Guide to exercises

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July 13, 2015

1 Queue theory

$\rho = \frac{\lambda}{\mu}$
WIP $= L_q$
In the priority formulas, $E(S) = W_s$

1.1 Occurences and average throughput time:

1. Calculate W_s for every stage.

   - For M/M/1, we use the formula written in the provided table $W_s = \frac{1}{\mu - \lambda}$
   - For M/M/c with $c > 1$, we first need to calculate L_q using the provided table (intersection between ρ and n. of operators c), then we use the other formula. If the exact ρ is not present, you can linearly interpolate between adjacent values.
   - If you have priority rules at the stage, the W_s is calculated with the appropriate formula
   - Take care of the unit of measurement. If, for example, λ and μ are provided as $\left[\frac{\text{pcs}}{\text{h}}\right]$, the resulting W_s is in hours and you need to multiply for 60 to obtain the results in minutes.

2. Calculate occurences for every stage.

3. The average throughtput time is the weighted average of W_s, weighted by the occurence of each path

Remove scrap from occurences:
Calculate the occurences of all the paths including the scrap one
Remove the scrap paths, and sum the occurences of the non-scrap path = Σ
Remodulate the occurences of the non scrap path, so to make their sum = 100 %
$\text{occ}_{\text{scrap}} : \Sigma = \text{occ}_{\text{no scrap}} : 100$

1.2 Inactivity time

Inactivity time $= P_0 \cdot 60 \left[\frac{\text{min}}{\text{h}}\right] = \frac{\text{min of inactivity}}{\text{h}}$ (see EX 3: Verdi spa)

1.3 Probability of $W_s < x$

(see EX 4 Speed spa question 3)
$P(W_s < x) = ?$
Little law: $\boxed{W_s = \frac{n}{\lambda}}$
n = number of customers in the whole system (the same letter used in the formula handout)
$P(W_s < x) = P(W_s \cdot \lambda < x \cdot \lambda) = P(n < x \cdot \lambda) = 1 - P(n \geq x \cdot \lambda)$
From the formula handout, $P(n \geq k) = \rho^k$
$= 1 - \rho^{(x \cdot \lambda)}$

1.4 Priority

Customers of class 1 have a service admission priority higher than customers of class 2
$E(S) = W_s$

- Preemtive
  $E(S_1) = \frac{1/\mu}{1-\rho_1}$
  $E(S_2) = \frac{1/\mu}{(1-\rho_1)(1-\rho_1-\rho_2)}$
- Non preemtive
  $E(S_1) = \frac{(1+\rho_2)/\mu}{1-\rho_1}$
  $E(S_2) = \frac{(1-\rho_1(1-\rho_1-\rho_2))/\mu}{(1-\rho_1)(1-\rho_1-\rho_2)}$

2 Yield Management

2.1 Protection level: Marginal analysis

X_1 = demand for full price unit
C_u = cost to underestimate the demand of a full price unit (=Mf - Md)
C_o = cost to overestimate the demand of a full price unit (=Md)
$C_o \rightarrow$ PL troppo alto
$C_u \rightarrow$ PL troppo basso
$P(\text{underest.}) \cdot C_u \geq P(\text{overesti.}) \cdot C_o \Rightarrow$

$$\boxed{\mathrm{P}(X_1 < S_1) \leq \frac{\mathrm{C}_u}{\mathrm{C}_u + \mathrm{C}_o}} = \alpha$$

α is the area under the normal distribution between $-\infty$ and Φ_α
From the normal distribution table we obtain Φ_α
$S_1 = \mu + \Phi_\alpha \cdot \sigma$

2.2 From probability to variance

Sometimes the variance of the distribution is not given. Instead, they give us the mean μ and a certain probability $P(a \leq x \leq b)$
$P(a \leq x \leq b) = p \Rightarrow P(x \leq b) = p + \frac{1-p}{2} = \alpha$, we have added the probability of a single tail of the distribution
In then normal distribution, α is the area before z_α, from the normal distribution table we get z_α

$$P\left(\frac{x-\mu}{\sigma} \leq \frac{b-\mu}{\sigma}\right) = P\left(Z \leq \frac{b-\mu}{\sigma}\right) = P\left(Z \leq z_\alpha\right) \text{ for construction}$$

$$\frac{b-\mu}{\sigma} = z_\alpha \Rightarrow \sigma = \frac{b-\mu}{z_\alpha}$$

2.3 Overbooking

Average values approach: $BL = \frac{C}{p}$
BL = booking limit
C = capacity
p = probability that the one who has booked a ticket, then buys it

