PART A- Optical waveguides

Photonic Devices

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Chapter 2: optical waveguides

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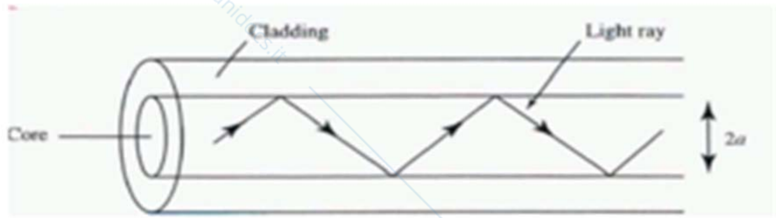
What is an optical waveguide?

- An optical waveguide is a spatially inhomogeneous structure for guiding light. The structure is defined for restricting the spatial region in which light can propagate. Usually, a waveguide contains a region of increased refractive index (the core), compared with the surrounding medium (called *cladding*)

https://www.rpphotonics.com/waveguides.html

Example: optical fiber

circular dielectric waveguides



- An optical waveguide is one of the fundamental elements of photonic devices

Lesson A1:

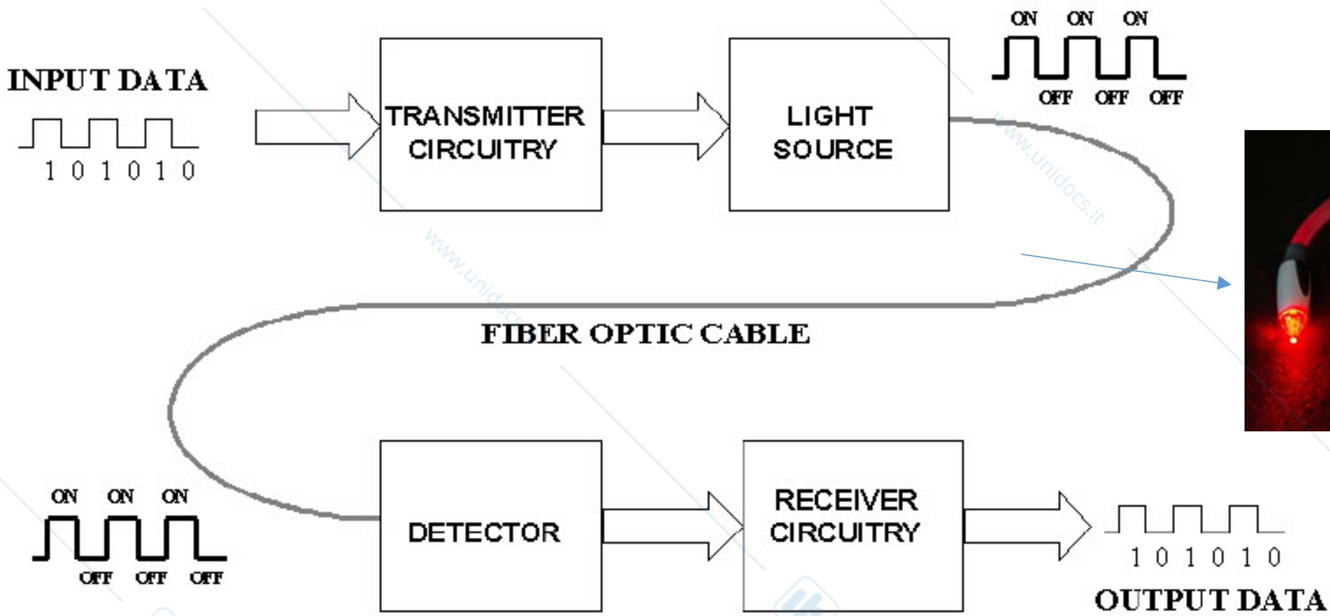
- Motivations
- Basic Principles

Motivations

Examples of optical waveguides : optical fibers

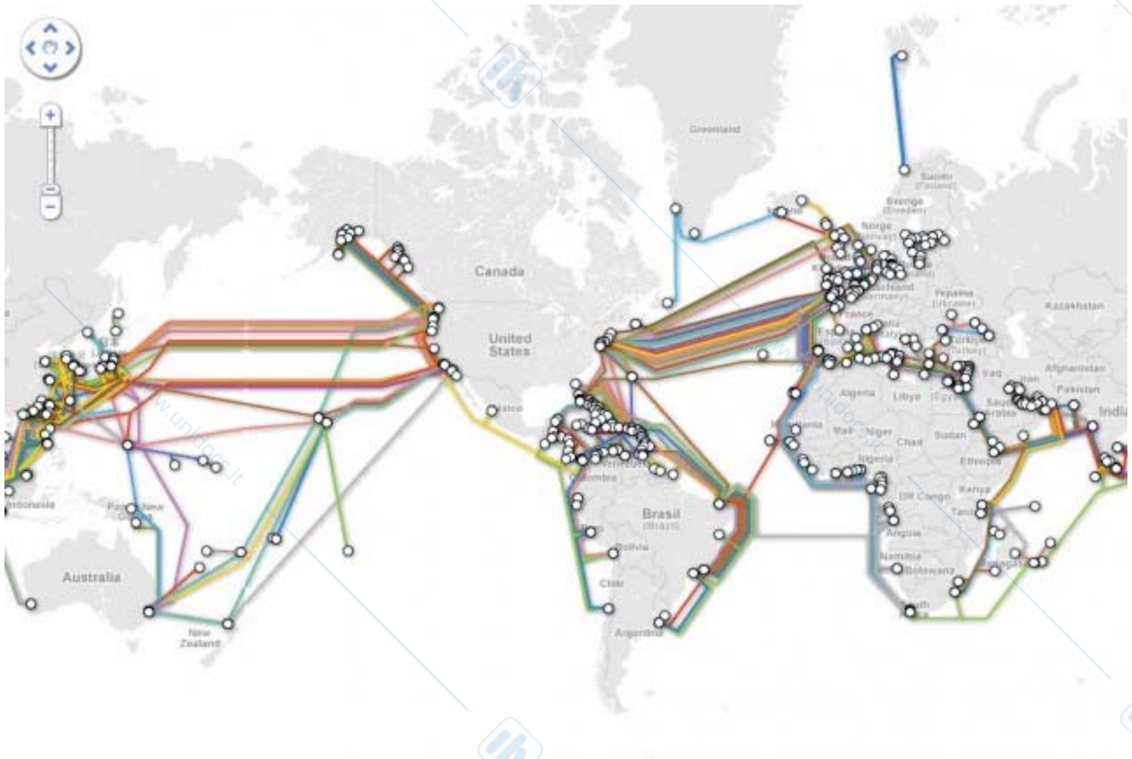
An example of optical waveguide is the optical fiber that finds many applications in optical communication systems.

Very simple schematic of an optical transmission link



Optical fibers for long distance link:

submarine optical cables permit very long distance optical communication



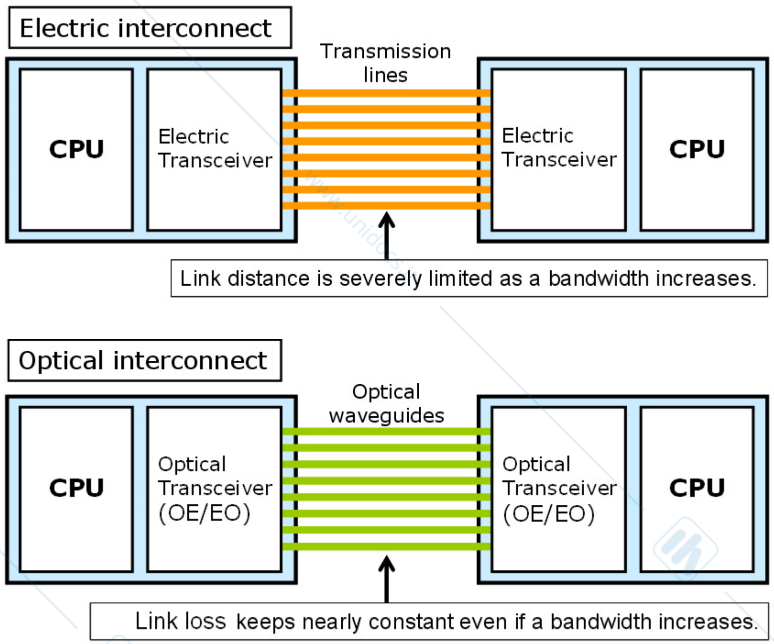
"If we were to unravel all of the glass fibers that wind around the globe, we would get a single thread over one billion kilometers long – which is enough to encircle the globe more than 25 000 times – and is increasing by thousands of kilometers every hour."

