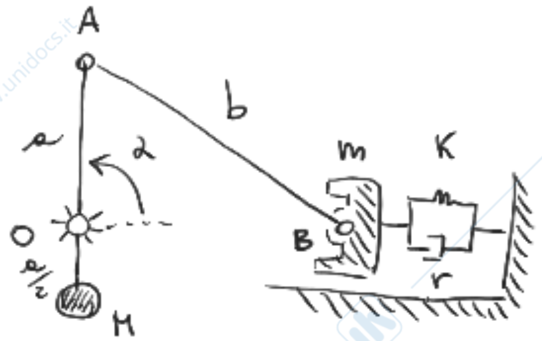
Ese20_vibrazioni



$\alpha_0 = \pi/2$

$b = 2a$

$$\frac{d}{dt}\frac{\partial E_c}{\partial \dot\alpha} - \frac{\partial E_c}{\partial \alpha} + \frac{\partial D}{\partial \dot\alpha} + \frac{\partial V}{\partial \alpha} = Q_\alpha$$

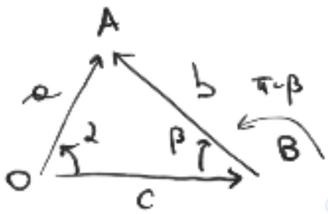
Non lineare

$$J(\alpha)\ddot\alpha + \frac{1}{2}\frac{\partial J}{\partial \alpha}\dot\alpha^2 + r\frac{\partial \Delta l}{\partial \alpha}\dot\alpha + mg\frac{\partial h}{\partial \alpha} + k\Delta l\frac{\partial \Delta l}{\partial \alpha} = F\frac{\partial x_F}{\partial \alpha}$$

linearizzata

$$J(\alpha_0)\ddot\alpha + r(\alpha_0)\dot\alpha + (K_p + K_{k0} + K_{k1})\alpha = F(\alpha_0)$$

$$E_c = \frac{1}{2}M v_M^2 + \frac{1}{2}m v_B^2 \qquad v_M = \frac{a}{2}\dot\alpha$$



(1) $\begin{cases} a\cos\alpha + b\cos\beta = c \\ a\sin\alpha = b\sin\beta \end{cases}$ $\qquad c, \beta$

(2) $\begin{cases} \beta = \arcsin\left(\frac{a}{b}\sin\alpha\right) \\ c = a\cos\alpha + b\cos\Big[\underbrace{\arcsin\left(\frac{a}{b}\sin\alpha\right)}_{\beta}\Big] \end{cases}$ $\qquad \beta(\alpha_0) = \pi/6 \text{ rad} \qquad c(\alpha_0) = b\frac{\sqrt{3}}{2}$

v_B? $\qquad v_B = \dot c$

$$\begin{cases} -a\dot\alpha\sin\alpha - b\dot\beta\sin\beta = \dot c \\ a\dot\alpha\cos\alpha = b\dot\beta\cos\beta \end{cases} \Rightarrow \begin{cases} \dot c = -a\sin\alpha\,\dot\alpha - b\sin\beta\,\frac{a\cos\alpha}{b\cos\beta}\dot\alpha \\ \dot\beta = \frac{a\cos\alpha}{b\cos\beta}\dot\alpha \end{cases}$$

$$\begin{cases} \dot c = -a(\sin\alpha + \cos\alpha\tan\beta)\,\dot\alpha \\ \dot\beta = \frac{a\cos\alpha}{b\cos\beta}\dot\alpha \end{cases} \qquad \begin{cases} \dot c = \Lambda_c(\alpha)\,\dot\alpha \\ \dot\beta = \Lambda_\beta(\alpha)\,\dot\alpha \end{cases}$$

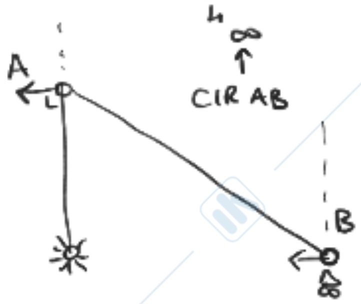
$$\dot c = \frac{d(c(\alpha))}{dt} = \frac{\partial c(\alpha)}{\partial \alpha}\frac{\partial \alpha}{\partial t} = \frac{\partial c}{\partial \alpha}\dot\alpha = \Lambda_c(\alpha)\,\dot\alpha$$

$$v_B = \dot c = \Lambda_c(\alpha)\,\dot\alpha$$

$$E_c = \frac{1}{2}M\frac{a^2}{4}\dot\alpha^2 + \frac{1}{2}m\Lambda_c^2(\alpha)\dot\alpha^2 = \frac{1}{2}\underbrace{\left(M\frac{a^2}{4} + m\Lambda_c^2(\alpha)\right)}\dot\alpha^2$$

$$= \frac{1}{2}J^*(\alpha)\,\dot\alpha^2$$

$$J^*(\alpha_0) = M\frac{a^2}{4} + m\,\Lambda_c^2(\alpha_0) = M\frac{a^2}{4} + m\left(a\left(\sin\alpha_0 + \cos\alpha_0 \tan\beta_0\right)\right)^2$$
$$= M\frac{a^2}{4} + m a^2$$



