QUALITY ENGINEERING

19/07/2019

General recommendations:

- avoid (if not required) theoretical introductions or explanations covered during the course;
- always state the assumptions, formulas/expressions and the final results (when using hypothesis tests provide the numerical value of the test statistic and the test conclusion in terms of p-value);
- show (qualitatively) all the plots.
- Duration: 2h (+ 10' for manual data entry)

Instructions on how to access the software (Minitab and Excel) on the workstation:
Open the browser (Internet Explorer); Go on favourites and select "Polimi Virtual Desktop"; Enter your credentials. Do not use the link to Excel SW on the desktop.

Instructions on how to install the Solver in Excel:
Open Excel; go on "File\Options"; click Add-Ins, and then in the Manage box, select Excel Add-ins; Click Go; in the Add-Ins available box, select the Solver (Risolutore) and then click OK.

Exercise 1 (max score 12)

A scientific instrument installed on the International Space Station is kept at a low temperature by means of a liquid cooling system. The average temperature measured every day for 40 consecutive days are reported in the table below. From day 18 to day 22 the instrument was used for some extra experiments, whereas normal operating conditions were applied in all the remaining days.

Day	T (°C)	Day	T (°C)	Day	T (°C)	Day	T (°C)
1	6,00	11	4,54	21	6,97	31	2,93
2	5,80	12	4,35	22	6,85	32	2,78
3	6,10	13	3,80	23	3,04	33	2,67
4	5,15	14	4,60	24	3,31	34	2,14
5	5,02	15	4,78	25	3,01	35	2,70
6	5,00	16	4,76	26	2,70	36	2,25
7	5,57	17	4,16	27	2,43	37	1,68
8	5,12	18	7,14	28	2,53	38	2,40
9	5,46	19	7,13	29	2,74	39	1,77
10	4,25	20	6,84	30	2,82	40	1,24

a) Perform a Bartlett's test (95% confidence) for the temperature data at lag 1 (show also the p-value) and comment the result.

b) Design a suitable approach to monitor the temperature of the instrument, assuming an average number of samples before a false alarm equal to 100. In case of alarms, check if the alarms are still present if probabilistic control limits are implemented.

c) In the next 10 days, new measurements are performed in normal operating conditions. Is the temperature of the instrument in-control? (show also the values of the fits and residuals for the new data)

Day	T (°C)
41	1,04
42	0,95
43	0,87
44	0,77
45	0,82
46	0,89
47	0,65
48	0,71
49	0,84
50	0,55

Exercise 2 (max score 14)

A company produces aluminium billets. One quality control consists of randomly measuring the diameter of $n = 5$ billets every hour.
The table shows the sample means computed in 24 consecutive samplings.

Sample	Xbar	Sample	Xbar	Sample	Xbar	Sample	Xbar
1	32,00	**7**	32,24	**13**	32,20	**19**	32,20
2	32,04	**8**	32,28	**14**	32,16	**20**	32,24
3	32,08	**9**	32,04	**15**	32,12	**21**	32,16
4	32,04	**10**	32,16	**16**	32,20	**22**	32,08
5	32,00	**11**	32,08	**17**	32,24	**23**	32,12
6	32,04	**12**	32,04	**18**	32,16	**24**	32,20

Assume that the desired average time between false alarms is equal to 500 hours.

a) Design a traditional control chart for monitoring the process mean assuming that the mean and the standard deviation of the original data are: $\mu = 32{,}15$ and $\sigma = 0{,}49\ mm$;
b) ONLY FOR MECHANICAL ENGINEERING STUDENTS: Is the assumed value of σ appropriate to this process data? Justify with a statistical test.
c) ONLY FOR MANAGEMENT ENGINEERING STUDENTS: Design a control chart for small shifts of the mean aimed at minimizing the time to detect a shift of the mean to $32{,}35\ mm$ (use the standard deviation estimated from the data instead of $\sigma = 0{,}49\ mm$).

Exercise 3 (max score 4)

