Ese 08

$J = \frac{1}{2} MR^2$

Calcolo del moto noto C

1 corpo rigido	3 gdl
1 contatto	1 gdlV
1 rotol. senza strisc.	1 gdlV
	1 gdl

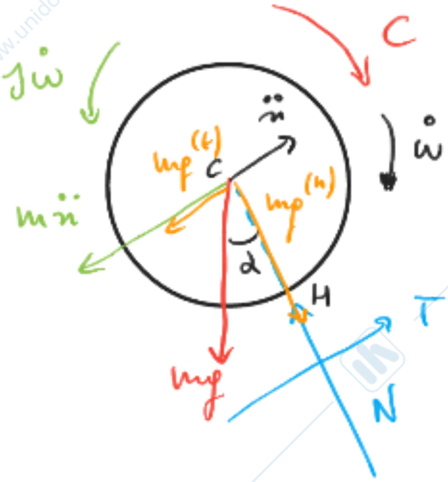
$\downarrow$

$x, \dot{x}, \ddot{x}$

$\dot{x} = \omega R$

$\ddot{x} = \dot{\omega} R$

Equilibri dinamici



$\begin{cases} \Sigma F_x = 0 \\ \Sigma F_y = 0 \end{cases}$ oppure $\begin{cases} \Sigma F_{\vec{n}} = 0 \\ \Sigma F_{\vec{t}} = 0 \end{cases}$

$\Sigma F_{\vec{t}} = 0 \qquad T - m\ddot{x} - \underbrace{mg \sin\alpha}_{mg^{(t)}} = 0 \qquad (1)$

$\Sigma F_{\vec{n}} = 0 \qquad N - \underbrace{mg \cos\alpha}_{mg^{(n)}} = 0 \qquad (2)$

$\Sigma M_C = 0 \qquad C - J\dot{\omega} - TR = 0 \qquad (3)$

oppure

$\Sigma M_H = 0 \qquad C - J\dot{\omega} - m\ddot{x}R - mg \sin\alpha R = 0 \qquad (4)$

$(1), (2), (3) \rightarrow T, N, \ddot{x}$

$(4) \rightarrow \ddot{x}$

$$C - J\frac{\ddot{x}}{R} - m\ddot{x}R - mg\sin\alpha R = 0$$

$$\frac{C}{R} - mg\sin\alpha = \ddot{x}\left(m + J/R^2\right)$$

$$\ddot{x} = \frac{C/R - mg\sin\alpha}{(m + J/R^2)}$$

$\uparrow$ $[m/s^2]$

m_{eq}

$\to kg + \frac{kg\, m^2}{m^2} = kg$

C/R $\quad$ N

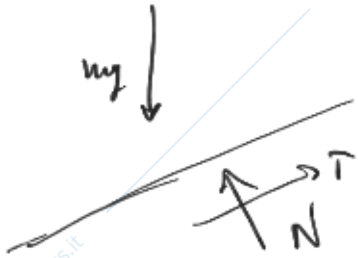
$\uparrow$ F_m

mg $\quad$ N

$\uparrow$ F_{res}

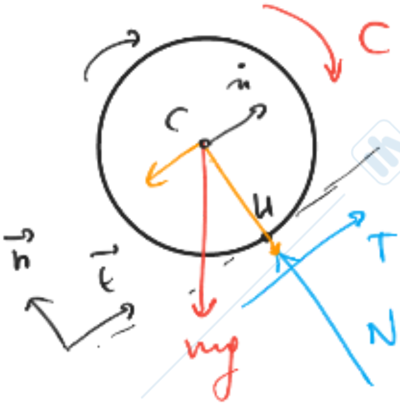
$$\dot{\omega} = \frac{\ddot{x}}{R}$$

$N = mg\cos\alpha$



$T = m\ddot{x} + mg\sin\alpha$

Bilancio di Potenze, th E_c $\qquad \sum W = \frac{dE_c}{dt}$



$$\sum W = m\vec{g}\cdot\vec{v}_c + \vec{C}\cdot\vec{\omega} + \cancel{\vec{N}\cdot\vec{v}_H} + \cancel{\vec{T}\cdot\vec{v}_H}$$

