Ese19_vibrazioni

Linearizzazione delle eq di moto $\frac{d}{dt}\frac{\partial E_c}{\partial \dot q} - \frac{\partial E_c}{\partial q} + \frac{\partial D}{\partial \dot q} + \frac{\partial V}{\partial q} = Q_q$

q variabile indipendente

$E_c = \frac{1}{2} m v^2 \qquad v = \Lambda(q)\dot q$

↑ Jacobiano

$E_c = \frac{1}{2} \underbrace{m\, \Lambda(q)^2}\, \dot q^2 = \frac{1}{2} m(q) \dot q^2$

$\frac{\partial E_c}{\partial \dot q} = m(q)\dot q \qquad \frac{d}{dt}\left(m(q)\dot q\right) = \frac{d}{dt}(m(q))\,\dot q + m(q)\ddot q$

$= \frac{\partial}{\partial q} m(q) \cdot \frac{\partial q}{\partial t}\dot q + m(q)\ddot q$

$\frac{d}{dt}\frac{\partial E_c}{\partial \dot q} = \frac{\partial m(q)}{\partial q}\dot q^2 + m(q)\ddot q$

$\frac{\partial E_c}{\partial q} = \frac{\partial}{\partial q}\left(\frac{1}{2} m(q)\dot q^2\right) = \frac{1}{2}\frac{\partial m(q)}{\partial q}\dot q^2$

$\frac{d}{dt}\frac{\partial E_c}{\partial \dot q} - \frac{\partial E_c}{\partial q} = m(q)\ddot q + \frac{1}{2}\frac{\partial m(q)}{\partial q}\dot q^2$

linearizzazione $\qquad f(x,y,z) \simeq f(x_0,y_0,z_0) + \left.\frac{\partial f}{\partial x}\right|_{x_0,y_0,z_0}(x-x_0) + \left.\frac{\partial f}{\partial y}\right|_{x_0,y_0,z_0}(y-y_0) + \left.\frac{\partial f}{\partial z}\right|_{x_0,y_0,z_0}(z-z_0) + o()$

$\boxed{|_0 = \left(q=q_0 \quad \dot q = 0 \quad \ddot q = 0\right)}$ ← condiz all'equilibrio

$m(q)\ddot q \simeq \cancel{\left(m(q)\ddot q\right)}\Big|_0^{\,0} + \frac{\partial}{\partial q}\left(m(q)\ddot q\right)\Big|_0 (q-q_0) + \frac{\partial}{\partial \ddot q}\left(m(q)\ddot q\right)\Big|_0 \ddot q$

$m(q)\ddot q \simeq \cancel{\left(\frac{\partial m}{\partial q}\ddot q\right)}\Big|_0^{\,0}(q-q_0) + m(q)\Big|_0 \ddot q$

$m(q)\ddot q \simeq m(q_0)\ddot q$

$\frac{1}{2}\frac{\partial m}{\partial q}\dot q^2 \simeq \cancel{\left(\frac{1}{2}\frac{\partial m}{\partial q}\dot q^2\right)}\Big|_0^{\,0} + \overset{\partial/\partial q}{\downarrow}\cancel{\left(\frac{1}{2}\frac{\partial^2 m}{\partial q^2}\dot q^2\right)}\Big|_0^{\,0}(q-q_0) + \overset{\partial/\partial \dot q}{\downarrow}\cancel{\left(\frac{\partial m}{\partial q}\dot q\right)}\Big|_0^{\,0}\dot q$