-- **Charles K. Kao** Nobel Prize in Physics, 2009 *for groundbreaking achievements concerning the transmission of light in fibers for optical communication"*

Optical waveguide in short distance links:

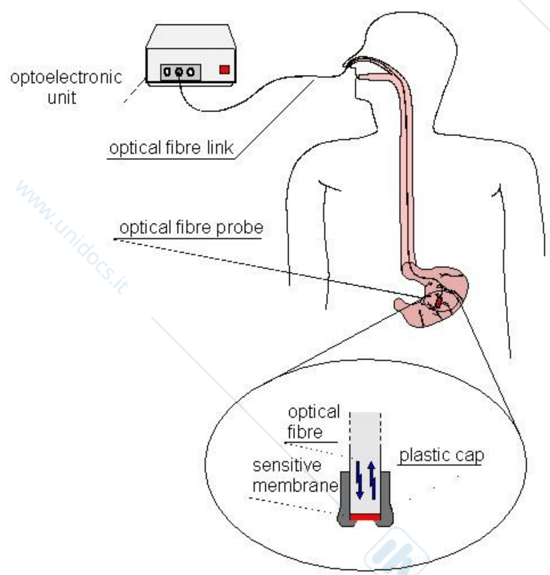
The use of light for transferring information at short distance is generally referred as “Optical interconnect”

As shown in this example, the physical medium for connecting the two transceivers can be an optical fiber or another optical waveguide.



Optical fibers for non-telecom applications:

It is well know that optical fibers find applications in several other fields; for example in medicine or to realize various types of sensors etc..



Optical waveguides in photonic devices (I):

Optical waveguides are also the fundamental “building block” of many optoelectronic devices such as semiconductor lasers or optical modulators

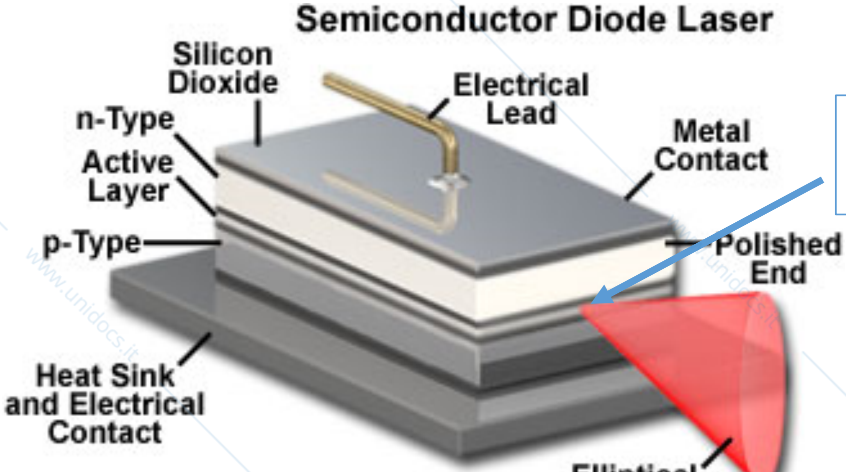
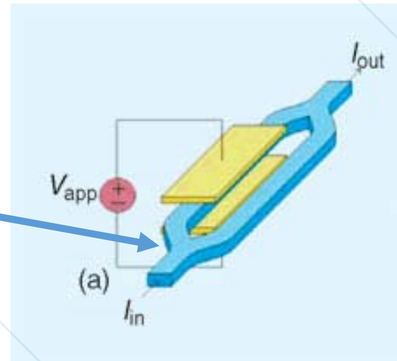
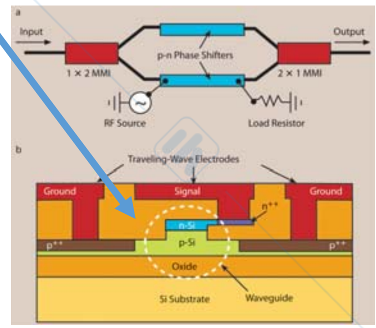


Figure 9

Here we have an optical waveguide!

Optical Modulator

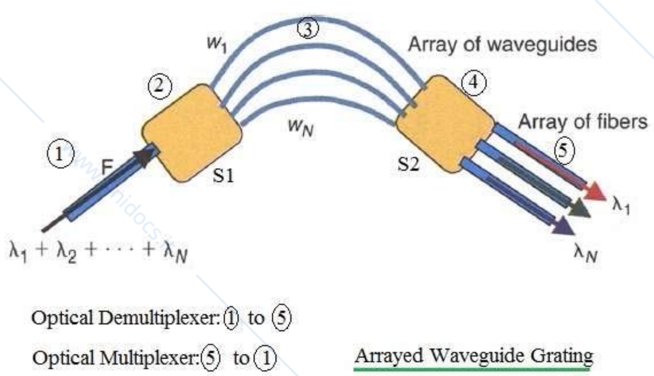




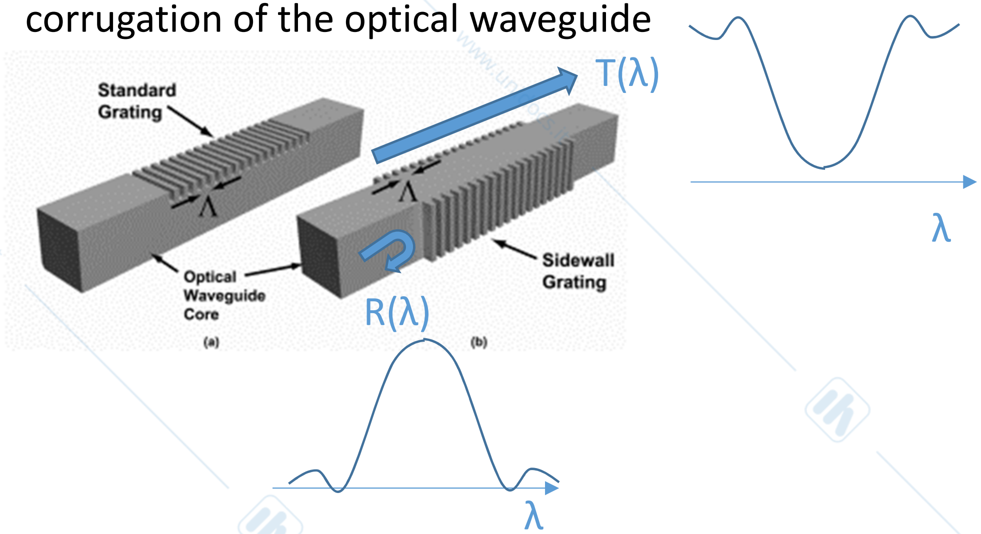
Optical waveguides in photonic devices (II):

Optical waveguides are also fundamental for devices that process (ie: switch, split, filter, …) optical signals in photonic integrated circuits. Here are some examples:

