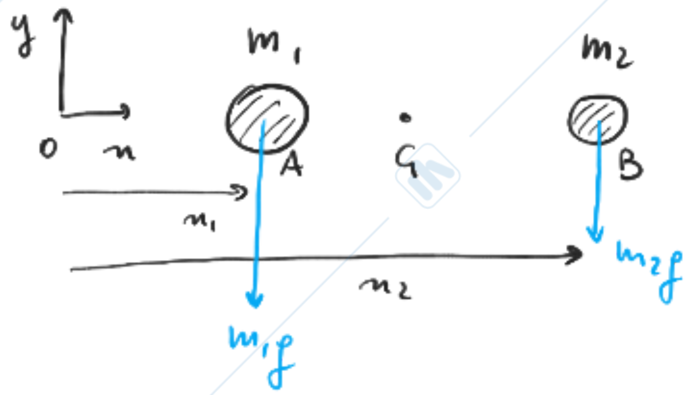
Geometria delle masse

Baricentro e centro di massa



G punto t.c. $\sum M_G = 0$

$$(A-G)\wedge(m_1\vec{g}) + (B-G)\wedge(m_2\vec{g}) = 0$$

$$-(x_1-x_G)\,m_1\cancel{g} - (x_2-x_G)\,m_2\cancel{g} = 0$$

$$x_G(m_1+m_2) - x_1 m_1 - x_2 m_2 = 0$$

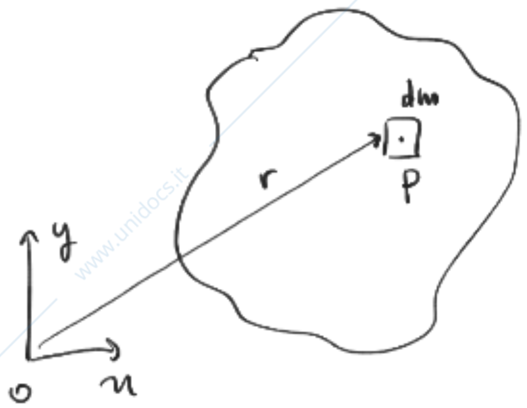
$$x_G = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

$$x_G = \frac{\sum m_i x_i}{\sum m_i}$$

Momento d'inerzia

$$J = m_1 x_1^2 + m_2 x_2^2 = \sum m_i r_i^2$$

Caso continuo



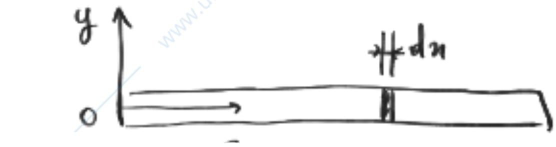
$$x_G = \frac{1}{\int_M dm}\cdot\int_M x_P\,dm$$

$$dm = \rho\,dV = \rho s\,dA = \rho s\,dx\,dy$$

$$x_G = \frac{1}{M}\int_A \rho s\,x_P\,dx\,dy$$

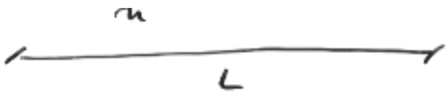
$$J_O = \int_M r^2\,dm = \int_A r^2 \rho s\,dx\,dy$$

Aste



Baricentro?

$$x_G = \frac{1}{M}\int^{L} x\,\rho A\,dx$$

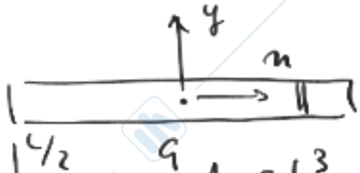


$dm = \rho A\, dx$

$M = \rho A L$

$$= \frac{1}{M}\rho A \frac{x^2}{2}\Big|_0^L = \frac{\rho A L^2}{2M} = \frac{ML}{2\cancel{M}}$$

$$= L/2$$



Momento d'inerzia O, G

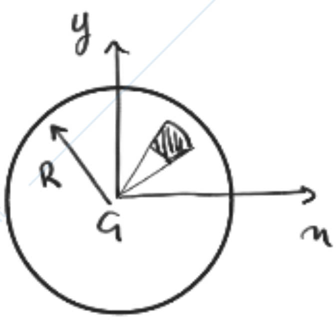
$$J_G = \int_M r^2 dm = \int_{-L/2}^{L/2} x^2 \rho A\, dx = \rho A \frac{x^3}{3}\Big|_{-L/2}^{L/2} = \frac{\rho A}{3}\frac{2L^3}{8}$$

$$= \frac{\rho A L^3}{12} = \frac{ML^2}{12}$$

$$J_O = \int_M r^2 dm = \int_0^L x^2 \rho A\, dx = \rho A \frac{x^3}{3}\Big|_0^L = \frac{\rho A L^3}{3} = \frac{ML^2}{3}$$

$$J_O = J_G + M\left(\frac{L}{2}\right)^2 = \frac{ML^2}{12} + \frac{ML^2}{4} = \frac{ML^2}{3}$$