Marginal approach: $P(\text{Ovb} \geq NS) = \frac{C_o}{C_u + C_o}$

$\text{Ovb} \sim \text{Bin}(n, p)$

2.4 Protection level: Heuristic EMSR

(Expected Marginal Seat Revenue)
n = number of available rates
f_i = unit revenue associated with rate i
$f_1 \geq f_2 \geq \cdots \geq f_n \Rightarrow$ **1 is the most expensive rate, n is the cheapest rate**
μ_i = Average demand for rate i
σ_i^2 = Variance in demand for rate i
θ_i = Level of protection for class i and more expensive classes (1 to i)
D_i = Available demand to pay rate i or more expensive

$$\overline{f}_i = \frac{\sum_{j=1}^{i} \mu_j f_j}{\sum_{j=1}^{i} \mu_j}$$

$$D_i \sim N \text{ with: } \begin{cases} \overline{\mu}_i = \sum_{j=1}^{i} \mu_j \\ \overline{\sigma_i}^2 = \sum_{j=1}^{i} \sigma_j^2 \end{cases}$$

$$F(z_\alpha) = 1 - \frac{f_{i+1}}{\overline{f}_i} = 1 - \frac{\text{next more expensive rate}}{\text{its weighted average mean rate}} \Rightarrow$$

$\Rightarrow$From normal distribution table find z_α
$\theta_i = \overline{\mu_i} + z_\alpha \cdot \overline{\sigma_i}$ = this is the protection level for all classes from 1 (more expensive) to i

2.4.1 EMSR example

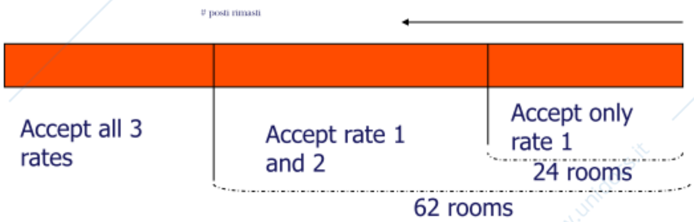
Class	Rate	Mean	Variance
1	100 E	30	50
2	80 E	30	80
3	40 E	50	120

Class	$\overline{f}_i$	$\overline{\mu}(i)$	$\overline{\sigma}^2(i)$
1	100 E	30	50
2	90 E	60	130
3	67.3 E	110	250

Protection level for class 1:
$F_1(z_\alpha) = 1 - \frac{80}{100} = 0.2 \Rightarrow z_\alpha = -0.84 \Rightarrow$

$\theta_1 = 30 - 0.84 \cdot \sqrt{50} = 24.06 \approx 24$ rooms

Protection level for class 1 and 2 togheter:
$F_2(z_\alpha) = 1 - \dfrac{40}{90} = 0.5556 \Rightarrow z_\alpha = 0.14 \Rightarrow$
$\theta_2 = 60 + 0.14 \cdot \sqrt{130} = 61.6 \approx 62$ rooms



3 Lean Management

CT = how much time passed btw two exits, two pieces

3.1 EPE definition

EPE of stage i: $\text{EPE}_i = \dfrac{T_{SU}}{T_{A,i} - T_{P,i}}$
Where T_{SU} is the time to set up all the range
$T_{SU} = t_{SU} \cdot$ number of products in the range
$T_{A,i} = T_{\text{AV,COMPANY}} \cdot \%$ dedication of the stage to the product family
$T_P = D \cdot \dfrac{\text{CT}}{\text{A}}$
If you calculate the EPE for a product family, T_{AV} is the time dedicated only to that product family.
For example, suppose you have family A and B, with family B to produce in a fixed amount. EPE for family A is:
$T_{AV,\text{family A}} = T_{AV,TOT} - T_{\text{family B}} = T_{AV,TOT} - T_{P,\text{family B}} - T_{\text{SU, family B}}$
If $T_{\text{P,family A}} < T_{\text{AV,family A}}$, this means EPE < 0, try to reduce single setup time (t_{SU}) in order to also reduce the total setup time for family B ($T_{\text{SU,family B}}$) and have more time available for family A (see EX LEAN 3, ex. 2: shirts)

3.2 Future state

8 questions:

1. What is the takt time of the production family?
   Takt time of stage $i = TT_i = \frac{T_{AV,i}}{D} \left[\frac{\text{sec}}{\text{pcs}}\right]$
   $T_{AV,i}$ = time plant opening − schedule stops
   D = total demand for all products of all family

2. The company needs to make for a finished good supermarket or directly for shipping?

   - Delivery time to the customer
     Lead time of the reaction (from the pacemaker on) < customer availability to wait
   - Product features
     - Good value
     - Obsolescence level
     - Standardization level of the product
     - Range (low number of variants favours stock)
   - Demand predictability
   - Demand stability

3. Where to put the flow?

   - How to create the continous flow?
   - How many resources are needed and how to allocate them to the various product families?
   - What actions are needed? To what extent will these interventions be?