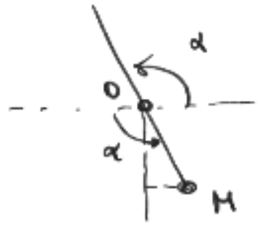
AB $\alpha = \alpha_0 = \pi/2 \rightarrow$ Trasla

$$\dot c(\alpha_0) = -a\left(\sin\alpha_0 + \cos\alpha_0 \tan\beta_0\right)\dot\alpha = -a\dot\alpha \quad [m/s]$$
$$\dot\beta(\alpha_0) = \frac{a\cos\alpha_0}{b\cos\beta_0}\dot\alpha = 0 \quad [rad/s]$$

$$J^*(\alpha_0)\,\ddot\alpha = \left(M\frac{a^2}{4} + m a^2\right)\ddot\alpha$$ forze inerzia eq linearizzata

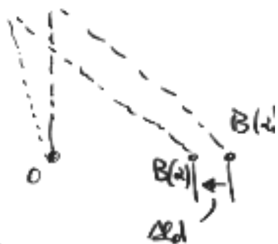
$$V = V_g + V_K$$

$$V_g = M g h_M + \cancel{m g h_m}$$ $h_m =$ costante



$$h_M = -\frac{a}{2}\sin\alpha \qquad \frac{\partial h_M}{\partial\alpha} = -\frac{a}{2}\cos\alpha$$
$$V_g = -Mg\frac{a}{2}\sin\alpha$$
$$K_g = Mg\left.\frac{\partial^2 h_M}{\partial\alpha^2}\right|_{\alpha_0} = Mg\frac{a}{2}\sin\alpha\Big|_{\alpha_0} = Mg\frac{a}{2} > 0$$ stabilizzante

$$V_K = \frac{1}{2}k\,\Delta l^2 \qquad \Delta l = \Delta l_0 + \Delta l_d$$
$$\Delta l_d = c(\alpha_0) - c(\alpha)$$



$$K_{K0} = k\,\Delta l_0 \left.\frac{\partial^2 \Delta l_d}{\partial\alpha^2}\right|_{\alpha_0}$$

$$K_{K1} = k\left.\left(\frac{\partial \Delta l_d}{\partial\alpha}\right)^2\right|_{\alpha_0} \qquad \frac{\partial \Delta l_d}{\partial\alpha} = \frac{\partial}{\partial\alpha}\left(c(\alpha_0) - c(\alpha)\right) = -\frac{\partial c(\alpha)}{\partial\alpha}$$

$$\frac{\partial \Delta l_d}{\partial\alpha} = -\frac{\partial c}{\partial\alpha} = -\frac{\partial}{\partial\alpha}\left(a\cos\alpha + b\cos\left(\arcsin\left(\frac{a}{b}\sin\alpha\right)\right)\right)$$

$$\dot c = \Lambda_c(\alpha)\,\dot\alpha = \frac{\partial c}{\partial\alpha}\dot\alpha$$

$$\frac{\partial c}{\partial\alpha} = \frac{\dot c}{\dot\alpha} = -a\left(\sin\alpha + \cos\alpha\tan\beta\right) \qquad \left.\frac{\partial c}{\partial\alpha}\right|_{\alpha_0} = -a$$

$$K_{K1} = k\left(\left.\frac{\partial \Delta l_d}{\partial\alpha}\right|_{\alpha_0}\right)^2 = k a^2$$

$$\frac{\partial^2 \Delta l_d}{\partial\alpha^2} = \frac{\partial}{\partial\alpha}\left(\Lambda_c(\alpha)\right)$$ oppure

$\partial\alpha^2 \qquad \partial\alpha$

$$\ddot{c} = \Lambda_c(\alpha)\,\ddot{\alpha} + \dot{\Lambda}_c(\alpha)\,\dot{\alpha}$$

$$\ddot{c} = \Lambda(\alpha)\,\ddot{\alpha} + \frac{\partial\Lambda_c}{\partial\alpha}\,\dot{\alpha}^2$$

$$\uparrow \qquad \frac{\partial\Lambda_c}{\partial\alpha} = \frac{\partial}{\partial\alpha}\left(\frac{\partial c}{\partial\alpha}\right) = \frac{\partial^2 c}{\partial\alpha^2}$$

$$\frac{d}{dt}\begin{cases} -a\,\dot{\alpha}\sin\alpha - b\dot{\beta}\sin\beta = \dot{c} \\ a\,\dot{\alpha}\cos\alpha = b\,\dot{\beta}\cos\beta \end{cases}$$