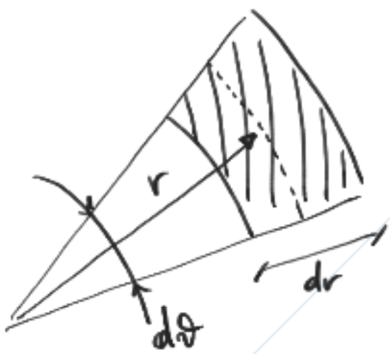
Disco



$$J_G = \int_M r^2 dm = \int_0^R\int_0^{2\pi} \rho s\, r^3\, dr\, d\vartheta$$

$$= \rho s \frac{r^4}{4}\Big|_0^R 2\pi$$

$$= \frac{\rho s R^4 \pi}{2}$$



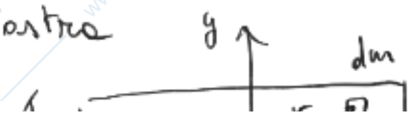
$dA = r\, d\vartheta\, dr$

$dm = \rho s\, dA = \rho s\, r\, d\vartheta\, dr$

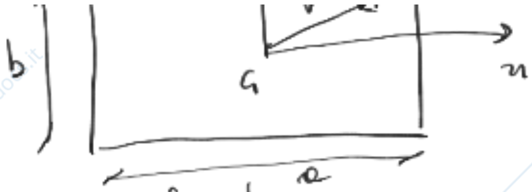
$$M = \pi R^2 \rho s$$

$$J_G = \frac{1}{2} M R^2$$

Piastra



$J_G = \int r^2 dm$



J_M

$dm = \rho s \, dx \, dy$

$r^2 = x^2 + y^2$

$$J_G = \int_{-a/2}^{a/2} \int_{-b/2}^{b/2} \rho s (x^2+y^2) \, dx \, dy = \rho s \iint x^2 + y^2 \, dx \, dy$$

$$= \rho s \int_{-a/2}^{a/2} \left(x^2 y + \frac{y^3}{3} \right) \Big|_{-b/2}^{b/2} dx$$

$$= \rho s \int_{-a}^{a} \left(x^2 (b) + \frac{b^3}{12} \right) dx$$

$$= \rho s \left(\frac{x^3}{3} b + \frac{b^3}{12} x \right) \Big|_{-a/2}^{a/2}$$

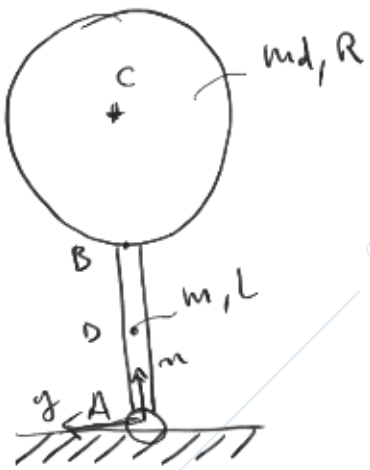
$$= \rho s \left(\frac{a^3 b}{12} + \frac{b^3 a}{12} \right)$$

$$= \rho s \frac{ab}{12} (a^2 + b^2)$$

$$= \frac{M}{12} (a^2 + b^2)$$

$M = \rho s A = \rho s a b$

Sistemi composti



$$J_A = J_d + m_d \overline{CA}^2 + J_a + m \left(\frac{L}{2} \right)^2$$

$J_d = \frac{1}{2} m_d R^2 \qquad \overline{CA} = L + R$

$J_a = \frac{m}{12} L^2$

$$J_A = \frac{1}{2} m_d R^2 + m_d (L+R)^2 + \frac{m}{12} L^2 + m \frac{L^2}{4}$$

Baricentro?

$$x_G = \frac{x_D m + x_C m_d}{m + m_d} = \frac{L/2 \, m + (L+R) m_d}{m + m_d}$$

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