$\simeq 0$

$\frac{d}{dt}\frac{\partial E_c}{\partial \dot q} - \frac{\partial E_c}{\partial q} = m(q)\ddot q - \frac{1}{2}\frac{\partial m}{\partial q}\dot q^2 \simeq m(q_0)\ddot q = m\,\Lambda(q_0)^2\ddot q$

$$V = V_g + V_k$$

$$V_g = mg\,h(q) \qquad \frac{\partial V_g}{\partial q} = mg\frac{\partial h}{\partial q}$$

$$\frac{\partial h}{\partial q} \simeq \frac{\partial h}{\partial q}\Big|_0 + \frac{\partial^2 h}{\partial q^2}\Big|_0 (q-q_0)$$

$$\frac{\partial V_g}{\partial q} = mg\frac{\partial h}{\partial q} \simeq \underset{\substack{\uparrow \\ \text{costante} \\ \text{All'eq si elide}}}{mg\frac{\partial h}{\partial q}\Big|_0} + \underbrace{mg\frac{\partial^2 h}{\partial q^2}\Big|_0}_{K_g^*}(q-q_0)$$

$$\simeq mg\frac{\partial^2 h}{\partial q^2}\Big|_0 q_d \qquad\qquad q = q_0 + q_d$$

$$\simeq K_g^* q_d$$

$$V_k = \frac{1}{2}k\Delta l^2 \qquad\qquad \Delta l = \Delta l_0 + \Delta l_d(q)$$

$$\frac{\partial V_k}{\partial q} = \frac{\partial}{\partial q}\left(\frac{1}{2}k\left(\Delta l_0 + \Delta l_d(q)\right)^2\right) = k\left(\Delta l_0 + \Delta l_d(q)\right)\frac{\partial \Delta l_d}{\partial q}$$

$$= k\Delta l_0\frac{\partial \Delta l_d}{\partial q} + k\Delta l_d\frac{\partial \Delta l_d}{\partial q}$$

$$k\Delta l_0\frac{\partial \Delta l_d}{\partial q} \simeq \underset{\substack{\uparrow \\ \text{costante} \\ \text{All'eq si elide}}}{\left(k\Delta l_0\frac{\partial \Delta l_d}{\partial q}\right)\Big|_0} + \underset{\substack{\uparrow \\ K_{k0}^*}}{\left(k\Delta l_0\frac{\partial^2 \Delta l_d}{\partial q^2}\right)\Big|_0} q_d$$

$$k\Delta l_d\frac{\partial \Delta l_d}{\partial q} \simeq \left(k\cancel{\Delta l_d}_{\,0}\frac{\partial \Delta l_d}{\partial q}\right)\Big|_0 + \left(k\cancel{\Delta l_d}_{\,0}\frac{\partial^2 \Delta l_d}{\partial q^2}\right)\Big|_0 q_d + \underset{\substack{\uparrow \\ K_{k1}^*}}{\left(k\left(\frac{\partial \Delta l_d}{\partial q}\right)^2\right)\Big|_0} q_d$$

$$\frac{\partial V_k}{\partial q} \simeq \left(k\Delta l_0\frac{\partial \Delta l_d}{\partial q}\right)\Big|_0 + \left(k\Delta l_0\frac{\partial^2 \Delta l_d}{\partial q^2}\right)\Big|_0 q_d + \left(k\left(\frac{\partial \Delta l_d}{\partial q}\right)^2\right)\Big|_0 q_d$$

$$\simeq \left(K_{k0}^* + K_{k1}^*\right) q_d$$

$$K_g^* = mg\frac{\partial^2 h}{\partial q^2}\Big|_0$$

$$\frac{\partial V}{\partial q} \simeq \left(K_g^* + K_{k0}^* + K_{k1}^*\right) q_d$$

$$K_{k0}^* = \left(k\Delta l_0\frac{\partial^2 \Delta l_d}{\partial q^2}\right)\Big|_0$$

$$K^*_{k1} = \left(k\left(\frac{\partial \Delta l_d}{\partial q}\right)^2\right)\Bigg|_0$$

$$D = \frac{1}{2} r \dot{\Delta l}^2 \qquad \Delta l = \Delta l_d(q) + \Delta l_0$$

$$\dot{\Delta l} = \dot{\Delta l}_d = \frac{\partial \Delta l_d}{\partial q}\frac{\partial q}{\partial t} = \frac{\partial \Delta l_d}{\partial q}\dot{q}$$

$$D = \frac{1}{2} r \left(\frac{\partial \Delta l_d}{\partial q}\right)^2 \dot{q}^2$$