Optical MUX/DEMUX



Optical filter or Bragg mirror: periodic corrugation of the optical waveguide

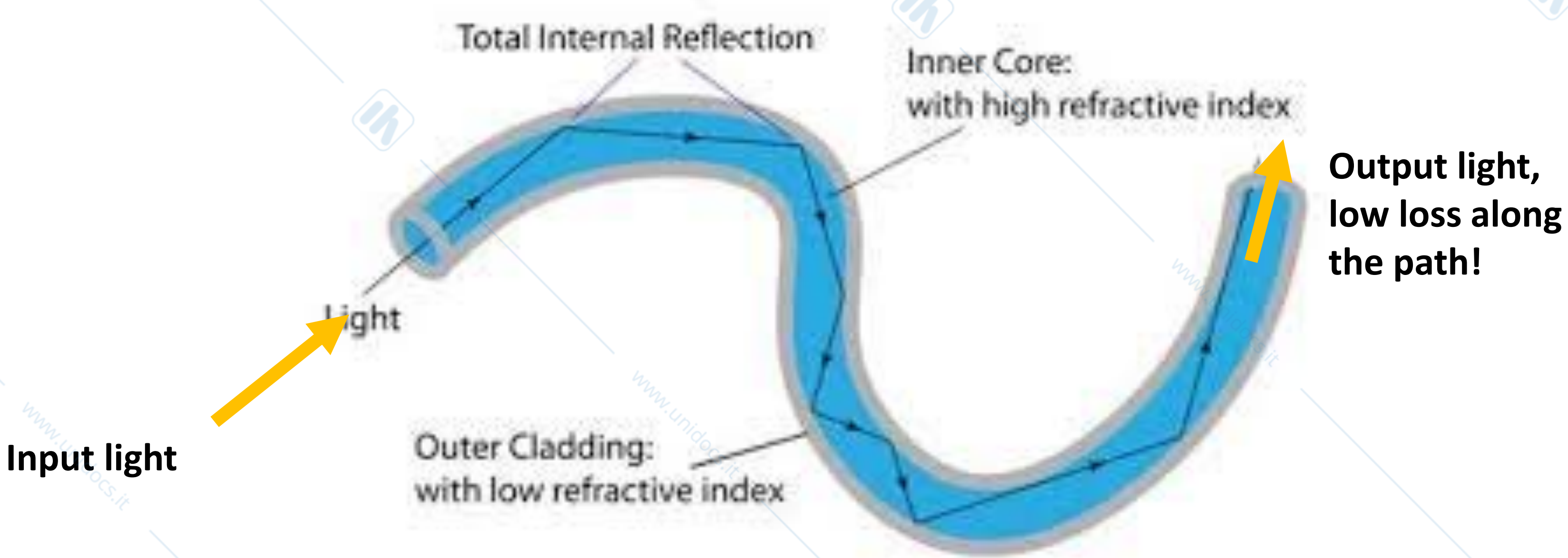


Basic Principles

OPTICAL WAVEGUIDES

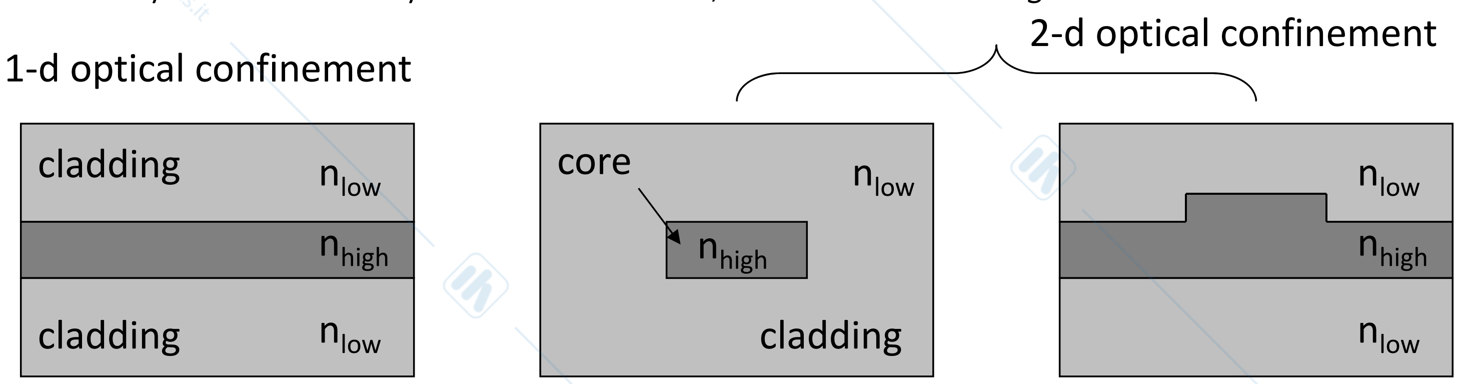
- Waveguide principle
- Optical waveguide fundamentals: the planar slab waveguide
- Method for the analysis
- Waveguide modes
- Channel waveguides
- Methods for the analysis
- Conclusions

Basic Principle



Geometry and terminology:

The basic structure of a dielectric optical waveguide consists of an high-index optical medium, called the *core*, which is transversely surrounded by low-index media, called the *cladding*.

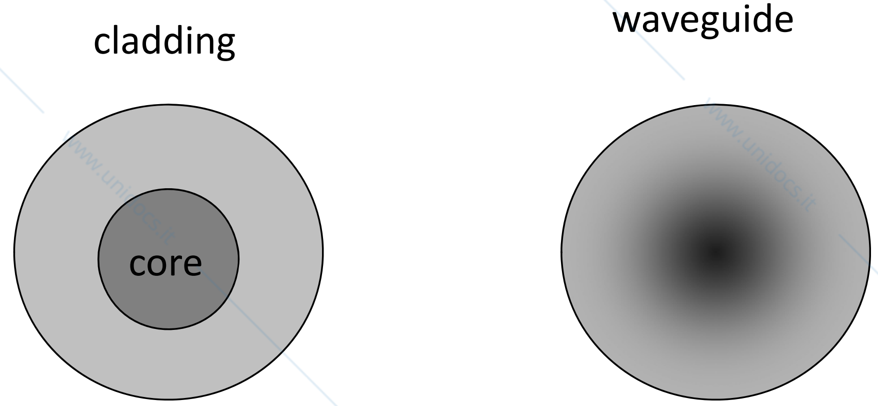


Slab waveguide

Channel/photonic wire waveguide

Rib/ridge waveguide

Planar waveguides:
Usually integrated waveguides
as for example in semiconductor



Step-index fiber

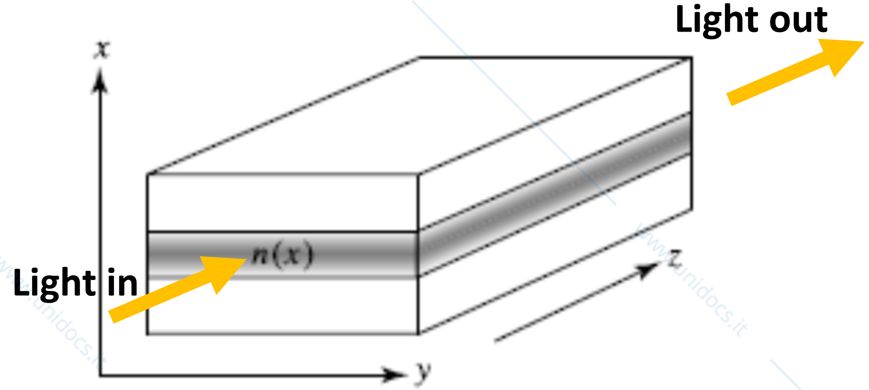
Graded-index (GRIN) fiber

Circular waveguides
as for examples optical fibers **(**optical fibers are in glass material)

Let’s start from a simple structure:

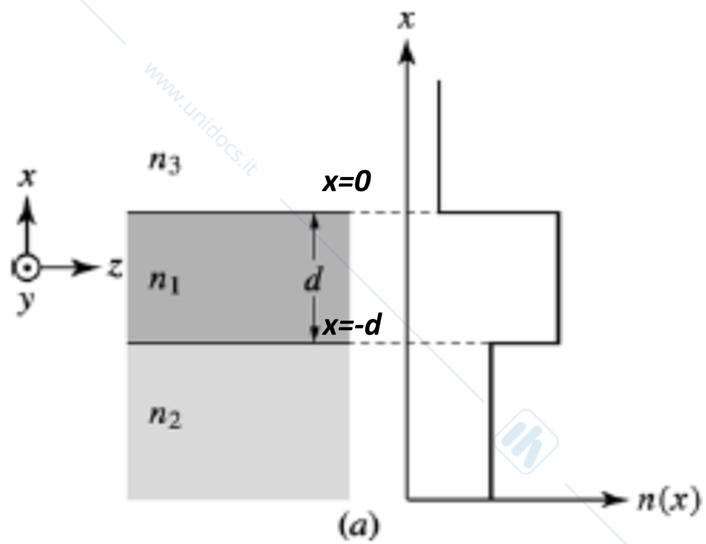
We consider a straight waveguide whose longitudinal direction is taken to be the *z* direction.
The characteristics of a waveguide are determined by the transverse (x,y) profile of its dielectric constant, which is independent of the *z* coordinate. For a waveguide made of optically isotropic media, we can simply characterize the waveguide with a transverse profile of the index of refraction, $n(x, y)$.

Step index slab waveguide



A guided optical wave propagates in the

waveguide along its longitudinal direction;
ie: z direction in this figures



An example: Silicon on silica waveguides

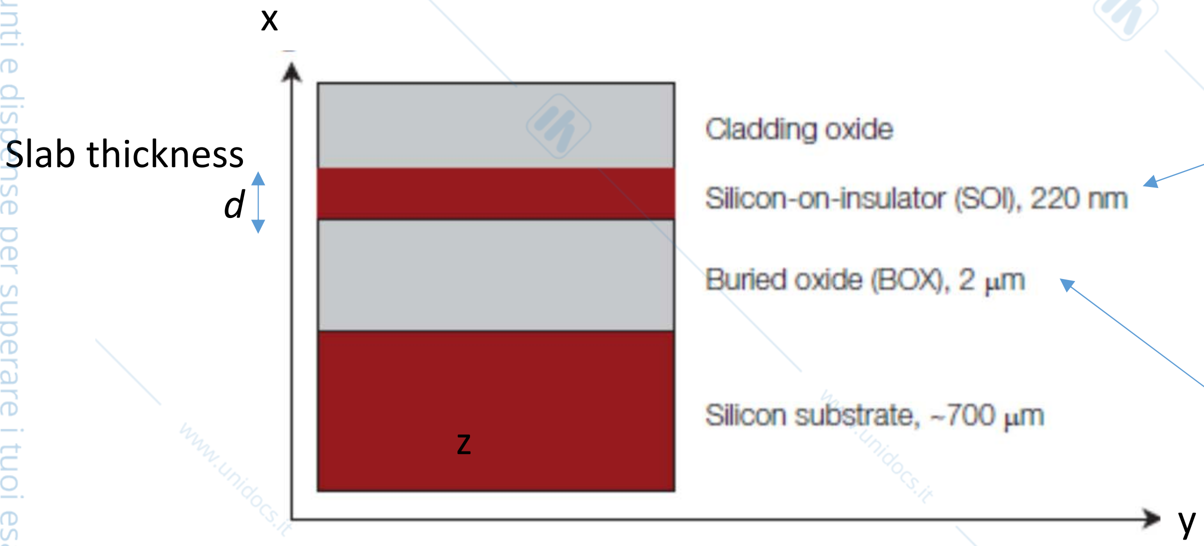


Figure 3.1 Cross-sectional view of silicon-on-insulator (SOI) wafer.