$\vec{v}_H = 0$

$$\sum W = (-mg\sin\alpha\,\vec{t} - mg\cos\alpha\,\vec{n})\cdot\dot{x}\,\vec{t} + (-C\vec{k})\cdot\left(-\frac{\dot{x}}{R}\vec{k}\right)$$

$$= -mg\sin\alpha\,\dot{x} + \frac{C}{R}\dot{x}$$

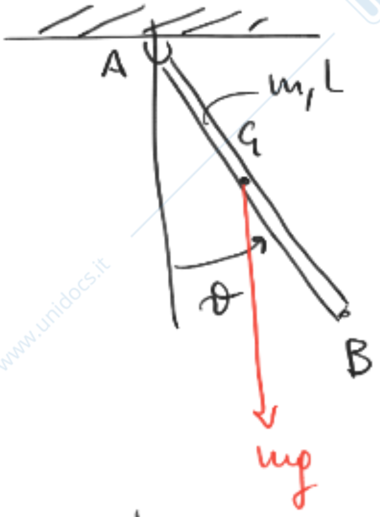
$$E_c = \left(\frac{1}{2}m v_{ct}^2 + \cancel{\frac{1}{2}m v_{cn}^2} + \frac{1}{2}J\omega^2\right) \qquad v_c = \dot{x}, \quad \omega = \dot{x}/R$$

$$= \frac{1}{2}m\dot{x}^2 + \frac{1}{2}J\left(\frac{\dot{x}}{R}\right)^2$$

$$= \frac{1}{2}\left(m + J/R^2\right)\dot{x}^2$$

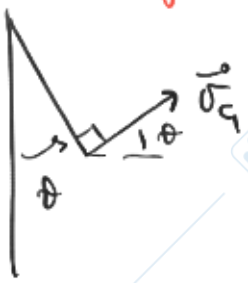
$$\sum W = \frac{dE_c}{dt} \qquad \frac{C}{R}\cancel{\dot{x}} - mg\sin\alpha\,\cancel{\dot{x}} = \left(m + \frac{J}{R^2}\right)\ddot{x}\,\cancel{\dot{x}}$$

$$\ddot{x} = \frac{C/R - mg\sin\alpha}{m + J/R^2}$$



$$\Sigma W = \frac{dE_c}{dt}$$

$$m\vec{g}\cdot\vec{v}_G = \frac{d}{dt}\left(\frac{1}{2} m v_{Gx}^2 + \frac{1}{2} m v_{Gy}^2 + \frac{1}{2} J_G \omega^2\right)$$



$$\vec{v}_G = \vec{\omega} \wedge (G-A)$$
$$= \dot{\vartheta}\,\frac{L}{2}\left(\cos\vartheta\,\vec{i} + \sin\vartheta\,\vec{j}\right)$$

$$\begin{cases} v_{Gx} = \dot{\vartheta}\,\frac{L}{2}\cos\vartheta \\ v_{Gy} = \dot{\vartheta}\,\frac{L}{2}\sin\vartheta \end{cases}$$

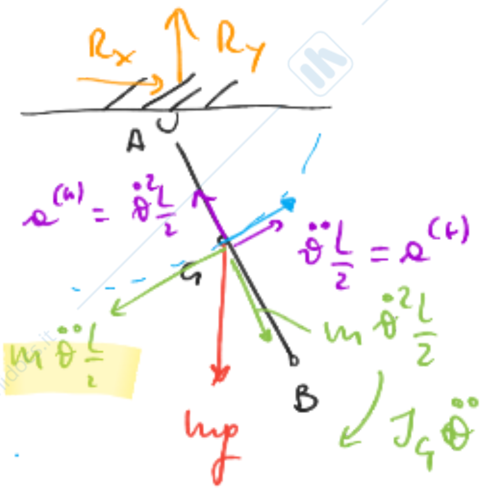
$$W_{(mg)} = \left(-mg\,\vec{j}\right)\cdot\left(v_{Gx}\,\vec{i} + v_{Gy}\,\vec{j}\right)$$
$$= -mg\,v_{Gy} = -mg\,\frac{L}{2}\,\dot{\vartheta}\sin\vartheta$$

$$E_c = \frac{1}{2} m v_{Gx}^2 + \frac{1}{2} m v_{Gy}^2 + \frac{1}{2} J_G \omega^2$$
$$= \frac{1}{2} m\left(\dot{\vartheta}^2 \frac{L^2}{4}\cos^2\vartheta + \dot{\vartheta}^2 \frac{L^2}{4}\sin^2\vartheta\right) + \frac{1}{2} J_G \dot{\vartheta}^2$$
$$= \frac{1}{2} m \frac{L^2}{4}\dot{\vartheta}^2 + \frac{1}{2} J_G \dot{\vartheta}^2$$
$$= \frac{1}{2}\underbrace{\left(m\frac{L^2}{4} + J_G\right)}_{J_A}\dot{\vartheta}^2$$

