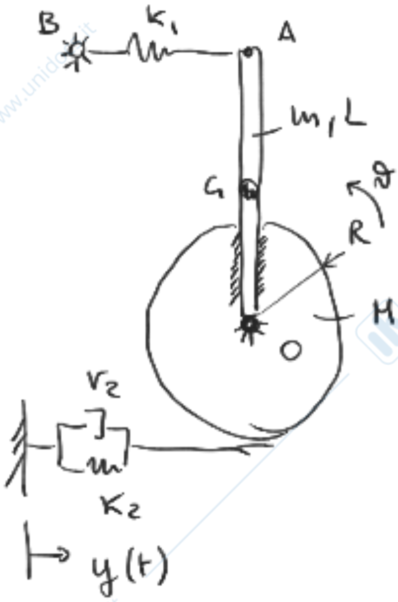
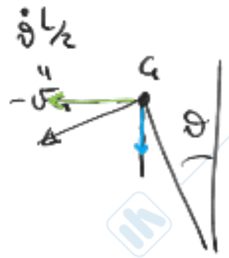
Ese22



$$E_c = \frac{1}{2} m v_{Gx}^2 + \frac{1}{2} m v_{Gy}^2 + \frac{1}{2} J_G \omega^2 + \frac{1}{2} J_O \omega_O^2$$

$$\omega = \dot\vartheta$$
$$\omega_O = \dot\vartheta$$



$$v_{Gx} = -\dot\vartheta \frac{L}{2} \cos\vartheta$$

$$v_{Gy} = -\dot\vartheta \frac{L}{2} \sin\vartheta$$

$$\frac{1}{2} m v_{Gx}^2 + \frac{1}{2} m v_{Gy}^2 = \frac{1}{2} m \left(v_{Gx}^2 + v_{Gy}^2\right)$$
$$= \frac{1}{2} m \left(\left(\dot\vartheta \frac{L}{2}\right)^2 \cos^2\vartheta + \left(\dot\vartheta \frac{L}{2}\right)^2 \sin^2\vartheta\right)$$
$$= \frac{1}{2} m \left(\dot\vartheta \frac{L}{2}\right)^2$$

$$E_c = \frac{1}{2}\left(\boxed{m\left(\frac{L}{2}\right)^2 + J_G} + J_O\right)\dot\vartheta^2 \qquad J_G = \frac{mL^2}{12} \qquad J_O = \frac{1}{2} M R^2$$

$$= \frac{1}{2} J^* \dot\vartheta^2 \qquad J^* \text{ costante}$$

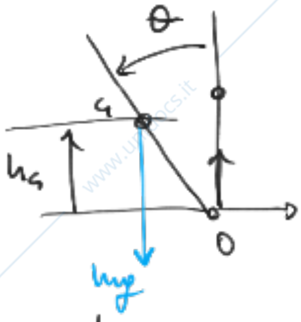
$$\boxed{J_G + m\frac{L^2}{4} = J_O}$$

$$\frac{d}{dt}\frac{\partial E_c}{\partial \dot\vartheta} - \frac{\partial E_c}{\partial \vartheta} = J^* \ddot\vartheta$$

$$V = V_g + V_K$$

$$V_g = mg\, h_G \qquad h_G = \frac{L}{2}\cos\vartheta$$

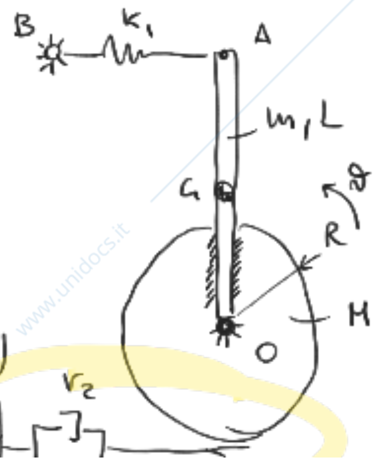
$$K_{KO} = mg \left.\frac{\partial^2 h_G}{\partial \vartheta^2}\right|_{\vartheta_0}$$