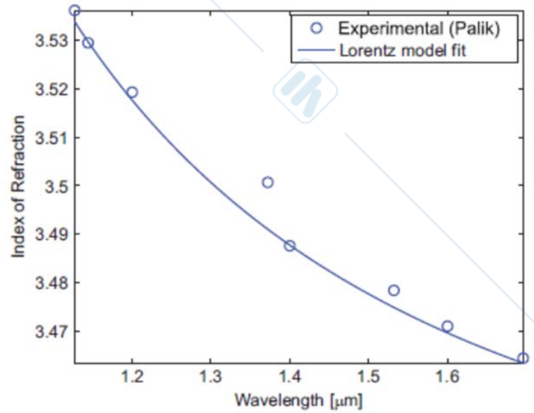


Figure 3.2 Index of refraction of silicon at room temperature, $T = 300$ K. Data fit using Lorentz model.

Silicon dioxide (also termed glass or silica) has index of refraction about 1.44 at 1550 nm

Silicon photonics wafers

The wafers commonly used for silicon photonics are termed “silicon-on-insulator”. These are the same as used in the electronics industry. For silicon photonics, the typical 200 mm (8”) wafer consists of a 725 µm silicon substrate, 2 µm of oxide (buried oxide, or BOX), and 220 nm of crystalline silicon, as shown in the figure of previous slide. It is in the top crystalline silicon layer that waveguides and devices are defined. Hence, the material properties of this silicon are important for designing optical (and optoelectronic) devices.

The 220 nm thickness has become a standard used in particular by multi-project wafer foundries and foundry service providers (e.g. Imec, LETI, IME). However, it should be noted that other thicknesses are also in use (e.g. some company as Luxtera, Kotura, Skorpios use much thicker crystalline silicon layer).

Calculation of the optical waveguide modes

Definition of waveguide mode (I)

A waveguide mode is a transverse field pattern, solution of the Maxwell equation, whose amplitude and polarization profiles remain constant along the longitudinal *z* coordinate.

Therefore, the electric and magnetic fields of a mode can be written in the following form:

Transverse field profile

$$\mathbf{E}_\nu(\mathbf{r}, t) = \mathcal{E}_\nu(x, y) \exp(i\beta_\nu z - i\omega t),$$
$$\mathbf{H}_\nu(\mathbf{r}, t) = \mathcal{H}_\nu(x, y) \exp(i\beta_\nu z - i\omega t),$$

where *v* is the *mode index* and ***β**$_v$* **is the propagation constant** of the mode.

Definition of waveguide mode (I)

- For the transverse field profile we will use in general the following notation:

$$\mathcal{E}_\nu(x, y) = Ex(x,y)\,\hat{\boldsymbol{x}} + Ey(x,y)\,\hat{\boldsymbol{y}} + Ez(x,y)\hat{\boldsymbol{z}}$$

$$\mathcal{H}_\nu(x, y) = Hx(x,y)\,\hat{\boldsymbol{x}} + Hy(x,y)\,\hat{\boldsymbol{y}} + Hz(x,y)\hat{\boldsymbol{z}}$$

We start from Maxwell's equations (I):

$$\nabla \times \mathbf{E} = -\mu_0 \frac{\partial \mathbf{H}}{\partial t},$$

$$\nabla \times \mathbf{H} = \epsilon \frac{\partial \mathbf{E}}{\partial t}.$$

In this analysis we consider only the real part of the refractive index and we neglect the imaginary part (=> the waveguide medium has therefore neither gain nor loss).

Where $\varepsilon = \varepsilon_0 \cdot n^2_{1,2,3}$

Maxwell’s equation (II)

Because the electromagnetic field has the form reported in slide 20, substituting in the Maxwell equation we have the following system

$$\frac{\partial \mathcal{E}_z}{\partial y} - i\beta\mathcal{E}_y = i\omega\mu_0\mathcal{H}_x,$$

$$i\beta\mathcal{E}_x - \frac{\partial \mathcal{E}_z}{\partial x} = i\omega\mu_0\mathcal{H}_y,$$

$$\frac{\partial \mathcal{E}_y}{\partial x} - \frac{\partial \mathcal{E}_x}{\partial y} = i\omega\mu_0\mathcal{H}_z,$$

and

$$\frac{\partial \mathcal{H}_z}{\partial y} - i\beta\mathcal{H}_y = -i\omega\epsilon\mathcal{E}_x,$$

$$i\beta\mathcal{H}_x - \frac{\partial \mathcal{H}_z}{\partial x} = -i\omega\epsilon\mathcal{E}_y,$$

$$\frac{\partial \mathcal{H}_y}{\partial x} - \frac{\partial \mathcal{H}_x}{\partial y} = -i\omega\epsilon\mathcal{E}_z.$$

In the case of slab waveguide …

To find the modes of slab waveguide we can make the following simplifications:

- No variation of **ε** in *y*- direction and infinitely long in *y*-direction =>

$\frac{\partial}{\partial y} = 0$ => the mode profiles do not depend on *y*- coordinate.

- Classification of the modes:

TE (transverse electric) modes:

- It is a mode with $E_z=0$. In the slab case (substituting in the system of slide 22) we also get that: $E_x=0$ and $H_y=0$. The non- zero components are $E_y(x,y) \neq 0$ for the electric field; for the magnetic field they are $H_x(x,y) \neq 0$ and $H_z(x,y) \neq 0$

TM (transverse magnetic) modes:

- It is a mode with $H_z=0$. In the slab case (substituting in the system of slide 22) we also get that: $H_x=0$ and $E_y=0$. The non- zero components are $H_y(x,y) \neq 0$ for the magnetic field and, for the electric field, $E_x(x,y) \neq 0$ and $E_z(x,y) \neq 0$.

Summary for TE modes and slab case

- TE modes have only 3 field components: Ey,Hx,Hz
- The slab is infinitely extended in *y*- and *z*-directions
- We search for solutions of the form:

$$Ey = Ey(x) \cdot e^{-j\beta z} \cdot e^{j\omega t}$$

- We want to find:

Ey(x), Hx,z(x) and β for any value of ω=>

- Exercize: $\omega = 2\pi f \quad f = \frac{c}{\lambda} \quad for\ \lambda = 1.55\ \mu m$ calculate f. *f*= …… ?

Calculation of TE modes of slab waveguide

Substituting in Maxwell equations (I)…

$$\begin{cases} -j\beta H_x - \dfrac{\partial H_z}{\partial x} = j\omega\epsilon_0 n^2 E_y \\ j\beta E_y = -j\omega\mu_0 H_x \\ \dfrac{\partial E_y}{\partial x} = -j\omega\mu_0 H_z \end{cases}$$

Hx,z components can be calculated from *Ey(x).*

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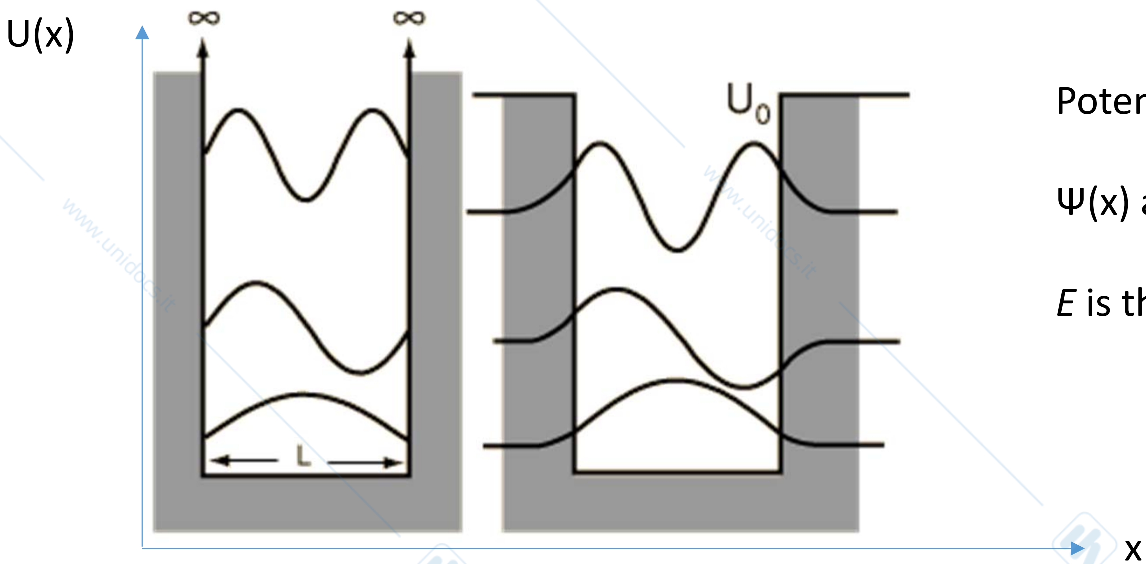
Substituting in Maxwell equations (II)…

$$\frac{\partial^2 E_y}{\partial x^2} + (n^2 k_0^2 - \beta^2) E_y = 0 \quad with \quad k_0^2 = \omega^2 \epsilon_0 \mu_0 = \left(\frac{2\pi}{\lambda}\right)^2$$

- 1 Dimensional second order differential equation
- Wave equation
- The solution provides *Ey(x)* and β

Similarity with other wave equations: **Schrodinger equation**

$$\frac{\partial^2 \Psi}{\partial x^2} = -\frac{2m}{\hbar^2}[E - U(x)]\Psi(x)$$



Potential well of infinite (left) and finite (right) height.