$$\frac{\partial D}{\partial \dot{q}} = r\left(\frac{\partial \Delta l_d}{\partial q}\right)^2 \dot{q} \simeq \cancel{\left(r\left(\frac{\partial \Delta l_d}{\partial q}\right)^2 \underset{\dot{q}=0}{\dot{q}}\right)\Bigg|_0} + \cancel{\left(r\frac{\partial}{\partial q}\left(\left(\frac{\partial \Delta l_d}{\partial q}\right)^2\right)\dot{q}\right)\Bigg|_0 q_d} +$$

$$+ \left(r\left(\frac{\partial \Delta l_d}{\partial q}\right)^2\right)\Bigg|_0 \dot{q}$$

$$\frac{\partial D}{\partial \dot{q}} \simeq r(q_0)\,\dot{q} \qquad r(q_0) = r\left(\frac{\partial \Delta l_d}{\partial q}\right)^2\Bigg|_0$$

$$Q_q = \frac{\delta L}{\delta q} = \frac{F\,\delta x_F}{\delta q} = F\frac{\partial x_F}{\partial q} \qquad F = F(t)$$

$$Q_q \simeq F(t)\frac{\partial x_F}{\partial q}\Bigg|_0 + \cancel{\left(F(t)\cdot\frac{\partial^2 x_F}{\partial q^2}\right)\Bigg|_0 q_d} + \cancel{\left(\frac{\partial F}{\partial t}\frac{\partial x_F}{\partial q}\right)\Bigg|_0 (t-t_0)}$$

K equivalente funzione del tempo

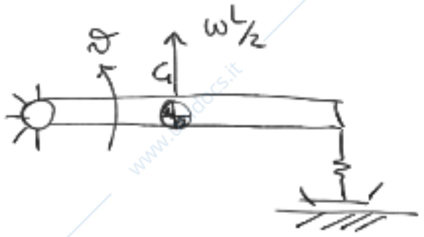
Eq non lineare

$$m(q)\ddot{q} - \frac{1}{2}\frac{\partial m}{\partial q}\dot{q}^2 + r\left(\frac{\partial \Delta l_d}{\partial q}\right)^2\dot{q} + mg\frac{\partial h}{\partial q} + k(\Delta l_0 + \Delta l_d)\frac{\partial \Delta l_d}{\partial q} = \frac{\partial x_F}{\partial q}F(t)$$

Eq linearizzata $\qquad q = q_0 + q_d \qquad \dot{q} = \dot{q}_d \qquad \ddot{q} = \ddot{q}_d$

$$m(q_0)\ddot{q}_d + r\left(\frac{\partial \Delta l_d}{\partial q}\right)^2\Bigg|_0 \dot{q}_d + \left[mg\frac{\partial^2 h}{\partial q^2}\Bigg|_0 + k\Delta l_0\frac{\partial^2 \Delta l_d}{\partial q^2}\Bigg|_0 + k\left(\frac{\partial \Delta l_d}{\partial q}\right)^2\Bigg|_0\right] q_d = \frac{\partial x_F}{\partial q}\Bigg|_0 F$$

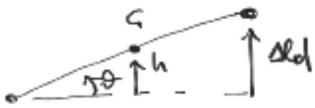
$$m^*\ddot{q}_d + r^*\dot{q}_d + k^* q_d = \underline{\Lambda_{F0}\,F(t)}$$



$$E_c = \frac{1}{2}mv^2 + \frac{1}{2}J\omega^2 = \frac{1}{2}\left(m\frac{L^2}{4} + J\right)\dot{\theta}^2$$

$$= \frac{1}{2}J^*\dot{\theta}^2 \qquad v = \frac{L}{2}\dot{\theta}$$

$$\frac{d}{dt}\frac{\partial E_c}{\partial \dot\vartheta} - \frac{\partial E_c}{\partial \vartheta} = J^*\ddot\vartheta$$