$$J_A = J_G + m\left(d_{AG}\right)^2$$
$$= J_G + m\frac{L^2}{4}$$

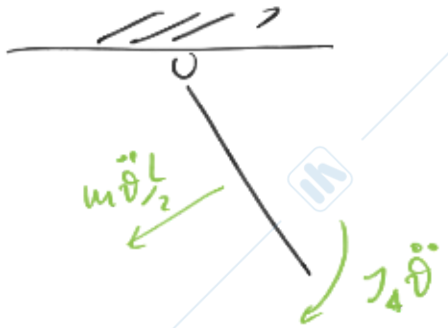
$$-mg \sin\vartheta \frac{L}{2} \cancel{\dot\vartheta} = \left(m\frac{L^2}{4} + J_G\right)\ddot\vartheta \cancel{\dot\vartheta}$$

$$\ddot\vartheta = -\frac{mg\frac{L}{2}\sin\vartheta}{J_G + mL^2/4} = -\frac{mg\, L/2 \sin\vartheta}{J_A}$$



$$\Sigma M_A = 0 \rightarrow -mg\frac{L}{2}\sin\vartheta + - m\ddot\vartheta\frac{L}{2}\frac{L}{2} - J_G\ddot\vartheta = 0$$

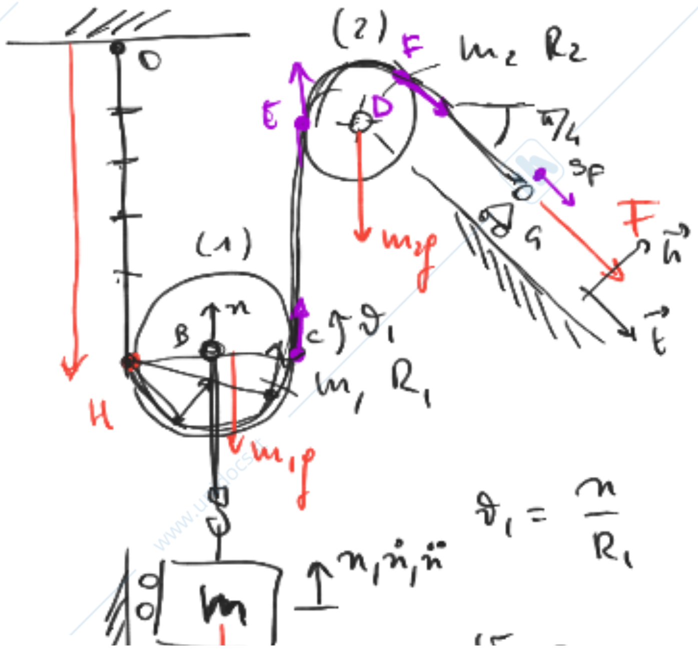
$$-mg\frac{L}{2}\sin\vartheta = \underbrace{\left(m\frac{L^2}{4} + J_G\right)}_{J_A}\ddot\vartheta$$



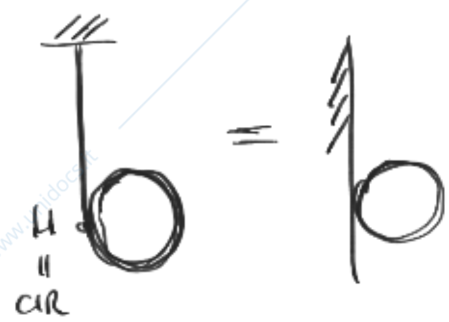
$$\Sigma F_x = 0 \quad R_x - m\ddot\vartheta\frac{L}{2}\cos\vartheta + + m\dot\vartheta^2\frac{L}{2}\sin\vartheta = 0$$

$$R_x = m\ddot\vartheta\frac{L}{2}\cos\vartheta + - m\dot\vartheta^2\frac{L}{2}\sin\vartheta$$

$$\Sigma F_y = 0 \quad R_y - mg - m\ddot\vartheta\frac{L}{2}\sin\vartheta + - m\dot\vartheta^2\frac{L}{2}\cos\vartheta = 0$$



1) Statiche → calcolo F
2) Nota F(t) → calcolo movimento



1) $\downarrow mg$

$v_H = 0$ $\qquad$ $v_E = v_C = 2\dot{x}$

$v_B = \dot{\theta}_1 R_1 = \dot{x}$ $\qquad$ $v_E = -\dot{\theta}_2 R_2$

$v_C = \dot{\theta}_1 2R_1 = 2\dot{x}$ $\qquad$ $\dot{\theta}_2 = -\frac{v_E}{R_2} = -\frac{2\dot{x}}{R_2}$

$v_F = v_E = 2\dot{x}$

$$\delta L = m\vec{g}\cdot\delta\vec{y}_m + m_1\vec{g}\cdot\delta\vec{y}_B + \cancel{m_2\vec{g}\cdot\delta\vec{y}_D}^{\,0} + \vec{F}\cdot\delta\vec{s}_F$$

$\delta\vec{y}_m = \delta x\,\vec{j}$ $\qquad$ $-mg\,\vec{j}\cdot\delta x\,\vec{j} = -mg\,\delta x$

