Appunti NMS B:

① Definisci z

② Definisci x

casi particolari:

- se sono paralleli metti l'asse come vuoi.
- ultimo elemento metti l'asse come vuoi.
- se non sono ne paralleli ne perpendicolari l'asse va messo nella direzione della retta che li congiunge.

② costruzione tabella:

[x]	[x]	[z]	[z]
α [angolo]	a [traslazione] [in-1]	d [traslazione] [Kn]	θ [angolo]

④ transformation matrix:

$$^{0}\hat{A}_1 = \hat{Rot}(x,\alpha)\ \hat{Tras}(x,a)\ \hat{Tras}(z,d)\ \hat{Rot}(z,\theta)$$

$$^{i-1}\hat{A}_i = \begin{bmatrix} \cos\theta_i & -\sin\theta_i & 0 & a_{i-1} \\ \sin\theta_i\cdot\cos\alpha_{i-1} & \cos\theta_i\cdot\cos\alpha_{i-1} & -\sin\alpha_{i-1} & -d_i\cdot\sin\alpha_{i-1} \\ \sin\theta_i\cdot\sin\alpha_{i-1} & \cos\theta_i\cdot\sin\alpha_{i-1} & \cos\alpha_{i-1} & d_i\cdot\cos\alpha_{i-1} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{3}\hat{A}_E = \begin{array}{c|c} & \begin{matrix} i_e & j_e & k_e & \end{matrix} \\ \begin{matrix} i_3 \\ j_3 \\ k_3 \\ \end{matrix} & \begin{bmatrix} \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

⑤ Angular velocity

$$\bar{\omega}_i = \bar{\omega}_{i-1} + \dot{q}_i(1-\delta_i)\bar{k}_i$$

$\delta_i = 0$ se si ha un joint di rotazione

$\delta_i = 1$ se si ha un joint di traslazione

$\omega_e = \omega_3$

$V_e = V_3$

⑥ origin point

$O_0 O_1$ = v_{ai} in tabella prendi i valori e moltiplichi
(i_{n-1} e k_n)

⑦ velocity of the origin O_i

$$v_n = v_{n-1} + \omega_{n-1} \times O_{n-1}O_n + \dot{q}_n \delta_n k_n .$$

$q_E = 0$
$\delta_E = 0$

⑧ Angular acceleration:

$$\dot{\bar{\omega}}_i = \dot{\bar{\omega}}_{i-1} + \ddot{q}_i (1-\delta_i) \bar{K}_i + \dot{q}_i (1-\delta_i) \bar{\omega}_{i-1} \times \bar{K}_i$$

⑨ linear acceleration

$$\dot{\bar{V}}_i = \dot{\bar{V}}_{i-1} + \dot{\bar{\omega}}_{i-1} \times (\overline{O_{i-1}O_i}) + \bar{\omega}_{i-1} \times [\bar{\omega}_{i-1} \times (\overline{O_{i-1}O_i})]$$
$$+ 2\dot{q}_i \delta_i \bar{\omega}_{i-1} \times \bar{K}_i$$

Position and rotation:

$$ {}^{1}\hat{A}_0 = ({}^{0}\hat{A}_1)^{-1} = \begin{bmatrix} {}^{0}A_1^T & -{}^{0}A_1^T \cdot {}^{0}p_1 \\ [0] & 1 \end{bmatrix} $$

translation:

$$ \hat{Tras}(1) = \begin{bmatrix} 1 & 0 & 0 & s_x \\ 0 & 1 & 0 & s_y \\ 0 & 0 & 1 & s_z \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

In the global system:

$$ {}^{0}\hat{B}_1 = \hat{Tras}(s) \cdot {}^{0}\hat{A}_1 = \begin{bmatrix} {}^{0}A_1 & \bar{p} + \bar{s} \\ [0] & 1 \end{bmatrix} $$

In the local system:

$$ {}^{0}\hat{B}_1 = {}^{0}\hat{A}_1 \cdot \hat{Tras}(s) = \begin{bmatrix} {}^{0}A_1 & {}^{0}A_1 {}^{1}\bar{s} + {}^{0}\bar{p} \\ [0] & 1 \end{bmatrix} $$

ROTATION MATRIX

about [x]

$$Rot(x,\theta) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c\theta & -s\theta & 0 \\ 0 & s\theta & c\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

about [y]

$$Rot(y,\theta) = \begin{bmatrix} c\theta & 0 & s\theta & 0 \\ 0 & 1 & 0 & 0 \\ -s\theta & 0 & c\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

about [z]

$$Rot(z,\theta) = \begin{bmatrix} c\theta & -s\theta & 0 & 0 \\ s\theta & c\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

N.B.