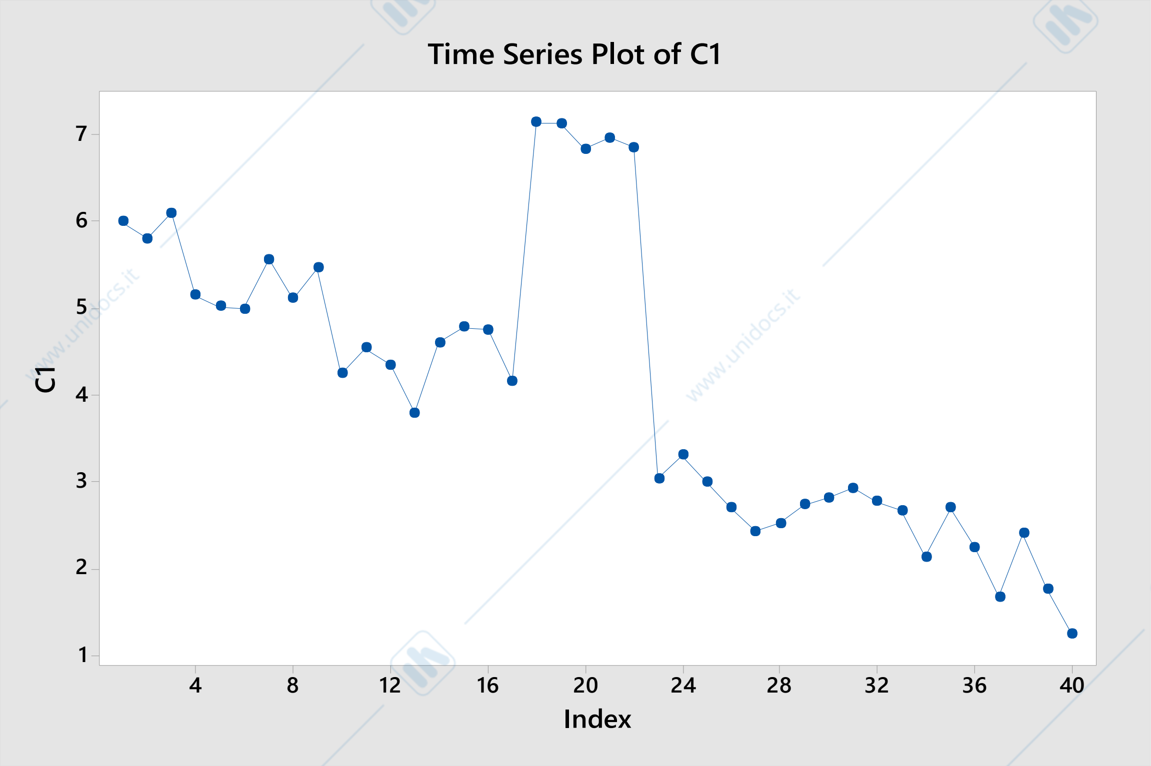
A univariate S control chart is used to monitor the variability of a dimensional quality characteristic in a turning process (sample size n=6). The quality characteristic in known to be normally distributed with a mean value of 0.4 mm and a standard deviation of 0.03 mm. Considering a false alarm rate α=0.01:

a) Compute the Type II error for the S chart when the process mean shifts to 0.8;
b) Compute the Type II error for the S chart when the standard deviation of the process becomes 0.07 mm.

SOLUTIONS

Exercise 1

The time series of the process is the following. There seems to be a decreasing trend with a shift in days when an extra experiment was performed.



The runs test confirms that data are not random:

