\alpha - b\ddot{\beta}\,\mathrm{sen}\,\beta - b\dot{\beta}^2\cos\beta \\ a_y = +a\ddot{\alpha}\cos\alpha - a\dot{\alpha}^2\,\mathrm{sen}\,\alpha + b\ddot{\beta}\cos\beta - b\dot{\beta}^2\,\mathrm{sen}\,\beta \end{cases}$$

Oss.

Signf fisico



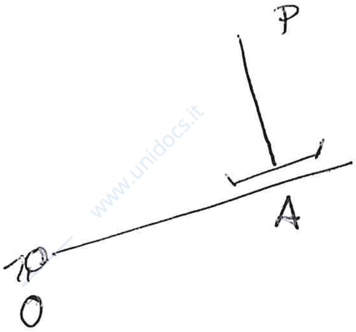


$$\vec{a}_P = \vec{a}_A + \vec{a}_{PA}$$

$$a_P = a_A^{(t)} + a_A^{(n)} + a_{PA}^{(t)} + a_{PA}^{(n)}$$

Matric

$$\begin{vmatrix} a_x \\ a_y \end{vmatrix} = \underbrace{[\Lambda(\alpha,\beta)] \begin{vmatrix} \ddot{\alpha} \\ \ddot{\beta} \end{vmatrix}}_{\text{effetto dell'acc di giunto}} + \underbrace{[\dot{\Lambda}(\alpha,\beta)] \begin{vmatrix} \dot{\alpha} \\ \dot{\beta} \end{vmatrix}}_{\text{effetto delle variaz di conf. del sist}}$$



accoppiam. a manicotto impone di

$$\beta = \alpha + \frac{\pi}{2}$$

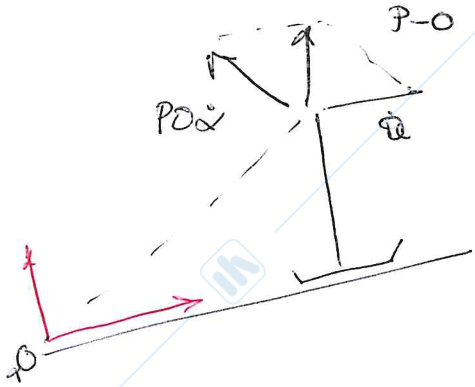
$$\dot\beta = \dot\alpha$$

$$(P-O) = a e^{i\alpha} + b e^{i\beta}$$

$$V_P = \dot a e^{i\alpha} + i a \dot\alpha e^{i\alpha} + b i \dot\beta e^{i\beta} \qquad \dot\beta = \dot\alpha$$

$$V_P = \dot a e^{i\alpha} + i \dot\alpha \left(\underbrace{a e^{i\alpha} + b e^{i\beta}}_{P-O}\right)$$

Signif. fisico



$$\bar V_P = \bar V_T + \bar V_R$$

$$V_P = \bar\omega \wedge (\overline{P-O}) + V_\rho$$

$$a_P = \ddot a e^{i\alpha} + 2 i \dot a \dot\alpha e^{i\alpha} + i a \ddot\alpha e^{i\alpha} - a \dot\alpha^2 e^{i\alpha} + b i \ddot\beta e^{i\beta} - b \dot\beta^2 e^{i\beta}$$

$$a_P = \underbrace{\ddot a e^{i\alpha}}_{V_{REL}} + \underbrace{2 \dot a \dot\alpha e^{i(\alpha + \frac{\pi}{2})}}_{V_{COR.}} + i \ddot\alpha \overbrace{\left(a e^{i\alpha} + b e^{i\beta}\right)}^{V_{TRAS}^{(t)}} - \underbrace{i \dot\alpha^2 \left(a e^{i\alpha} + b e^{i\beta}\right)}_{V_{TRAS}^{(n)}}$$