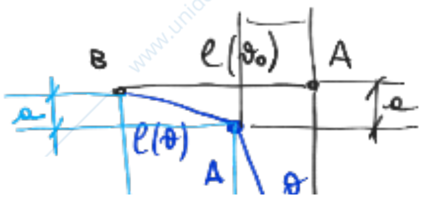
$$\frac{\partial h_G}{\partial \vartheta} = -\frac{L}{2}\sin\vartheta \qquad \frac{\partial^2 h_G}{\partial \vartheta^2} = -\frac{L}{2}\cos\vartheta \qquad \left.\frac{\partial^2 h_G}{\partial \vartheta^2}\right|_{\vartheta=0} = -\frac{L}{2}$$

$$K_{KO} = -mg\frac{L}{2} < 0 \quad \text{DESTABILIZZANTE}$$

$$V_K = \underline{\frac{1}{2} k_1 \Delta\ell_1^2} + \underline{\frac{1}{2} k_2 \Delta\ell_2^2}$$



$$\Delta\ell_1 = \Delta\ell_{10} + \Delta\ell_{1d}$$



$$\Delta\ell_d = \underbrace{\ell(\vartheta_0)}_{\text{costante}} - \ell(\vartheta)$$

$$\ell(\theta) = \sqrt{a^2 + b^2} = \sqrt{(L - L\cos\theta)^2 + (\ell(\theta_0) - L\sin\theta)^2}$$

$$K_{K1}^{(1)} = k_1 \left(\frac{\partial \Delta\ell_{1d}}{\partial\theta}\Big|_0 \right)^2$$

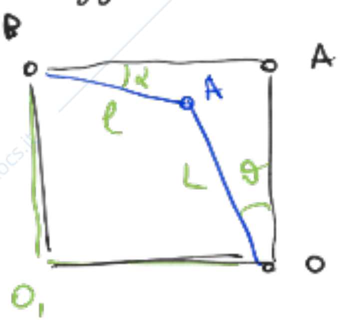
$$K_{K0}^{(1)} = k_1 \left(\frac{\partial^2 \Delta\ell_{1d}}{\partial\theta^2}\Big|_0 \right) \Delta\ell_{10}$$

$$\frac{\partial \Delta\ell_{1d}}{\partial\theta} = \frac{\partial}{\partial\theta}\big(\ell(\theta_0) - \ell(\theta)\big) = -\frac{\partial \ell(\theta)}{\partial\theta}$$

$$\frac{\partial \ell}{\partial\theta} = \frac{1}{2}\left[(L - L\cos\theta)^2 + (\ell(\theta_0) - L\sin\theta)^2\right]^{-1/2} \cdot$$
$$\cdot \left[2(L - L\cos\theta)(L\sin\theta) + 2(\ell(\theta_0) - L\sin\theta)(-L\cos\theta)\right]$$

$$\frac{\partial \ell}{\partial\theta}\Big|_{\theta=0} = \frac{1}{\cancel{2}}\left[\cancel{\ell(\theta_0)^2}\right]^{-1/2} \cdot \left[\cancel{2}\,\cancel{\ell(\theta_0)} \cdot (-L)\right]$$
$$= -L$$

$$\frac{\partial^2 \ell}{\partial\theta^2}$$



$$\begin{cases} \ell\cos\alpha + L\sin\theta = x_{0,0} \\ \ell\sin\alpha + L\cos\theta = y_{0,B} \end{cases} \quad (1)$$

$$\frac{d}{dt}\left(\begin{cases} \dot\ell\cos\alpha - \ell\dot\alpha\sin\alpha + L\dot\theta\cos\theta = 0 \\ \dot\ell\sin\alpha + \ell\dot\alpha\cos\alpha - L\dot\theta\sin\theta = 0 \end{cases}\right. \quad (2)$$

All'equilibrio $\theta = 0$ $\alpha = 0$

$$\begin{cases} \dot\ell + L\dot\theta = 0 \\ \ell\dot\alpha = 0 \end{cases} \qquad \begin{matrix} \dot\ell(\theta_0) \cong -L\dot\theta \\ \dot\alpha(\theta_0) = 0 \end{matrix}$$

$$\boxed{\dot\ell = \Lambda_\ell(\theta)\,\dot\theta} \qquad \Lambda_\ell(\theta_0) = -L = \frac{\partial \ell}{\partial\theta}\Big|_{\theta_0}$$

