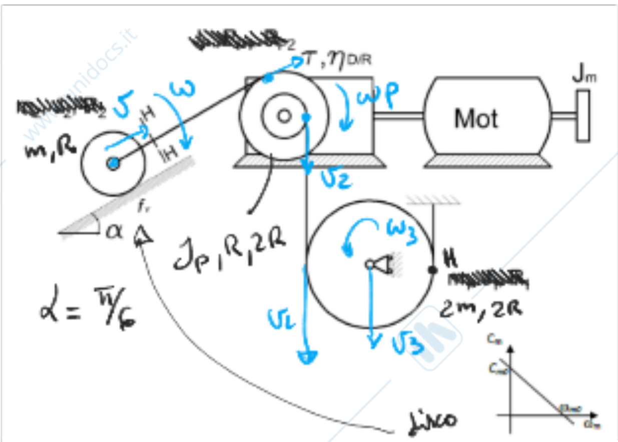
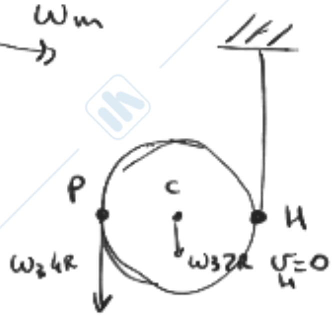
Ese16.1_MTU



Si chiede di calcolare:

1. l'accelerazione del centro della massa m_2 nella condizione di spunto in salita;
2. la velocità nella condizione di salita a regime della massa m_2 (indicare la velocità del centro di m_2);
3. il tiro della fune nella sezione H-H nella condizione di moto del punto 1.



$$\omega = v/R \qquad \omega_p = \frac{v}{2R} \qquad v_2 = \omega_p R = \frac{v}{2} \qquad v_3 = \frac{v_2}{2} = \frac{v}{4} \qquad \omega_3 = \frac{v_3}{2R}$$

$$\omega_p = \tau\omega_m \qquad \omega_m = \frac{\omega_p}{\tau} = \frac{v}{2R\tau} \qquad \omega_3 = \frac{v}{8R}$$

1) Utilizzatore

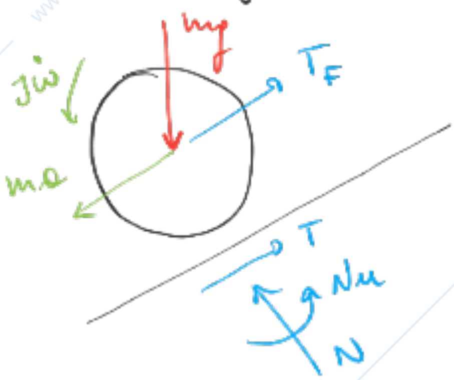
$$W_u - W_r = \frac{dE_{cu}}{dt}$$

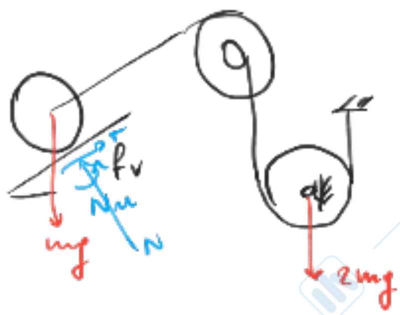
$$E_{cu} = \frac{1}{2} m v^2 + \frac{1}{2} J\omega^2 + \frac{1}{2} J_p\omega_p^2 + \frac{1}{2} 2m\, v_3^2 + \frac{1}{2} J_3\omega_3^2$$

$$E_{cu} = \frac{1}{2}\left(m + \frac{J}{R^2} + J_p\frac{1}{4R^2} + \frac{2m}{16} + \frac{J_3}{64R^2}\right)v^2$$