$V_g = mgh \qquad h = \frac{L}{2}\sin\vartheta$

$V_g = mg\frac{L}{2}\sin\vartheta$

$\frac{\partial V_g}{\partial \vartheta} = mg\frac{L}{2}\cos\vartheta$

$V_K = \frac{1}{2}K\Delta l^2 \qquad \Delta l = \Delta l_0 + \Delta l_d = \Delta l_0 + L\sin\vartheta$

$V_K = \frac{1}{2}K(\Delta l_0 + L\sin\vartheta)^2 \qquad \frac{\partial V_K}{\partial \vartheta} = K(\Delta l_0 + L\sin\vartheta)L\cos\vartheta$

Eq non lineare

$J^*\ddot\vartheta + mg\frac{L}{2}\cos\vartheta + K(\Delta l_0 + L\sin\vartheta)L\cos\vartheta = 0$

posizione di equilibrio $(\dot\vartheta = \ddot\vartheta = 0) \qquad \frac{\partial V}{\partial \vartheta} = 0$

$mg\frac{L}{2}\cos\vartheta + K\Delta l_0 L\cos\vartheta + KL^2\sin\vartheta\cos\vartheta = 0$

$\vartheta_0 = 0 \qquad mg\frac{L}{2}\cos\vartheta_0 + K\Delta l_0 L\cos\vartheta_0 + \cancel{KL^2\sin\vartheta_0\cos\vartheta_0} = 0$

$mg\frac{\cancel{L}}{2} + K\Delta l_0\cancel{L} = 0 \qquad \Delta l_0 = -\frac{mg}{2K}$

linearizzazione eq non lineare intorno a $\vartheta_0 = 0$

$\cos\vartheta \simeq \cos\vartheta_0 + (-\sin\vartheta_0)(\vartheta - \vartheta_0) = 1$

$\sin\vartheta \simeq \sin\vartheta_0 + (\cos\vartheta_0)(\vartheta - \vartheta_0) = \vartheta$

$J\ddot\vartheta + \cancel{mg\frac{L}{2}} + \cancel{K\Delta l_0 L} + KL^2\vartheta = 0$

$J\ddot\vartheta + KL^2\vartheta = 0$

$$m(q_0)\ddot q_d + r\left(\frac{\partial \Delta l_d}{\partial q}\right)^2\Big|_0 \dot q_d + \left[mg\frac{\partial^2 h}{\partial q^2}\Big|_0 + K\Delta l_0\frac{\partial^2 \Delta l_d}{\partial q^2}\Big|_0 + K\left(\frac{\partial \Delta l_d}{\partial q}\right)^2_0\right]q_d = \frac{\partial x_F}{\partial q}\Big|_0 F$$

$h = \frac{L}{2}\sin\vartheta \qquad \frac{\partial h}{\partial q} = \frac{L}{2}\cos\vartheta \qquad \frac{\partial^2 h}{\partial q^2} = -\frac{L}{2}\sin\vartheta \qquad \frac{\partial^2 h}{\partial q^2}\Big|_{\vartheta=0} = 0 \qquad K_g^* = 0$

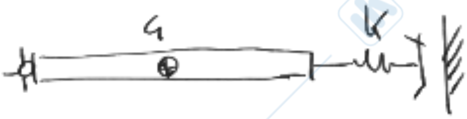
$\Delta l_d = L\sin\vartheta \qquad \frac{\partial \Delta l_d}{\partial \vartheta} = L\cos\vartheta \qquad \frac{\partial \Delta l_d}{\partial \vartheta}\Big|_{\vartheta=0} = L \qquad K_{K_1}^* = KL^2$

$\partial^2 \Delta l_d \, | \qquad |$

$$\frac{\partial^2 \ }{\partial\theta^2}\bigg|_{\theta=0} = -L\sin\theta\bigg|_{\theta=0} = 0 \qquad k^*_{k_0} = 0$$