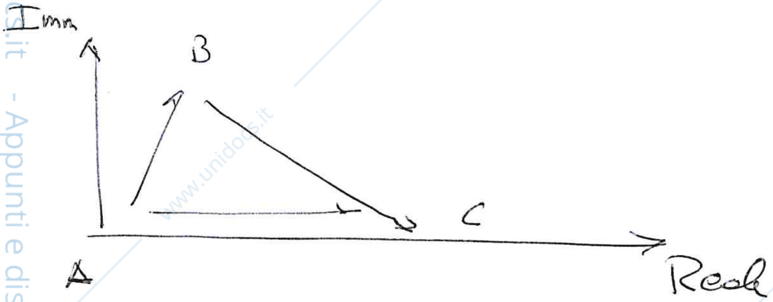
×

CATENA CINEMATICA CHIUSA

MANOVELL. ORD. CENTRATO : Moto rotatorio in traslatorio alternato rettilineo

Mot. C.I - Pompe - Compressori

Tutte le grandezze cinematiche dipendono da $\underline{\underline{\alpha(t)}}$



$$(C-A) = (B-A) + (C-B)$$

$$c = a e^{i\alpha} + b e^{i\beta}$$

Costanti = a b e γ

Variabili := α, β, c

$$\begin{cases} c = a \cos\alpha + b\cos\beta \\ 0 = a \operatorname{sen}\alpha + b \operatorname{sen}\beta \end{cases} \Rightarrow$$

Noto $\alpha(t)$

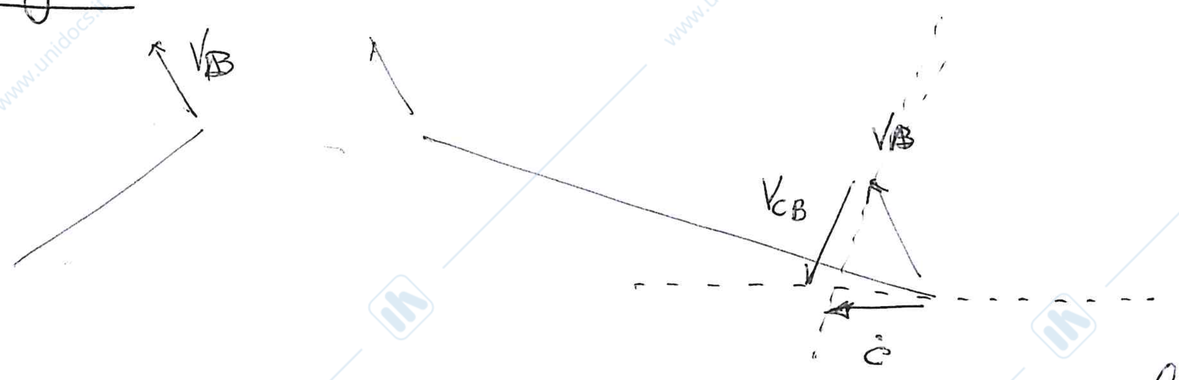
$$\begin{cases} \operatorname{sen}\beta = -\dfrac{a}{b} \operatorname{sen}\alpha \\ c = a\cos\alpha + b\sqrt{1 - \dfrac{a^2}{b^2}\operatorname{sen}^2\alpha} \end{cases}$$

VELOCITÀ

$$\dot{c} = a i \dot{\alpha} e^{i\alpha} + b i \dot{\beta} e^{i\beta}$$

$$\begin{cases} \dot{c} = -a\dot{\alpha}\operatorname{sen}\alpha - b\dot{\beta}\operatorname{sen}\beta \\ 0 = a\dot{\alpha}\cos\alpha + b\dot{\beta}\cos\beta \end{cases}$$

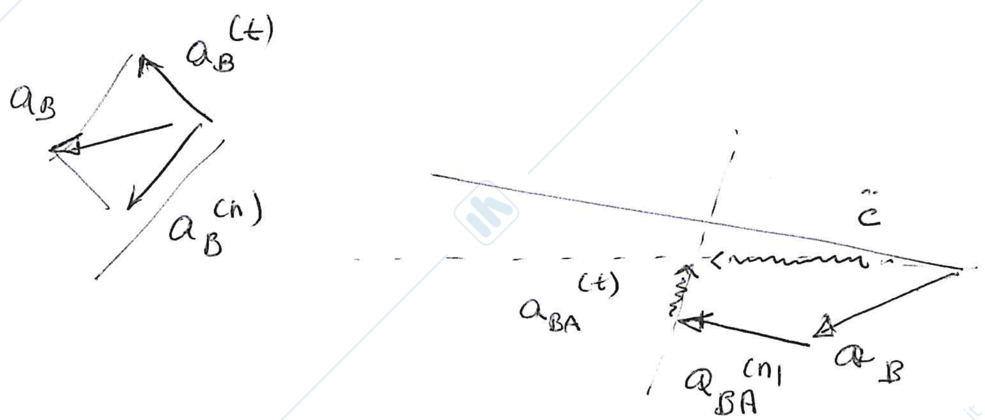
Sign. fisico



β non è più libero ma vincolato al corsoio

Accelerazione

$$\ddot{c} = i a \ddot{\alpha} e^{i\alpha} - a \dot{\alpha}^2 e^{i\alpha} + i b \ddot{\beta} e^{i\beta} - b \dot{\beta}^2 e^{i\beta}$$