$\delta\vec{y}_B = \delta x\,\vec{j}$ $\qquad$ $-m_1 g\,\vec{j}\cdot\delta x\,\vec{j} = -m_1 g\,\delta x$

$$\delta\vec{s}_F = 2\frac{\partial \vec{s}_F}{\partial x}\delta x = 2\,\delta x\,\vec{t}$$

$v_F = \boxed{2}\,\dot{x}$

$$\boxed{\frac{\partial v_F}{\partial \dot{x}} = \frac{\partial s_F}{\partial x} = 2}$$

$$\vec{F}\cdot\delta\vec{s}_F = F\vec{t}\cdot 2\,\delta x\,\vec{t} = 2F\,\delta x$$

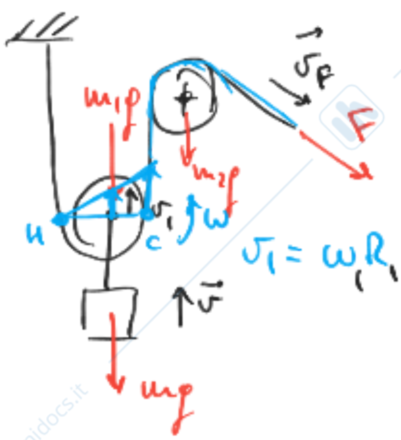
$\delta L = 0$ $\qquad$ $-(m+m_1)g\,\cancel{\delta x} + 2F\,\cancel{\delta x} = 0$

$$F = \frac{(m+m_1)g}{2}$$

Forza equilibrio statico è costante non dipende da x

2) $\qquad \sum W = \frac{dE_c}{dt}$

$$m\vec{g}\cdot\vec{v} + m_1\vec{g}\cdot\vec{v}_1 + \vec{F}\cdot\vec{v}_F = \sum W$$



$v = v_1 = \dot{x}$

$v_F = 2v_1 = 2\dot{x}$

1) $\quad -mg\dot{x} - m_1 g\dot{x} + F\cdot 2\dot{x} = \sum W$

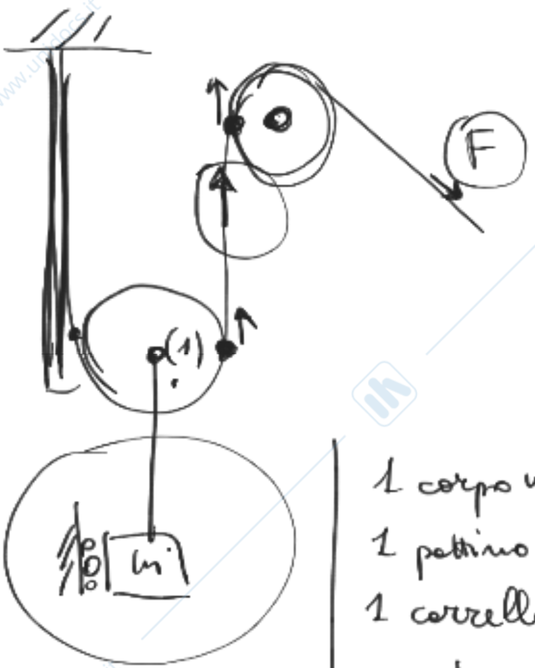
$$E_c = \frac{1}{2}mv^2 + \frac{1}{2}m_1 v_1^2 + \frac{1}{2}J_1\omega_1^2 + \frac{1}{2}J_2\omega_2^2 \qquad \omega_1 = \frac{\dot{x}}{R_1}$$

$$\omega_2 = \frac{v_F}{R_2} = \frac{2\dot{x}}{R_2}$$

2) $$= \frac{1}{2}\left(m + m_1 + \frac{J_1}{R_1^2} + \frac{4J_2}{R_2^2}\right)\dot{x}^2$$

$$-(m+m_1)g\cancel{\dot{x}} + F2\cancel{\dot{x}} = \left(m + m_1 + \frac{J_1}{R_1^2} + \frac{4J_2}{R_2^2}\right)\ddot{x}\cancel{\dot{x}}$$

$$\ddot{x} = \frac{2F - (m+m_1)g}{m + m_1 + \frac{J_1}{R_1^2} + \frac{4J_2}{R_2^2}}$$



1 corpo m 3 pdl
1 pattino −2 pdv
1 carrello $m - m_1$ → 1 pdv
1 rot senza strisc. → 1 pdv
1 contatto → 1 pdv
1 corpo (1) 3pdl
1 corpo (2) 3pdl
1 cerniera (2) 2pdv
1 fune 1 pdv

9 pdl
8 pdv
1pdl