$$E_{cu} = \frac{1}{2} m_u^* v^2 \qquad \frac{dE_{cu}}{dt} = m_u^* a v$$

$$W_u = -mg\sin\alpha\, v - N u\,\omega + 2mg\, v_3$$





$$\Sigma F_\perp = 0$$

$$N = mg\cos\alpha$$

$$u = f_v R$$

$$W_u = -mg\sin\alpha\, v - mg\cos\alpha\, f_v R\frac{v}{R} + 2mg\frac{v}{4}$$

$$= -mg\left(\sin\alpha + f_v\cos\alpha - 1/2\right)v$$

$$= -mg\left(1/2 + f_v\frac{\sqrt{3}}{2} - 1/2\right)v$$

$$= -mg f_v \frac{\sqrt{3}}{2} v$$

$$W_2 = W_u - \frac{dE_{cu}}{dt} = -\frac{\sqrt{3}}{2} m g f_v v - m_u^* a v$$

Supponiamo $a > 0$ $\quad W_2 < 0 \rightarrow$ DIRETTO

$$W_p = -(1-\eta_D) W_1$$

$$W_m - W_1 = \frac{dE_{cm}}{dt}$$

$$W_1 = W_m - \frac{dE_{cm}}{dt} = C_m \omega_m - J_m \dot{\omega}_m \omega_m$$

$$W_m + W_u + W_p = \frac{dE_c}{dt}$$

$$\cancel{C_m \omega_m} - \frac{\sqrt{3}}{2} m g f_v v - (\cancel{1} - \eta_D)(C_m \omega_m - J_m \dot{\omega}_m \omega_m) = m_u^* a v + \cancel{J_m \dot{\omega}_m \omega_m}$$

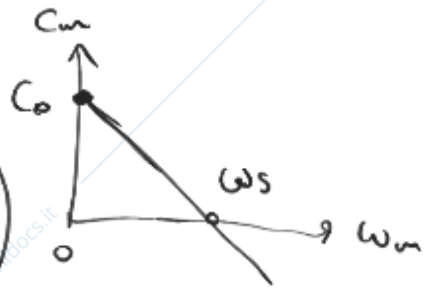
$$\eta_D C_m \omega_m - \frac{\sqrt{3}}{2} m g f_v v = m_u^* a v + \eta_D J_m \dot{\omega}_m \omega_m$$

$$\frac{\eta_D}{2R\tau} C_m \cancel{v} - \frac{\sqrt{3}}{2} m g f_v \cancel{v} = (m_u^* + \eta_D m_m^*) a \cancel{v} \qquad m_m^* = \frac{J_m}{(2R\tau)^2}$$

$$\frac{\eta_D}{2R\tau} C_m - \frac{\sqrt{3}}{2} m g f_v = (m_u^* + \eta_D m_m^*) a$$ ← eq di moto DIRETTO

Allo spunto $(v = 0)$ $\quad C_m = C_0$

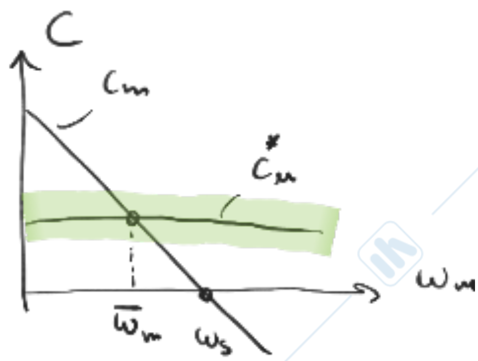
$$a_s = \left(\frac{\eta_D}{2R\tau} C_0 - \frac{\sqrt{3}}{2} m g f_v\right) \Big/ \left(m_u^* + \eta_D m_m^*\right)$$



2) v regime? $(a = 0)$

$$\frac{\eta_D}{2R\tau} C_m - \frac{\sqrt{3}}{2} m g f_v = 0$$

$$\begin{cases} \bar{C}_m = \dfrac{2R\tau}{\eta_D} \dfrac{\sqrt{3}}{2} m g f_v \\ \bar{C}_m = C_0 \left(1 - \dfrac{\bar{\omega}_m}{\omega_s}\right) \end{cases}$$

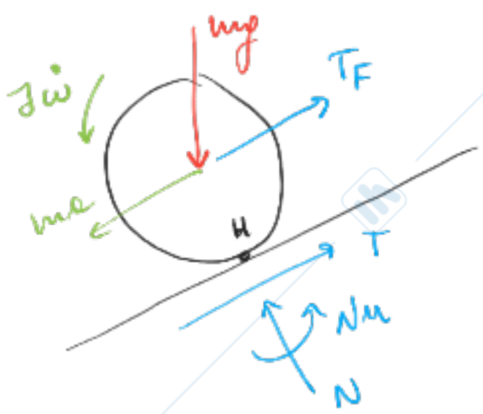


$$\frac{\bar{C}_m}{C_0} = 1 - \frac{\bar{\omega}_m}{\omega_s} \qquad \bar{\omega}_m = \omega_s \left(1 - \frac{\bar{C}_m}{C_0}\right)$$

$$\bar{\omega}_m = \omega_s \left(1 - \frac{\frac{2R\tau}{\eta_D} \frac{\sqrt{3}}{2} m g f_v}{C_0}\right)$$