body = local

Fixed = global

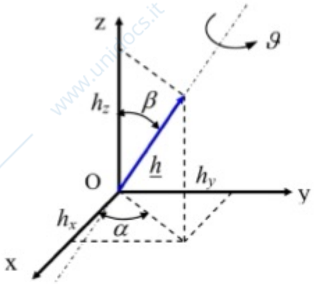
About the global:

$$^{0}\hat{B}_2 = Rot(x,\theta)\,^{0}A_1 = \begin{bmatrix} Rot(x,\theta)\,^{0}A_1 & Rot(x,\theta)\cdot{}^{0}\bar{p} \\ [0] & 1 \end{bmatrix}$$

About the local:

$$^{0}\hat{B}_2 = {}^{0}A_1 \cdot Rot(x,\theta) = \begin{bmatrix} ^{0}A_1 \cdot Rot(x,\theta) & ^{0}\bar{p} \\ [0] & 1 \end{bmatrix}$$

About an arbitrary Axis



$$\hat{Rot}(h,\vartheta)=\begin{bmatrix} h^2_x(1-\cos\vartheta)+\cos\vartheta & h_xh_y(1-\cos\vartheta)-h_z\sin\vartheta & h_xh_z(1-\cos\vartheta)+h_y\sin\vartheta & 0\\ h_xh_y(1-\cos\vartheta)+h_z\sin\vartheta & h^2_y(1-\cos\vartheta)+\cos\vartheta & h_yh_z(1-\cos\vartheta)-h_x\sin\vartheta & 0\\ h_xh_z(1-\cos\vartheta)-h_y\sin\vartheta & h_yh_z(1-\cos\vartheta)+h_x\sin\vartheta & h^2_z(1-\cos\vartheta)+\cos\vartheta & 0\\ 0&0&0&1\end{bmatrix}$$

inverse problem

$$\theta = \cos^{-1}\left(\frac{R_{11}+R_{22}+R_{33}-1}{2}\right)$$

if $\text{sen}\,\theta \neq 0$

$$\bar{h} = \frac{1}{2\sin\theta}\cdot\begin{bmatrix} R_{32}-R_{23}\\ R_{13}-R_{31}\\ R_{21}-R_{12}\end{bmatrix}$$

R_{32} → colonne !!!
↓
Righe !!!

if $\text{sen}\,\theta = 0$

$$h_x = \pm\sqrt{\frac{R_{11}+1}{2}} \qquad \text{sgn}(h_xh_y)=\text{sgn}(R_{21})$$

$$h_y = \pm\sqrt{\frac{R_{22}+1}{2}} \qquad \text{sgn}(h_xh_z)=\text{sgn}(R_{31})$$

$$h_z = \pm\sqrt{\frac{R_{33}+1}{2}} \qquad \text{sgn}(h_yh_z)=\text{sgn}(R_{32})$$

Eulero Angles

body - Axes zxz - 313

$$C = \mathcal{V}\,\mathrm{Rot}(z, e)\,\mathrm{Rot}(x, \theta)\,\mathrm{Rot}(z; \psi)$$

$$= \begin{bmatrix} c\varphi\cdot c\psi - s\varphi\cdot c\vartheta\cdot s\psi & -c\varphi\cdot s\psi - s\varphi\cdot c\vartheta\cdot c\psi & s\varphi\cdot s\vartheta \\ s\varphi\cdot c\psi + c\varphi\cdot c\vartheta\cdot s\psi & -s\varphi\cdot s\psi + c\varphi\cdot c\vartheta\cdot c\psi & -c\varphi\cdot s\vartheta \\ s\vartheta\cdot s\psi & s\vartheta\cdot c\psi & c\vartheta \end{bmatrix}$$

if $\mathrm{sen}\,\theta \neq 0$

$$\theta = \cos^{-1}(R_{33})$$

$$e = \left(\frac{R_{13}}{s\theta} ; \frac{R_{23}}{s\theta}\right)$$

$$\psi = \mathrm{Atan2}\left(\frac{R_{31}}{s\theta} ; \frac{R_{32}}{s\theta}\right)$$

if $\sin\theta = 0$

$\theta = 0°$ $(R_{33} = +1)$ $\Rightarrow$ $\varphi + \psi = \mathrm{Atan2}(R_{21}, R_{11})$

$\theta = 180°$ $(R_{33} = -1)$ $\Rightarrow$ $\varphi - \psi = \mathrm{Atan2}(R_{21}, R_{11})$

Angular velocity

$$\begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \begin{bmatrix} 0 & c\varphi & s\varphi\cdot c\theta \\ 0 & s\varphi & -c\varphi\, s\theta \\ 1 & 0 & c\theta \end{bmatrix} \begin{bmatrix} \dot\varphi \\ \dot\theta \\ \dot\psi \end{bmatrix}$$