Ψ(x) are the wave functions => eigenvectors

E is the energy of the confined states

Similarity with other wave equations: Transmission lines

$$\frac{d^2}{dz^2}V(z,\omega) + k^2 V(z,\omega) = 0$$

- $V(z)$ => $Ey(x)$ of slide 27
- k^2 => $(n^2 k_0^2 - \beta^2)$ of slide 27

⇒ Class exercise and LAB 1

Boundary conditions for slab case

- Boundary conditions: the solution must satisfy the boundary conditions at the two dielectric interfaces at $x=0$ and $x=-d$. It is required that the tangential **E** and **H** fields are continuous at the dielectric discontinuities.

- We thus require that E_y and H_z are continuous at $x=0$ and $x=-d$.

Solution of the wave equation of slide 27 for guided modes

- We want the electric field to be confined in the core region => **guided mode**
- We therefore impose

$$E_y = \begin{cases} Ae^{-\delta x} & if\ x > 0 \\ A\cos(k_x x) + B\sin(k_x x) & if\ -d < x < 0 \\ (A\cos(k_x d) - B\sin(k_x d))e^{\gamma(x+d)} & if\ x < -d \end{cases}$$

δ real number $\longrightarrow$ positive
$\longrightarrow E_y$ vanishes for $x \to \infty$

γ real number $\longrightarrow$ positive
$\longrightarrow E_y$ vanishes for $x \to -\infty$

Ey in this form satisfies boundary conditions!!

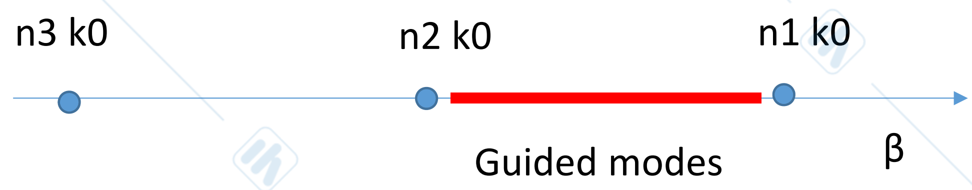
Substituting in the wave equation:

$$\delta = \sqrt{\beta^2 - n_3^2 k_0^2} \qquad \gamma = \sqrt{\beta^2 - n_2^2 k_0^2}$$

$$k_x = \sqrt{n_1^2 k_0^2 - \beta^2}$$

(See next slide for the calculation of *kx*)

Note that δ, γ and *kx* must be real and positive => this impose constrains on β values!!!



Calculation of *kx:*

$$-A\left(\frac{\partial(\sin(k_x x))}{\partial x}\right)k_x + Bk_x\frac{\partial(\cos(k_x x))}{\partial x} + (n_1^2k_0^2 - \beta^2)E_y = 0$$

$$-\cancel{A}k_x^2\cancel{\cos(k_x x)} + \cancel{B}k_x^2\cancel{\sin(k_x x)} + (n_1^2k_0^2 - \beta^2)\cancel{E_y} = 0$$

$$k_x = \sqrt{n_1^2k_0^2 - \beta^2}$$

Boundary conditions for Hz (I):

- We calculate Hz from: $$\frac{\partial E_y}{\partial x} = -j\omega\mu_0 H_z$$

- We impose *Hz* to be continuous at the two interfaces:

$$\begin{cases} \delta A + k_x B = 0 & \text{in x=0} \\ [k_x \sin(k_x d) - \gamma \cos(k_x d)]A + [k_x \cos(k_x d) + \gamma \sin(k_x d)]B = 0 & \text{in x=-d} \end{cases}$$

Solution of the system of slide 34

- The non-trivial solution of the system requires:

$$in\ x = 0 \Longrightarrow \quad \frac{B}{A} = -\frac{\delta}{k_x}$$

$$in\ x = -d$$

$$\tan(k_x d) = \frac{k_x(\gamma + \delta)}{k_x^2 - \gamma\delta}$$

The unknown is kx, the solution provides kx and therefore β,δ, γ

F(kx·d)

Range of *kx·d* for guided modes:

- Let's write first F(kx·d)= $\frac{(k_x d) * [\gamma d + \delta d]}{k_x^2 d^2 - \gamma * d\delta d}$

- Let's calculate minimum and maximum *kx·d* for guided modes:

Since β in the range n_2k_0-n_1k_0 we have:

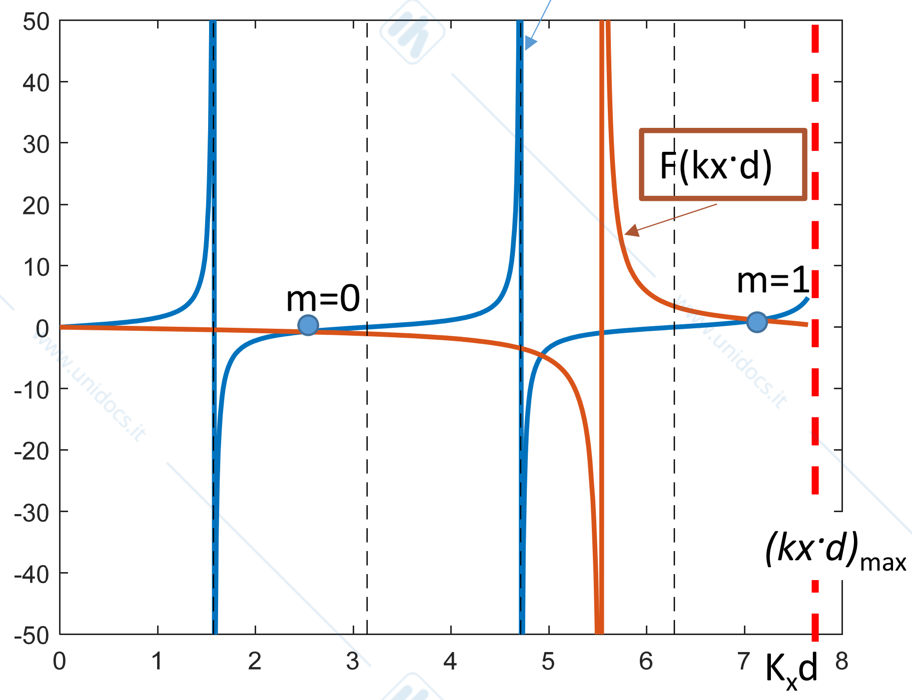
- When $\beta = n_1k_0$ => $k_{x_{min}} d = 0$

- When $\beta = n_2k_0$ =>

$$k_{x_{max}} d = \sqrt{n_1^2 k_0^2 - n_2^2 k_0^2} d = k_0 d \sqrt{n_1^2 - n_2^2}$$

Graphical solution

$$\tan(k_x d) = \frac{(k_x d) * [\gamma d + \delta d]}{k_x^2 d^2 - \gamma * d\delta d}$$



- Intersect points provide Kx=> β => δ

- Since we search for guided modes we consider only the intersect between 0 and $(kx \cdot d)_{max}$

- In this example we have only two guides modes.