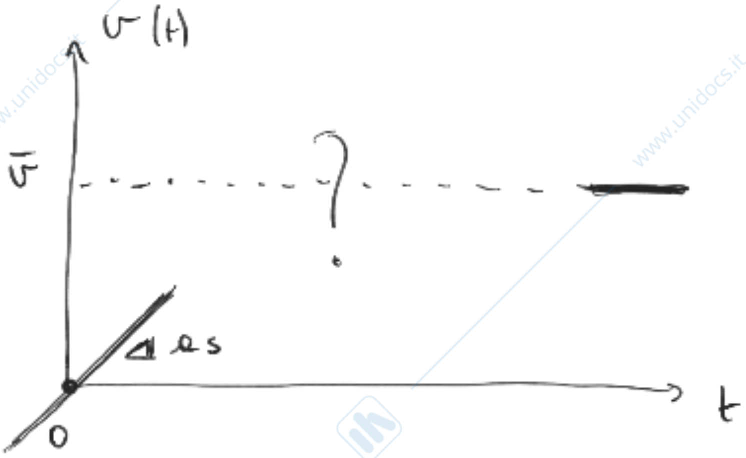
$$\bar{v} = \bar{\omega}_m 2R\tau$$

3) T_F ? $(a = a_s)$



$$\Sigma M_H = 0 \qquad J\dot\omega + maR - T_F R + Nu + mg\sin\alpha R = 0$$

$$T_F = \left(J\dot\omega + maR + mg\cos\alpha f_v R + mg\sin\alpha R\right)/R$$



$$a = \frac{dv}{dt}$$

$$\begin{cases} \frac{\eta_D}{2R\tau} C_m - \frac{\sqrt{3}}{2} mg f_v = m^* a \\ C_m = C_0 - \frac{C_0}{\omega_s}\omega_m \end{cases} \qquad m^* = m_u^* + \eta_D m_m^*$$

$$a = \frac{dv}{dt} = \dot v$$

$$\frac{\eta_D}{2R\tau} C_0 - \frac{\eta_D}{2R\tau}\frac{C_0}{\omega_s}\frac{v}{2R\tau} - \frac{\sqrt{3}}{2} mg f_v = m^* \dot v$$

$$m^*\dot v + \left(\frac{\eta_D}{2R\tau}\frac{C_0}{\omega_s}\frac{1}{2R\tau}\right) v = \frac{\eta_D}{2R\tau} C_0 - \frac{\sqrt{3}}{2} mg f_v$$

$m^*\dot v + r v = F_0$ eq diff lineare omogenea a coeff costanti 1° ordine

$$v(t) = v_{0A}(t) + v_p$$

F_0 = costante v_p = costante $= V_p$ $\dot v_p = 0$

$$\cancel{m^*\dot v_p} + r v_p = F_0 \qquad V_p = \frac{F_0}{r}$$

$$V_p = \frac{F_0}{r} = \left(\frac{\eta_D}{2R\tau} C_0 - \frac{\sqrt{3}}{2} mg f_v\right) \Big/ \left(\frac{\eta_D}{2R\tau}\frac{C_0}{\omega_s}\frac{1}{2R\tau}\right)$$

$$V_p = \frac{\cancel{\eta_D}}{\cancel{2R\tau}} \cancel{C_0} \frac{\cancel{2R\tau}\, \omega_s\, 2R\tau}{\cancel{\eta_D C_0}} - \frac{\sqrt{3}}{2} m p f_r \frac{2R\tau\, \omega_s\, 2R\tau}{\eta_D C_0}$$

$$= \omega_s 2R\tau - \omega_s 2R\tau \left(\frac{\frac{\sqrt{3}}{2} m p f_r 2R\tau}{\eta_D C_0} \right)$$

$$= 2R\tau\, \omega_s \left(1 - \frac{\overline{C_m}}{C_0} \right) = 2R\tau\, \overline{\omega_m} = \overline{v}$$

$$V_p = \overline{v}$$

$v_{OA}(t) = V_0 e^{\lambda t} \rightarrow m^* \dot{v} + r v = 0 \leftarrow$ Omogenea Associata