FIXED - AXES XYZ

$$C = Rot(x, \varphi)\, Rot(y, \theta)\, Rot(x, \psi)\, U$$

$$= \begin{bmatrix} c\varphi \cdot c\vartheta & c\varphi \cdot s\vartheta \cdot s\psi - s\varphi \cdot c\psi & c\varphi \cdot s\vartheta \cdot c\psi + s\varphi \cdot s\psi \\ s\varphi \cdot c\vartheta & s\varphi \cdot s\vartheta \cdot s\psi + c\varphi \cdot c\psi & s\varphi \cdot s\vartheta \cdot c\psi - c\varphi \cdot s\psi \\ -s\vartheta & c\vartheta \cdot s\psi & c\vartheta \cdot c\psi \end{bmatrix}$$

if $\cos\theta \neq 0$

$$\theta = \sin^{-1}(-R_{31})$$

$$\psi = \mathrm{Atan2}\left(\frac{R_{32}}{c\theta}, \frac{R_{33}}{c\theta}\right)$$

$$\varphi = \mathrm{Atan2}\left(\frac{R_{21}}{c\theta}, \frac{R_{11}}{c\theta}\right)$$

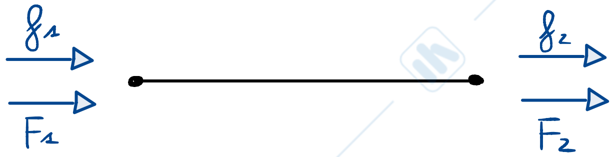
if $\cos\theta = 0$

$\theta = 90°$ $(R_{31} = -1)$ $\Rightarrow$ $\psi - \varphi = \mathrm{Atan2}(R_{12}, R_{22})$

$\theta = 270°$ $(R_{31} = 1)$ $\Rightarrow$ $\psi + \varphi = \mathrm{Atan2}(-R_{12}, R_{22})$

FORMULARY
NMS PART A

ROD ELEMENT

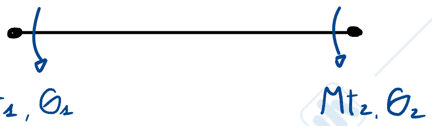


$$K_{ROD} = \frac{EA}{\ell}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

THERMAL EXPANSION

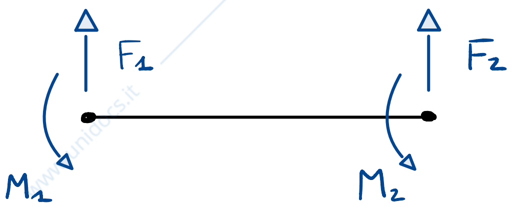
$$\Delta \ell = \ell \cdot \alpha \cdot \Delta T_m$$

TORSION ELEMENT



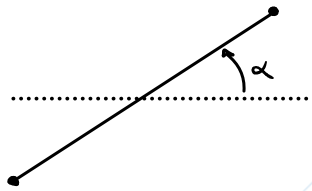
$$K_t = \frac{G J_t}{\ell}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

BEAM ELEMENT



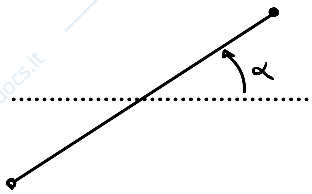
$$K = \begin{bmatrix} \frac{12EJ}{\ell^3} & \frac{6EJ}{\ell^2} & -\frac{12EJ}{\ell^3} & \frac{6EJ}{\ell^2} \\ \frac{6EJ}{\ell^2} & \frac{4EJ}{\ell} & -\frac{6EJ}{\ell^2} & \frac{2EJ}{\ell} \\ -\frac{12EJ}{\ell^3} & -\frac{6EJ}{\ell^2} & \frac{12EJ}{\ell^3} & -\frac{6EJ}{\ell^2} \\ \frac{6EJ}{\ell^2} & \frac{2EJ}{\ell} & -\frac{6EJ}{\ell^2} & \frac{4EJ}{\ell} \end{bmatrix}$$

ROD ELEMENT ROTATION



$$[T] = \begin{bmatrix} C\alpha & S\alpha & 0 & 0 \\ 0 & 0 & C\alpha & S\alpha \end{bmatrix}$$

BEAM ELEMENT ROTATION



$$[T] = \begin{bmatrix} C\alpha & -S\alpha & 0 & 0 & 0 & 0 \\ S\alpha & C\alpha & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & C\alpha & -S\alpha & 0 \\ 0 & 0 & 0 & S\alpha & C\alpha & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$