Having *(kx·d) of each mode,* we can then

- plot Ey(x) and the magnetic fields Hx and Hz for each guided mode
- calculate the propagation constant β of each guided mode

Example of guided mode profiles –
The field is almost localized in the high refractive index region

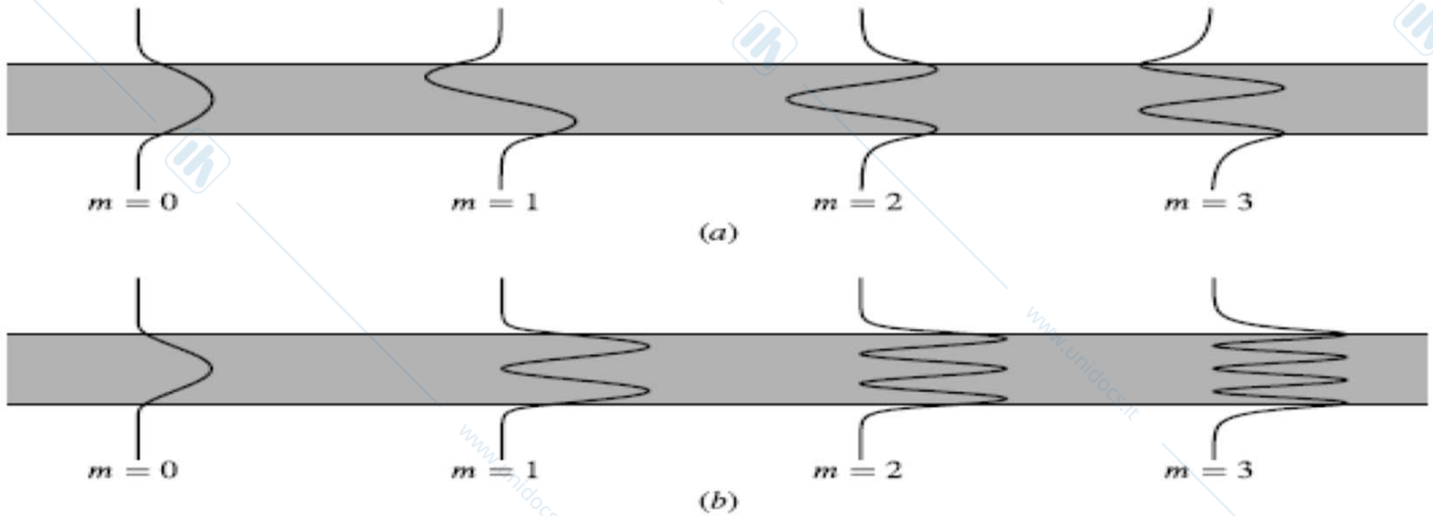
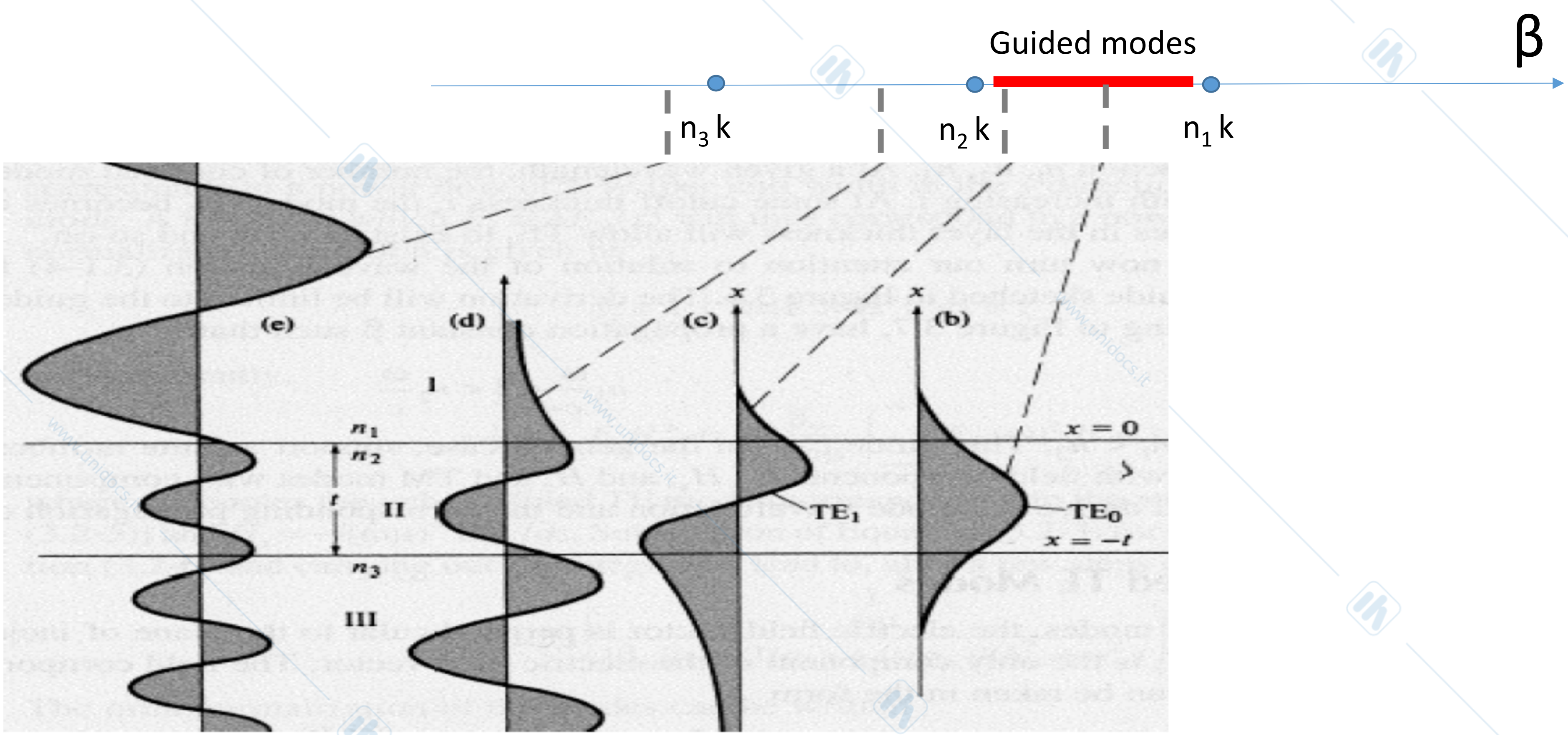


Figure 2.9 (*a*) Field patterns and (*b*) intensity distributions of the first few guided modes of a symmetric slab waveguide.

These are examples of modal field (a) and intensity distribution (b) of some modes in a planar multimode waveguide. If only the m=0 solution exists the waveguide is called mono-mode waveguide.

Radiated modes: when Kxd > $(Kxd)_{max}$

The Electric field no more localized in the high refractive index region



Effective refractive index

For any guided mode we find:

- Kx
- From Kx we find β
- From β we define the effective refractive index as:

$$n_{eff}=\beta/k_o$$

It turns that for guided modes

$$n2<neff<n1$$

We can then write $\beta=\omega/c\cdot neff=2\pi/\lambda\cdot neff$

The **effective refractive index** can be understood as the "refractive index" that the modal distribution in (x,y)-plane "sees" when propagating in the waveguide.

Summary slide-

1) We know now what an optical waveguide is.

2) The modes can be classified respect to the **two possible polarization states** characterized by the major components of the E and H field:

a) when **Ey and Hx** are the most relevant components, the mode is called:

quasi-TE or **TE mode**, where TE stands for Transverse Electric

b) when **Ex and Hy** are the most relevant components the mode is called:

quasi-TM or **TM mode**, where TM stands for Transverse Magnetic

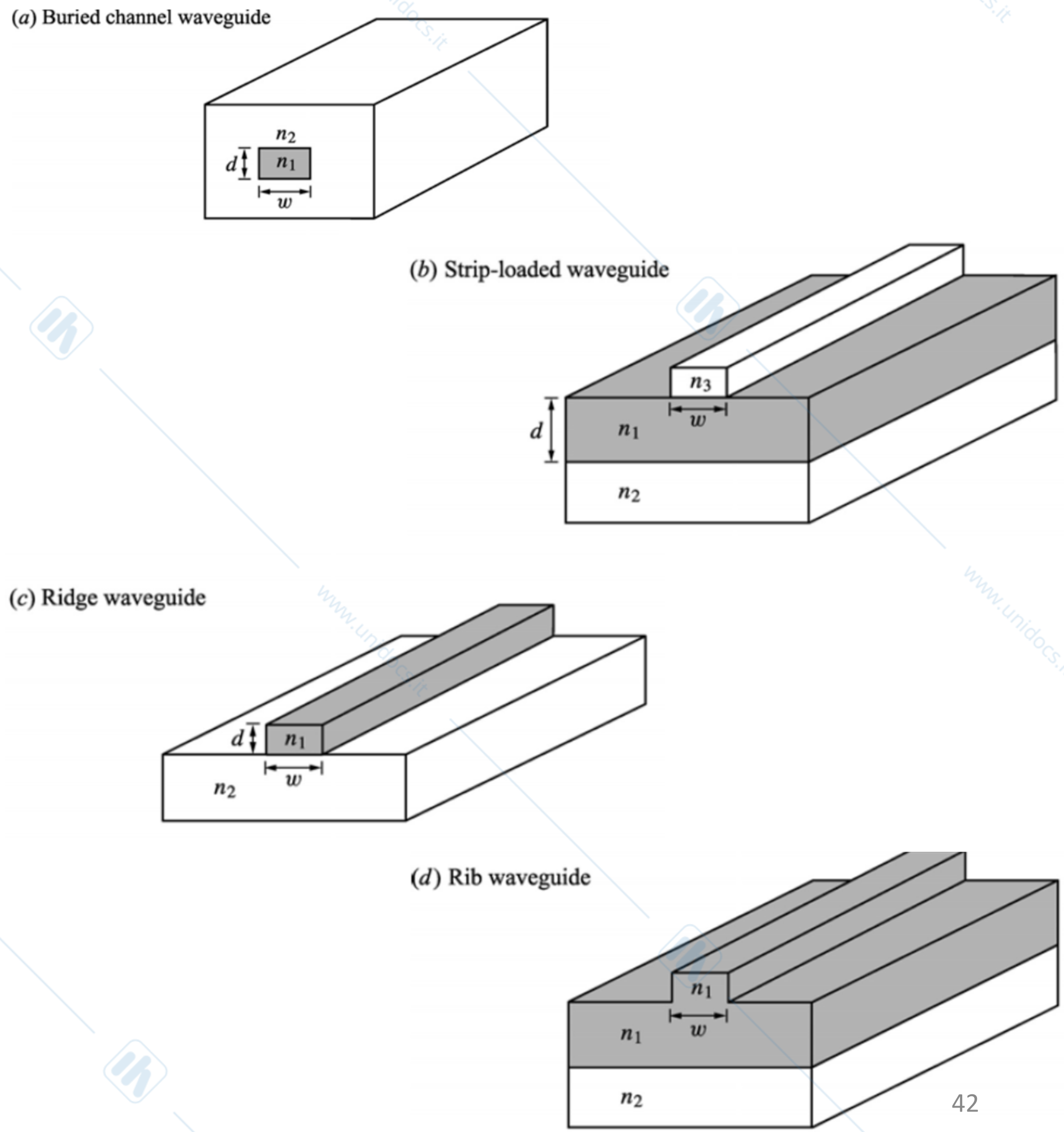
2) The guided modes are usually of major interest to describe the behavior and to design the photonic devices considered in this course. In particular reducing the waveguide lateral dimensions you can reduce the number of solutions to only 1 for each field polarization; in these conditions the waveguide is called mono-mode (or single mode) waveguide.