$$\frac{d}{dt}\big(eq(2)\big) \quad \begin{cases} \ddot\ell\cos\alpha - \dot\ell\dot\alpha\sin\alpha - \dot\ell\dot\alpha\sin\alpha - \ell\ddot\alpha\sin\alpha - \ell\dot\alpha^2\cos\alpha + \\ \quad + L\ddot\theta\cos\theta - L\dot\theta^2\sin\theta = 0 \\ \ddot\ell\sin\alpha + \dot\ell\dot\alpha\cos\alpha + \dot\ell\dot\alpha\cos\alpha + \ell\ddot\alpha\cos\alpha - \ell\dot\alpha^2\sin\alpha + \\ \quad - L\ddot\theta\sin\theta - L\dot\theta^2\cos\theta = 0 \end{cases}$$

All'equilibrio

$$\begin{cases} \ddot{\ell} - \ell\dot{\alpha}^2 + L\ddot{\vartheta} = 0 \\ 2\dot{\ell}\dot{\alpha} + \ell\ddot{\alpha} - L\dot{\vartheta}^2 = 0 \end{cases}$$

$\dot{\alpha} = 0 \quad (\vartheta = \vartheta_0)$

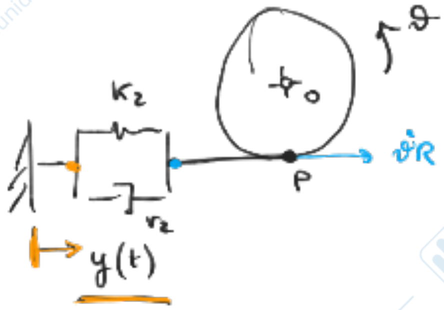
$$\begin{cases} \ddot{\ell} = -L\ddot{\vartheta} \\ \ddot{\alpha} = +\frac{L}{\ell}\dot{\vartheta}^2 \end{cases}$$

$$\ddot{\ell} = \Lambda_\ell(\vartheta_0)\ddot{\vartheta} + \underbrace{\left.\frac{\partial\Lambda}{\partial\vartheta}\right|_0}_{0}\dot{\vartheta}^2$$

$$\left.\frac{\partial\Lambda}{\partial\vartheta}\right|_0 = \left.\frac{\partial^2\ell}{\partial\vartheta^2}\right|_0 = 0$$

$k_{k0}^{(1)} = 0$

$$k_{k_1}^{(1)} = k_1 L^2$$



$$\Delta\ell_2 = \Delta\ell_{20} + \Delta\ell_{2d}$$

$$\Delta\ell_{2d} = R\vartheta - y$$

$$V_{k_2} = \frac{1}{2}k_2(\Delta\ell_{20} + R\vartheta - y)^2$$

$$\frac{\partial V_{k_2}}{\partial\vartheta} = k_2(\Delta\ell_{20} + R\vartheta - y)R$$

$$= \underbrace{\cancel{k_2 R\Delta\ell_{20}}}_{\text{costante}} + k_2R^2\vartheta - k_2Ry$$

$$D = \frac{1}{2}r_2\Delta\dot{\ell}_2^2 \qquad \Delta\dot{\ell}_2 = R\dot{\vartheta} - \dot{y}$$

$$\frac{\partial D}{\partial\dot{\vartheta}} = r_2(R\dot{\vartheta} - \dot{y})R = r_2R^2\dot{\vartheta} - r_2R\dot{y}$$

$$\boxed{r_1^* = r_1\left(\left.\frac{\partial\Delta\ell_{d1}}{\partial\vartheta}\right|_0\right)^2 = r_1L^2}$$

k_g

$$J^*\ddot{\vartheta} + r_2R^2\dot{\vartheta} - r_2R\dot{y} + \left(-mg\frac{L}{2} + k_1L^2 + k_2R^2\right)\vartheta - k_2Ry = 0$$

$$J^*\ddot{\vartheta} + r_2R^2\dot{\vartheta} + \left(-mg\frac{L}{2} + k_1L^2 + k_2R^2\right)\vartheta = r_2R\dot{y} + k_2Ry$$