$\dot{v}_{OA}(t) = \lambda V_0 e^{\lambda t}$

$$m^* \lambda V_0 e^{\lambda t} + r V_0 e^{\lambda t} = 0$$

$$(m^* \lambda + r) V_0 e^{\lambda t} = 0$$

$e^{\lambda t} \neq 0 \quad \forall t$

$V_0 = 0 \leftarrow$ soluzione banale

$\lambda = -\frac{r}{m^*} < 0$ Autovalore, polo $\in \mathbb{R} < 0$

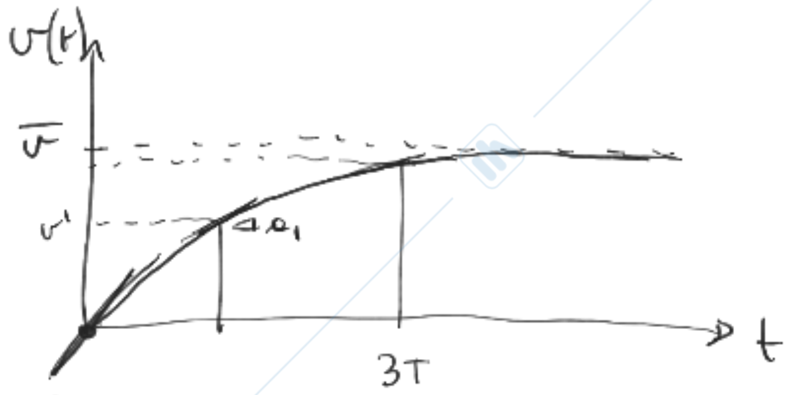
$$v_{OA}(t) = V_0 e^{-r/m^* t}$$

$$v(t) = v_p + v_{OA} = \overline{v} + V_0 e^{-r/m t}$$

Condiz. iniz $\quad v(t=0) = 0 \quad \overline{v} + V_0 = 0 \quad V_0 = -\overline{v}$

$$v(t) = \overline{v}\left(1 - e^{-t/T}\right)$$

$T = \frac{m}{r}$ costante di tempo del sistema

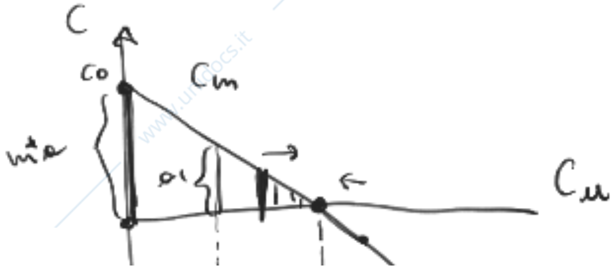


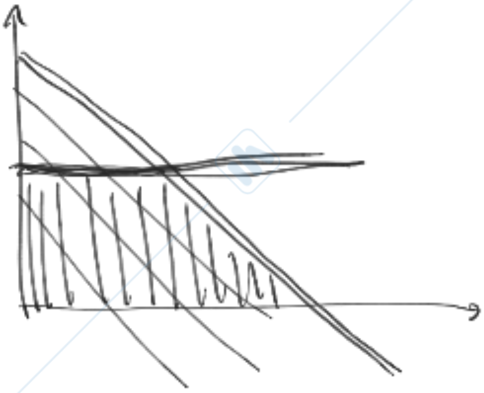
$1 - e^{-t/T}$

$t = T \quad \simeq \quad 0,65$

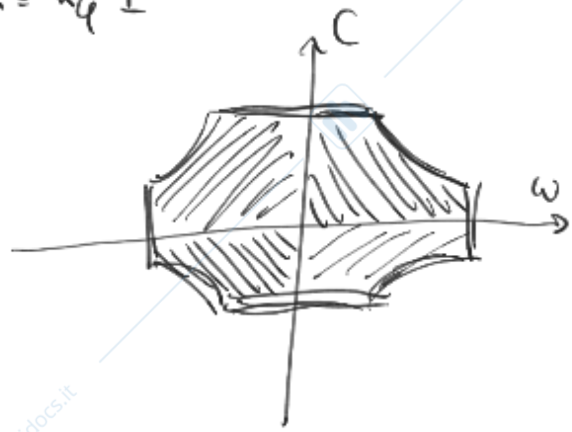
$t = 2T \quad \simeq \quad 0,83$

$t = 3T \quad \simeq \quad 0,95$





$C_m = k_\varphi I$



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