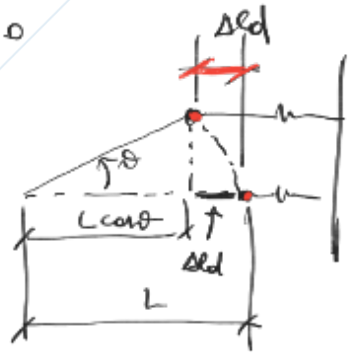
$$J\ddot\theta + K^*_{k_1}\theta = 0$$

$$J\ddot\theta + L^2 K\theta = 0$$



$$J^* = m\frac{L^2}{4} + J_G$$

$$h = \frac{L}{2}\sin\theta \qquad \frac{\partial^2 h}{\partial\theta^2}\bigg|_0 = -\frac{L}{2}\sin\theta\bigg|_0 = 0 \qquad k^*_p = 0$$



$$\Delta l = \Delta l_0 + \Delta l d = \Delta l_0 + (L - L\cos\theta)$$

$$\frac{\partial \Delta l d}{\partial\theta}\bigg|_{\theta=0} = L\sin\theta\bigg|_{\theta=0} = 0 \rightarrow k^*_{k_1} = 0$$

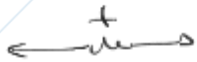
$$\frac{\partial^2 \Delta l d}{\partial\theta^2}\bigg|_{\theta=0} = L\cos\theta\bigg|_{\theta=0} = L \qquad k^*_{k_0} = k\Delta l_0 L$$

$$J\ddot\theta + k\Delta l_0 L\,\theta = 0$$

$$\theta = \theta_0 e^{\lambda t} \qquad (J\lambda^2 + k\Delta l_0 L)\,\theta_0 e^{\lambda t} = 0$$

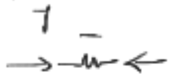
$$\lambda_{1/2} = \sqrt{-\frac{k\Delta l_0 L}{J}}$$

$\Delta l_0 > 0$



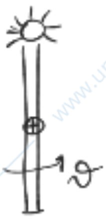
$$\lambda_{1/2} = \mp i\sqrt{\frac{k\Delta l_0 L}{J}}$$

$\Delta l_0 < 0$



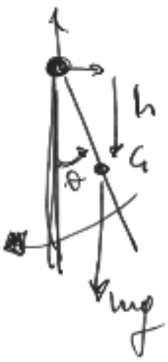
$\Delta l_0 < 0$

$\Delta l_0 > 0$



$$E_c = \frac{1}{2}\left(m\frac{L^2}{4} + J_G\right)\dot\theta^2 \qquad J^* = m\frac{L^2}{4} + J_G$$

$$k^*_p \qquad \frac{\partial^2 h}{\partial\theta^2}\bigg|_{\theta_0}$$

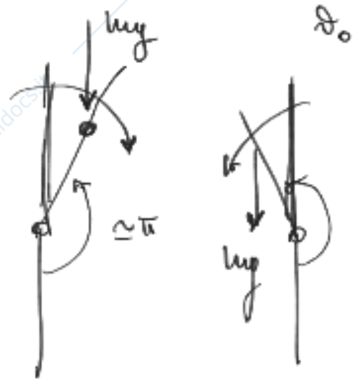


$$h = -\frac{L}{2}\cos\vartheta \qquad \frac{\partial h}{\partial \theta} = \frac{L}{2}\sin\vartheta \qquad \frac{\partial^2 h}{\partial \theta^2} = \frac{L}{2}\cos\vartheta$$

$$\vartheta_0 = 0 \qquad \left.\frac{\partial^2 h}{\partial \theta^2}\right|_{\vartheta_0=0} = \frac{L}{2} \qquad \underline{k_f^* = mg\frac{L}{2} > 0}$$

$$\vartheta_0 = \pi/2 \qquad \left.\frac{\partial^2 h}{\partial \theta^2}\right|_{\vartheta_0=\pi/2} = 0 \qquad k_f^* = 0$$

$$\vartheta_0 = \pi \qquad \left.\frac{\partial^2 h}{\partial \theta^2}\right|_{\vartheta_0=\pi} = -\frac{L}{2} \qquad \underline{\underline{k_f^* = -mg\frac{L}{2} < 0}}$$



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