   GENERAL METHODOLOGY:

   - Start from the final stage and go upstream thinking stage by stage where to put CONTINUOUS FLOW and where to decouple (with SUPERMARKET or FIFO). Departments that work with different number of shifts must be decoupled.
   - Use DeCAF
   - Fix intermediate targets (not necessarily all at once in a continuous flow, but also FIFO and supermarket)

In case the stage is not dedicated:
Calculate the total capacity that the stage put into the product family:
$C = \text{n. operators} \cdot T_{\text{AV}} \cdot$ % dedication to the product family
This is the total time that the stage put into the product family.
See if you can re-allocate the time among operators in order to dedicate some operators (e.g. 1 operator) entirely to the product family.
Pick up some operators to dedicate entirely to the product family.
This means to dedicate the stage to the profuct family.
The constraint is for the operators to work $< T_{\text{AV}}$ in a day.
$\frac{C}{\text{n. of desireded operators}} < T_{\text{AV}}$
(see EX LEAN 3, ex. 2: shirts)

DECAF (to be applied for the cell):

- **De**dicated: is the stage entirely dedicated to the product family? Yes/No
- **C**apable: $CT_{SYS} = \max(CT_i)$
  Condition $CT_{SYS} < TT$
  If there are different product family, you can use a weighted average for the CT with the demand of the different families as weights. The CT for the single family is still the max CT among the stages (see EX LEAN 2)
- **A**vailable: $A_{SYS} = \prod A_i$
  Condition: $\frac{CT_{SYS}}{A_{SYS}} < TT$
- **F**lexible: $\text{EPE}_{\text{cell}} \leq \text{EPE}_{\text{customer}}$

4. Where to enable a pull supermarket?
   Supermarket size $= EPE_{\text{upstream stage}} \cdot D \cdot 1.5$
   $EPE_{\text{supplier}} =$ interarrival time
   Security coefficient is 2 for external suppliers

   Put supermarket if:

   - Low availability of the stage: decoupling avoids that the stage enter as a factor in the calculation of A_{SYS}
   - Different reasons to setup and the combination of variants is high enough to require reductions in setup time that are not possible

yet (FOR THE EXAMINATION PUT SUPERMARKET if setup type and different number of results variants very high)

- Stage is not dedicated but shared
- Different shifts (turni di lavoro)
- For the exam, if you have a stage in one phase, all the upstream stages have a supermarket

> If two stages work on different shift, you can also put a FIFO lane between them.
> Its size is: $\text{FIFO} = \Delta\text{Shifts} \cdot D \cdot 1.5$
> Where D is the demand for the single shift of the slowest stage
> (see EX LEAN 4)

5. What only point in the production chain (the pacemaker process) does the company have to plan?
When we have supermarket (replenishment pull), the pacemaker is the last stage, typically the cell
When we have FIFOs (sequential pull), the pacemaker is the first stage before the lane starts

6. How should the company level the production mix at the pacemaker process?

7. What should be the increase of work to be released to the pacemaker process?

8. What improvements to the process will be required to get the Value Stream flow described by the future state?

<u>Number of operators:</u> $= \dfrac{WKC}{TT}$
<u>Batch</u>: $b =$ batch size

1. D/γ is the demand of the single product code
$b = \dfrac{D}{\gamma} \cdot EPE$

2. $\left(TT - \dfrac{CT}{A}\right)$ is the time left to do the setups
We start at $t = 0$,
at $t = b \cdot TT$ we do a setup and change to produce another product

code
$b \geq \dfrac{t_{SU}}{TT - \dfrac{CT}{A}}$

3. To calculate batch size, you first need to convert EPE from days to minutes. In order to do so, you multiply by the T_{AV} for that product family only (see above)
   γ = product range, number of different products
   x = batch size, number of products for each family to produce within the EPE
   $\gamma \cdot x \cdot \dfrac{CT}{A} + \gamma \cdot \text{CO} = \text{EPE}$
   and you solve for x

Convert stocks from days to pieces, you divide by customer demand
$\text{Stocks [days]} = \dfrac{\text{Stocks [pieces]}}{D}$
Note that, if for example you need 2 pieces for every demanded unit of final product, you have to divide by $2 \cdot D$

3.3 Sizing Capacity

1. **Workload vs Capacity method**

   If breakdowns are possible only when the machine is working:
   $$\underbrace{\frac{WKC}{A} \cdot D + TCO}_{\text{workload}} \leq C$$
   If breakdowns are also possible while machine is undergoing setups:
   $$\frac{WKC}{A} \cdot D + \frac{TCO}{A} \leq C$$
   WKC = Work Content

   TCO = Total Change Over time

   C = capacity

   The **work content** of a job is the amount of time you need to "put" into that job. If there is more than one operator, they share the work content equally.

2. **Workpace method**

   CT if workload is perfectly balanced between operators:
   CT = WKC / # of operators

CO if workload is perfectly balanced between operators:
CO = TCO / # of operators