$$J^*\ddot{\vartheta} + r^*\dot{\vartheta} + k^*\vartheta = r_y\dot{y} + k_y y$$

$$y(t) = Y_0e^{i\Omega t} \qquad \dot{y} = i\Omega Y_0 e^{i\Omega t}$$

$$J\ddot{\vartheta} + r\dot{\vartheta} + k\vartheta = (r_y i\Omega + k_y)\, Y_0\, e^{i\Omega t}$$

$$\vartheta = \vartheta_{OA} + \vartheta_P$$

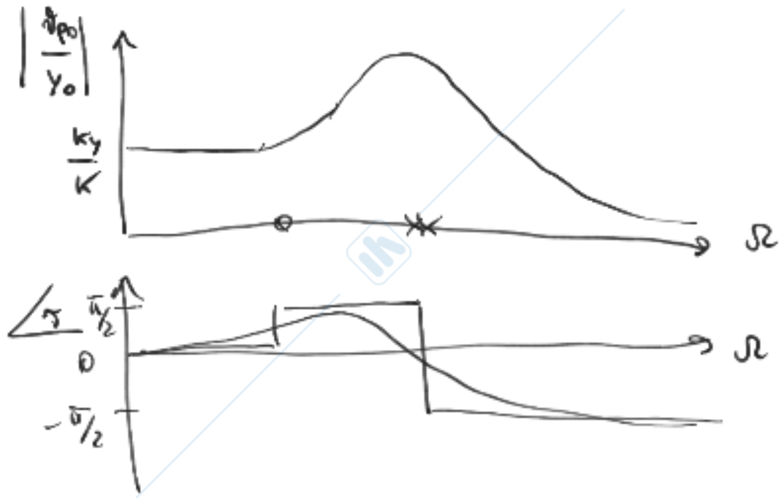
$$\vartheta_P = \vartheta_{P0}\, e^{i\Omega t} \qquad \dot{\vartheta}_P = i\Omega\vartheta_{P0} e^{i\Omega t} \qquad \ddot{\vartheta}_P = -\Omega^2 \vartheta_{P0} e^{i\Omega t}$$

$$(-J\Omega^2 + i\Omega r + k)\, \vartheta_{P0}\, \cancel{e^{i\Omega t}} = (r_y i\Omega + k_y)\, Y_0\, \cancel{e^{i\Omega t}}$$

$$\vartheta_{P0} = \frac{r_y i\Omega + k_y}{-J\Omega^2 + i\Omega r + k}\, Y_0$$

$$FRF = \frac{\vartheta_{P0}}{Y_0} = \frac{i\Omega r_y + k_y}{k - J\Omega^2 + i\Omega r} \qquad TF = \frac{\vartheta_P(s)}{Y(s)} = \frac{r_y s + k_y}{Js^2 + rs + k}$$

$$s = i\Omega$$



$$\vartheta_{P0}(\Omega \to 0) = \frac{k_y Y_0}{K}$$

$$\left|\frac{\vartheta_{P0}}{Y_0}\right|(\Omega \to 0) = \frac{k_y}{K}$$

$$\mu = \frac{k_y}{K}$$

$$\ell(\vartheta) \qquad \dot{\ell}(\vartheta) = \frac{d\,\ell(\vartheta)}{dt} = \frac{\partial \ell(\vartheta)}{\partial\vartheta}\frac{\partial\vartheta}{\partial t} = \underbrace{\frac{\partial \ell(\vartheta)}{\partial\vartheta}}_{\Lambda_\ell(\vartheta)}\dot{\vartheta}$$

$$\ddot{\ell}(\vartheta) = \frac{d}{dt}\left(\Lambda_\ell(\vartheta)\,\dot{\vartheta}\right) = \Lambda_\ell(\vartheta)\,\ddot{\vartheta} + \frac{d\Lambda_\ell}{dt}\dot{\vartheta}$$

$$= \Lambda_\ell \ddot{\vartheta} + \frac{\partial\Lambda}{\partial\vartheta}\frac{\partial\vartheta}{\partial t}\dot{\vartheta}$$

$$= \Lambda_\ell \ddot{\vartheta} + \frac{\partial\Lambda}{\partial\vartheta}\dot{\vartheta}^2 \qquad \Lambda = \frac{\partial\ell}{\partial\vartheta} \qquad \frac{\partial\Lambda}{\partial\vartheta} = \frac{\partial^2\ell}{\partial\vartheta^2}$$

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