3) Depending on the refractive index difference and the wavelength we can choose the waveguide thickness to have only one guided mode.

4) Very important mathematical property of the ensemble modes: the modes are a complete and orthogonal set of functions and therefore their linear combination allows to represent the most general electromagnetic field.

Channel waveguides

So far we have discussed the characteristics of slab waveguides. In practice, most waveguides used in device applications are non-slab waveguides. For a non-slab waveguide, the index profile $n(x, y)$ is a function of both transverse coordinates x and y. There are many different types of non-slab waveguides that are differentiated by the distinctive features of their index profiles. An important group of non-slab waveguides is the ***channel waveguide***, which include for example the *buried channel waveguides*, the *strip-loaded waveguides*, the *ridge waveguides*.



Analysis methods to find optical modes

- Finite Element Methods (FEM)

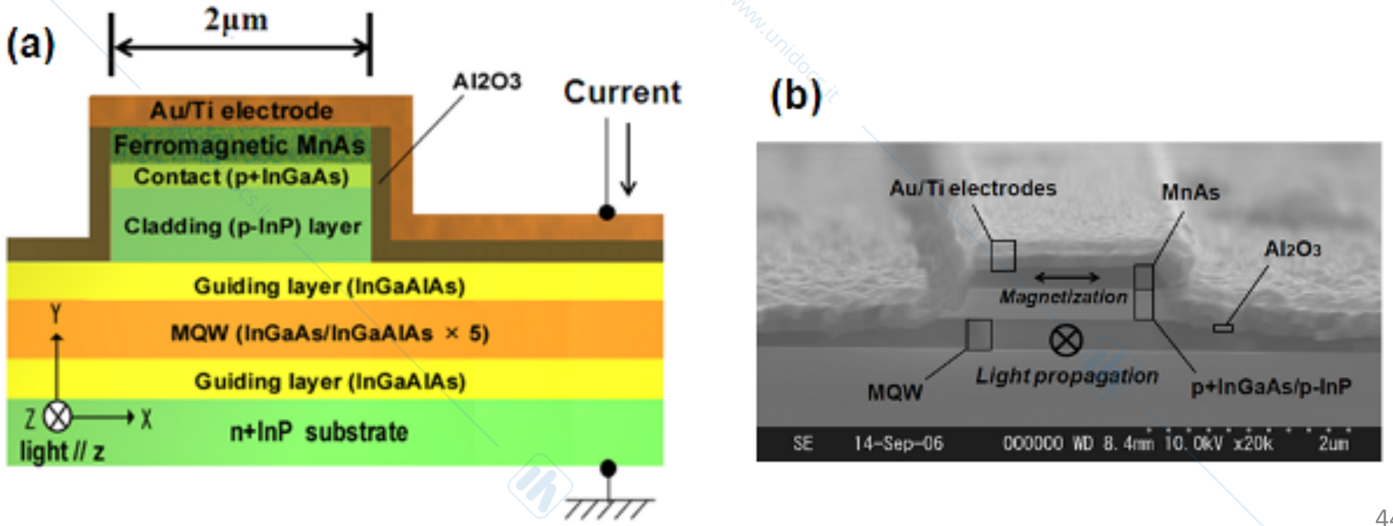
- Simplified method: Effective Refractive Index method

FEM- Maxwell equations for optical waveguide in Cartesian coordinates

$$\begin{cases} \dfrac{\partial \mathcal{E}_z}{\partial y} - i\beta\mathcal{E}_y = i\omega\mu_0\mathcal{H}_x, \\ i\beta\mathcal{E}_x - \dfrac{\partial \mathcal{E}_z}{\partial x} = i\omega\mu_0\mathcal{H}_y, \\ \dfrac{\partial \mathcal{E}_y}{\partial x} - \dfrac{\partial \mathcal{E}_x}{\partial y} = i\omega\mu_0\mathcal{H}_z, \\ \text{and} \\ \dfrac{\partial \mathcal{H}_z}{\partial y} - i\beta\mathcal{H}_y = -i\omega\epsilon\mathcal{E}_x, \\ i\beta\mathcal{H}_x - \dfrac{\partial \mathcal{H}_z}{\partial x} = -i\omega\epsilon\mathcal{E}_y, \\ \dfrac{\partial \mathcal{H}_y}{\partial x} - \dfrac{\partial \mathcal{H}_x}{\partial y} = -i\omega\epsilon\mathcal{E}_z. \end{cases}$$

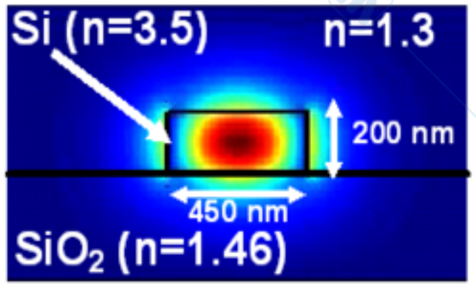
The refractive index is function of x- and y-

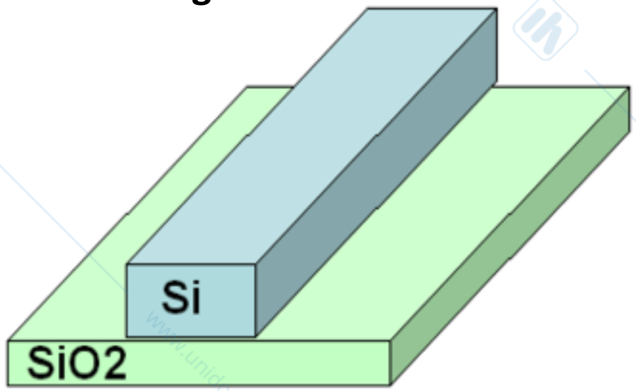
Example of a complicate channel waveguide



Solution examples (I)

Intensity profile of the fundamental mode of a Silicon Waveguide





Colors are used for a 2D plot of the intensity of the Electric field (red for high intensity and blue for low intensity)

Solution examples (II)

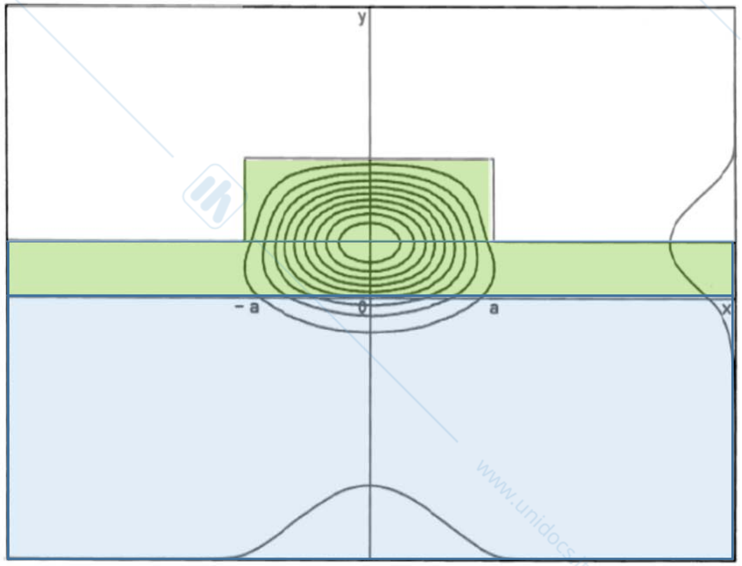


Figure 6.15 Optical intensity distribution of E_{11} mode in the rib waveguide. $n_c = 3.38$, $n_s = 3.17$, $n_a = 1.0$, $2a = 1.5$ µm, $h = 0.75$ µm, $t = 0.3$ µm and $\lambda = 1.55$ µm.