$$\begin{cases} -a\,\ddot{\alpha}\sin\alpha - a\,\dot{\alpha}^2\cos\alpha - b\ddot{\beta}\sin\beta - b\dot{\beta}^2\cos\beta = \ddot{c} \\ a\,\ddot{\alpha}\cos\alpha - a\,\dot{\alpha}^2\sin\alpha = b\,\ddot{\beta}\cos\beta - b\dot{\beta}^2\sin\beta \end{cases}$$

$$\ddot{\beta} = \frac{a\cos\alpha}{b\cos\beta}\,\ddot{\alpha} - \frac{a\sin\alpha\,\dot{\alpha}^2 + b\sin\beta\,\dot{\beta}^2}{b\cos\beta}$$

$$\ddot{\beta} = \frac{a\cos\alpha}{b\cos\beta}\,\ddot{\alpha} + \left(-a\sin\alpha + b\sin\beta\left(\frac{a\cos\alpha}{b\cos\beta}\right)^2\right)\frac{\dot{\alpha}^2}{b\cos\beta}$$

$$\ddot{\beta} = \Lambda_\beta(\alpha)\,\ddot{\alpha} + \frac{\partial\Lambda_\beta}{\partial\alpha}\,\dot{\alpha}^2$$

$$\ddot{c} = -a\sin\alpha\,\ddot{\alpha} - b\sin\beta\,\ddot{\beta} - a\cos\alpha\,\dot{\alpha}^2 - b\cos\beta\,\dot{\beta}^2$$

$$\ddot{c} = \left(-a\sin\alpha - b\sin\beta\,\Lambda_\beta\right)\ddot{\alpha} + \left(-b\sin\beta\,\frac{\partial\Lambda_\beta}{\partial\alpha} - a\cos\alpha + \right.$$

$$\left. - b\cos\beta\left(\frac{a\cos\alpha}{b\cos\beta}\right)^2\right)\dot{\alpha}^2$$

$$\ddot{c} = \Lambda_c(\alpha)\,\ddot{\alpha} + \frac{\partial\Lambda_c}{\partial\alpha}\,\dot{\alpha}^2$$

$$\left.\frac{\partial\Lambda_c}{\partial\alpha}\right|_{\alpha_0} = \left.\frac{\partial^2 c}{\partial\alpha^2}\right|_{\alpha_0} = \left(-\frac{\cancel{b}\sin\beta_0}{\cancel{b}\cos\beta_0}\left(-a\overset{1}{\sin\alpha_0} + b\sin\beta_0\left(\frac{a\,\overset{0}{\cancel{\cos\alpha_0}}}{b\cos\beta_0}\right)^2\right) + \right.$$

$$\left. - a\,\overset{0}{\cancel{\cos\alpha_0}} - b\cos\beta_0\left(\cancel{\frac{a\cos\alpha_0}{b\cos\beta_0}}\right)^2\right)$$

$$= a\tan\beta_0 = a\sqrt{3}$$

$$K_{k0} = k\,\Delta l_0\,a\sqrt{3}$$

$$\frac{\partial V}{\partial\alpha} \simeq \left(k_g + K_{k0} + K_{k1}\right)\tilde{\alpha} = \left(Mg\,\frac{a}{2} + k\,\Delta l_0\,a\tan\beta_0 + k\,a^2\right)\tilde{\alpha}$$

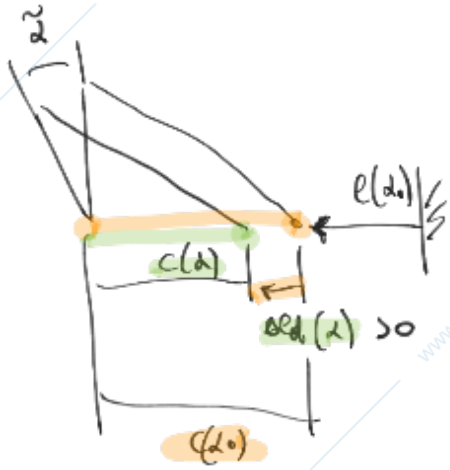
$\tilde{\alpha} = \alpha - \alpha_0$

$\tilde{\alpha} = \alpha - \alpha_0$

Eq di moto linearizzate

$$\left(M\frac{a^2}{4} + m a^2\right)\ddot{\tilde{\alpha}} + r a^2 \dot{\tilde{\alpha}} + \left(Mg\frac{a}{2} + k\Delta l_0 a \tan\beta + k a^2\right)\tilde{\alpha} = 0$$

$$D = \frac{1}{2} r \dot{\Delta l}^2 \qquad \frac{\partial D}{\partial \dot{\alpha}} = r\left(\frac{\partial \Delta l}{\partial \alpha}\right)^2 \dot{\alpha} = r a^2 \dot{\alpha}$$



$\tilde{\alpha} = \alpha - \alpha_0 > 0$

$\xleftarrow{+}$

$\Delta l = c(\alpha_0) - c(\alpha)$

Ultima modifica: mag 20, 2020