OSSERVAZ

$$c(\alpha) = a\cos\alpha + b\sqrt{1 - \frac{a^2}{b^2}\operatorname{sen}^2\alpha}$$

$$\sqrt{1+\varepsilon} \cong 1 + \frac{\varepsilon}{2}$$

$$c(\alpha) \cong a\cos\alpha + b - \frac{b}{2}\frac{a^2}{b^2}\operatorname{sen}^2\alpha$$

$$c(\alpha) \cong b + a\cos\alpha - \frac{a^2}{2b}\operatorname{sen}^2\alpha$$

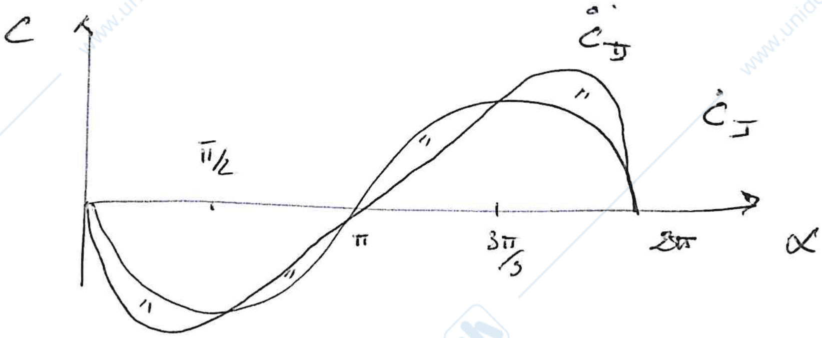
$$\dot c(\alpha) = -a\,\dot\alpha\,\operatorname{sen}\alpha - a\,\frac{a}{b}\,\operatorname{sen}\alpha\,\cos\alpha\,\dot\alpha$$

se $\frac{a^2}{b^2} = \varepsilon$ $\quad \frac{a}{b}$ piccolo rapporto tra manovella/biella

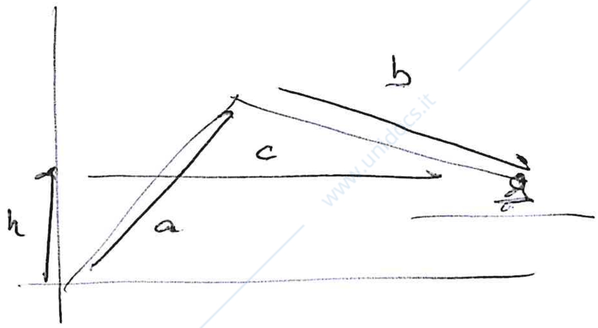
$\frac{a}{b} \ll 1 \qquad \dot c = -a\,\dot\alpha\,\operatorname{sen}\alpha$

$\frac{a}{b} < 1 \qquad \dot c = \left(-a\,\operatorname{sen}\alpha - \frac{a}{2}\,\frac{a}{b}\,\operatorname{sen}2\alpha\right)\dot\alpha$ I approx

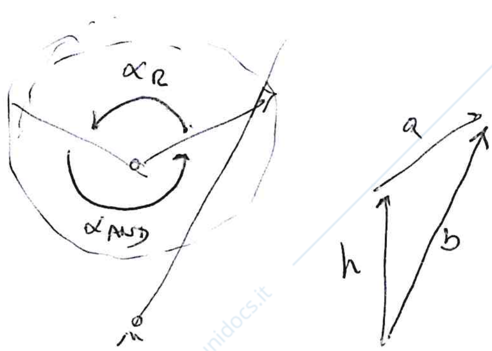
$\dot c = -a\left(\operatorname{sen}\alpha + \frac{\lambda}{2}\,\operatorname{sen}2\alpha\right)\dot\alpha$ II approx



Momen. der.



Glifo



Quadrilatero