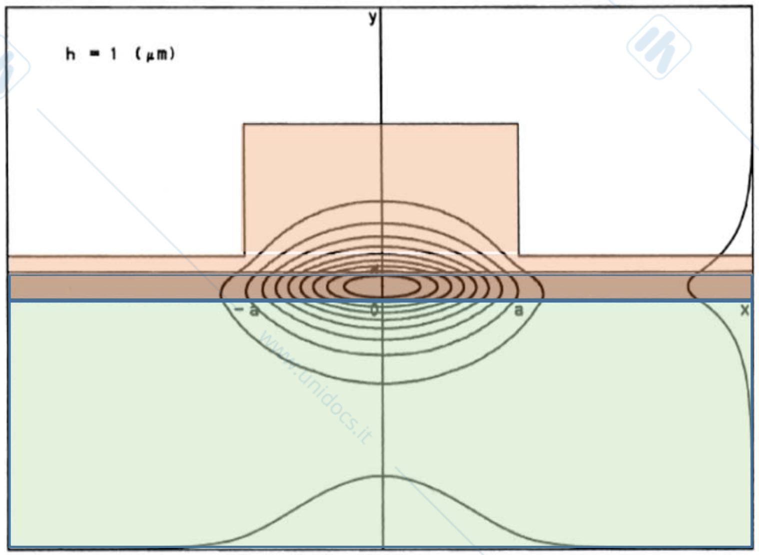


Figure 6.16 Optical intensity distribution of the E_{11} mode in the ridge waveguide, $n_c = 3.38$, $n_s = 3.17$, $n_r = n_s$, $n_a = 1.0$, $2a = 2$ µm, $d = 0.2$ µm, $h = 1$ µm, $t = 0.1$ µm and $\lambda = 1.55$ µm.

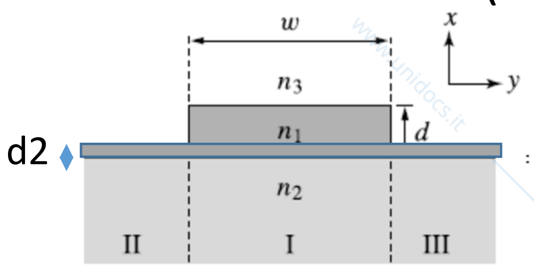
In all the 2D waveguides the field confinement in depth (y- in the figure above) can be attributed to the variation of the refractive index while the field confinement in the lateral (x- in the figure above) direction to a reduction of n_{eff} in the lateral part of the structure respect to its value in the central part.

Effective refractive index method (ERI)-I

- Concept: covert the problem of a channel waveguide in that of two slab waveguides
- The method is a good approximation if the waveguide satisfies the following two conditions:

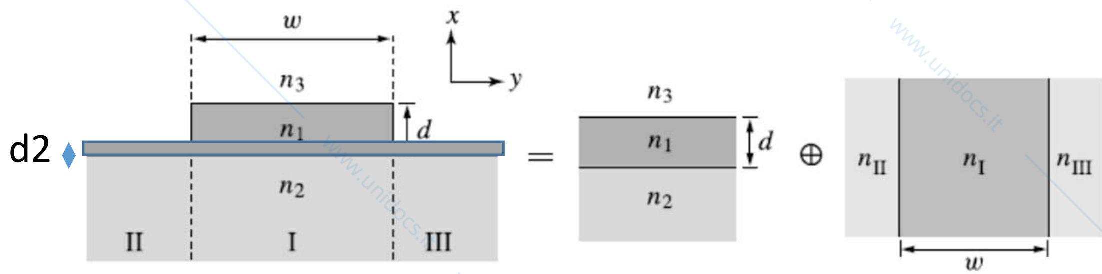
  (1) the waveguide width is larger than its thickness, $w > d$;

  (2) the waveguiding in the y direction (across its width) is not stronger than that in the x direction (across its thickness).



Effective refractive index method-II

- When these two conditions are satisfied, the characteristics of the guided modes are primarily determined by the layered structure perpendicular to the *x* direction, much like a slab waveguide of thickness *d*, but are modified by a lateral structure of width *w*.



$$\mathcal{E}_{mn}(x,y) = \mathcal{E}_m(x) \times \mathcal{E}_n(y)$$

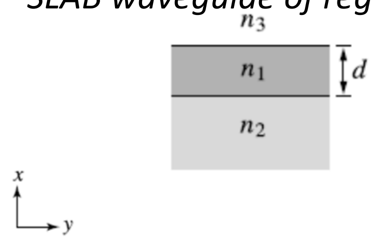
$$\mathcal{H}_{mn}(x,y) = \mathcal{H}_m(x) \times \mathcal{H}_n(y)$$

Figure 2.15 Basic concept of the effective index method.

Example: find TE_{mn} mode

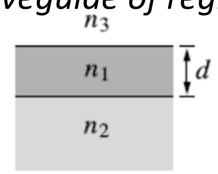
- **STEP I**: find TE_m and propagation constants $\beta_{I,II,III\ m}$ of the three slab waveguides of thickness d, d2 and d3

SLAB waveguide of region I



Calculate:
- $\beta_{I,\ m}$ and $n_{eff\ I}$
- Ey(x) of the TE mode in I

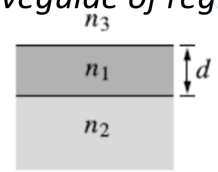
SLAB waveguide of region II



d= d2

Calculate:
- $\beta_{II,\ m}$ and $n_{eff\ II}$
- Ey(x) of the TE mode in II

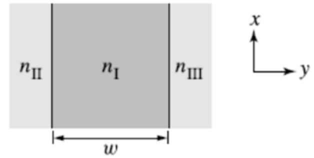
SLAB waveguide of region III



d= d3

Calculate:
- $\beta_{III,\ m}$ and $n_{eff\ III}$
- Ey(x) of the TE mode in III

- **STEP II**: Solve another slab waveguide as:



<u>**By finding now TM_n mode that provides $E_{ym}(y)$!!**</u>

Summary slide-

1) The slab waveguide is a simple structure for understanding the basic principles of optical waveguides

2) Channel waveguides are used in many photonic integrated circuits and devices (see next lessons)

3) To study channel waveguides we have two options:

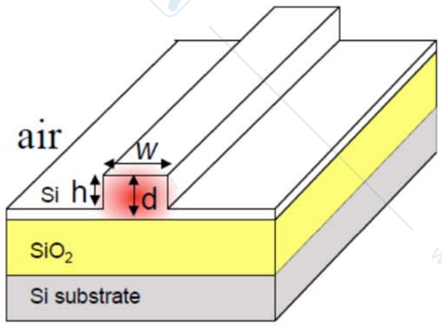
   - use rigorous FEM (there are several commercial CAD softwares)

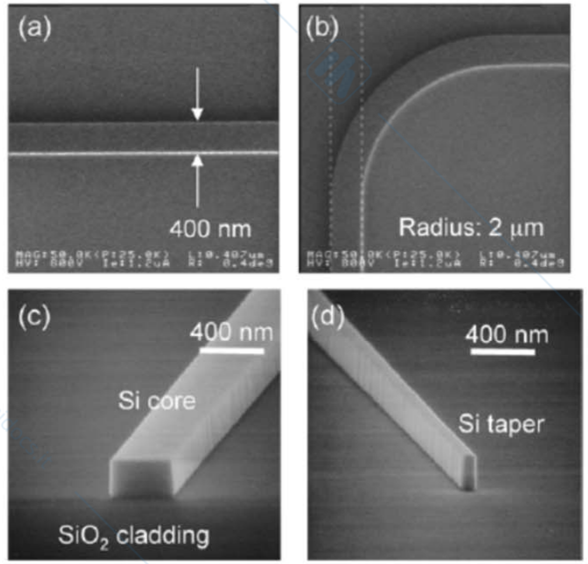
   - use simplified Effective Refractive index Method : very approximate method but you can write your own code in MATLAB.

See LAB1-Problem #1.a

Silicon optical waveguides (nanophotonic wires)

A silicon rib waveguide





- Guide light by total internal reflection in a few 100 nm cross-section (propagation loss typically few - ~1 dB/cm)

Tsuchizawa *et al.*, *IEEE J. Sel. Topic Quantum Electron.* **11**, 232-240 (2005).

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Next LESSONs

- Blackboard problems and presentation of LAB1 (first part)

Lesson A2:

- Guided modes parameters
- Optical Fibers

d

- Optical interconnect