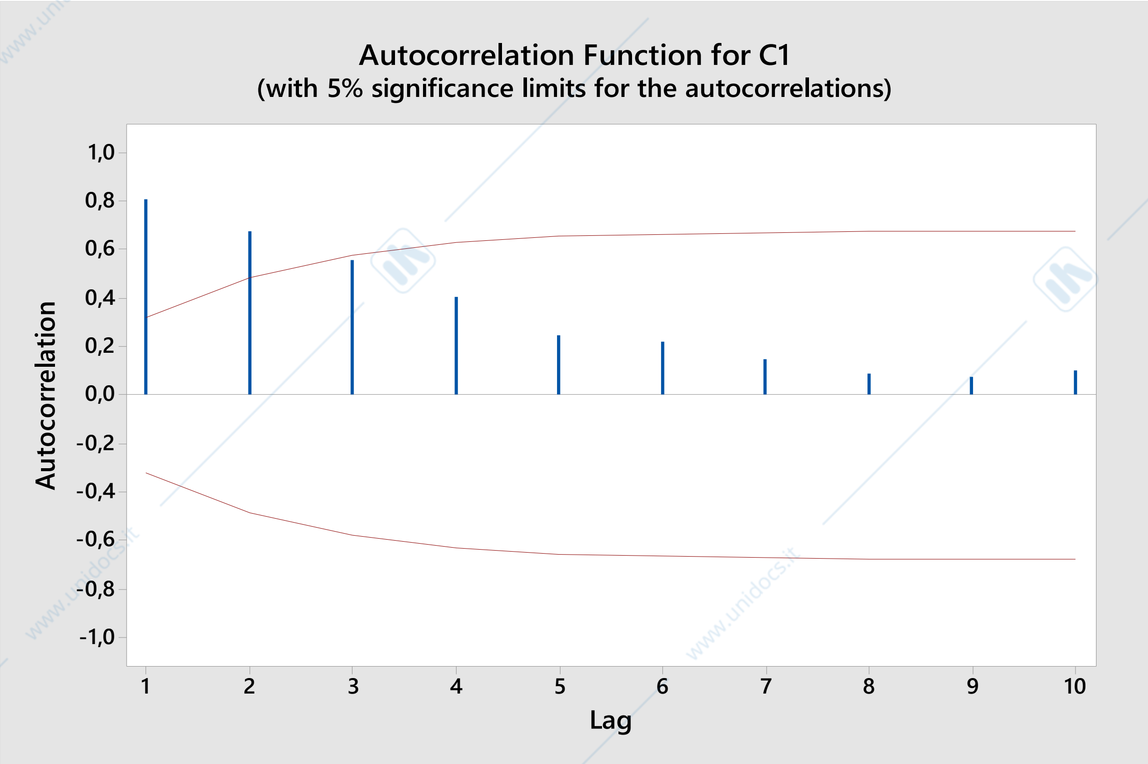
Test

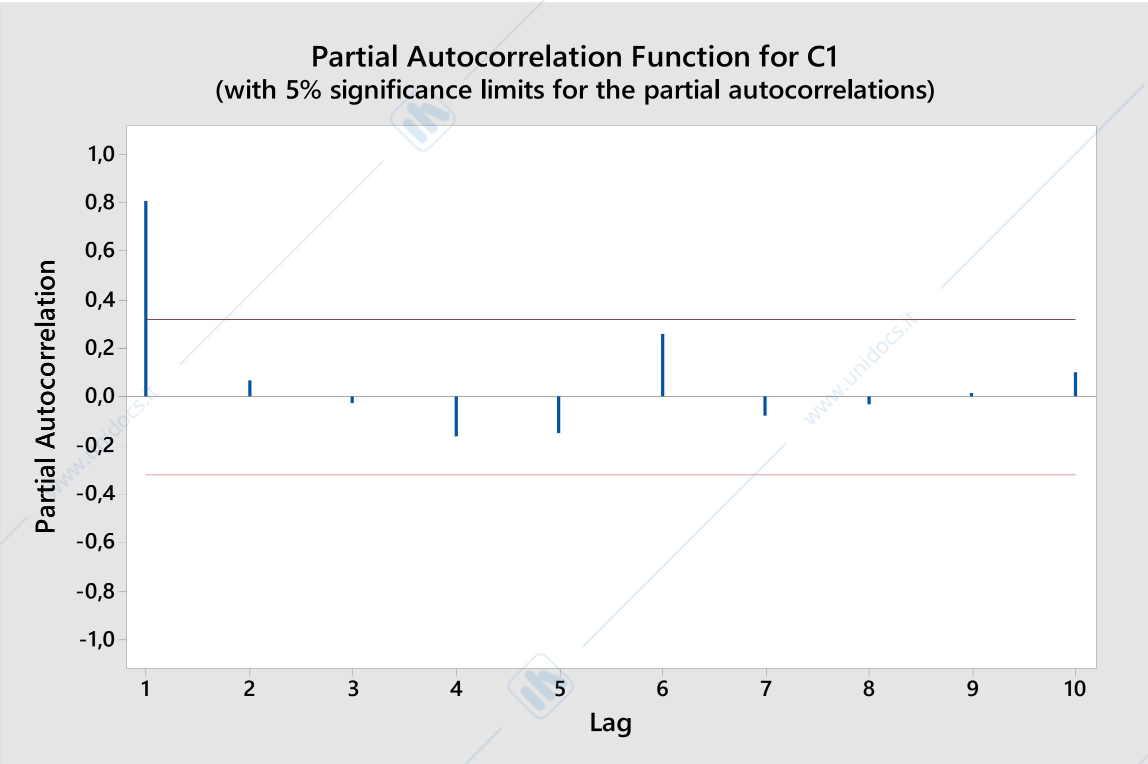
Null hypothesis	H_0: The order of the data is random
Alternative hypothesis	H_1: The order of the data is not random

Number of Runs

Observed	Expected	P-Value
4	20,95	0,000

The SACF and SPACF show that there is a non-stationary pattern, which is the result of the decreasing trend.





a) The Bartlett test at lag 1 is:

Test statistic at lag 1: r1 = 0.8092

The critical region is: $|r1| > \frac{z_{\alpha/2}}{\sqrt{n}} = 0{,}3099$, where n=40 and $z_{\alpha/2} = 1{,}96$

Since r1=0.8092>0.3099, the null hypothesis of lack of autocorrelation at lag 1 is rejected. The p-value of the test is p-value = 1.5e-7 (the test statistics approximately follows a normal distribution with zero mean and standard deviation equal to $\frac{1}{\sqrt{n}}$).

b) Model fitting

A dummy variable is needed to take into account the possible variation in days 18 to 22 caused by the extra experiments. The non-stationary pattern can be modelled by means of an trend model.

Regression Analysis: T versus day; dummy

Method

Categorical predictor coding (1; 0)

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	2	107,127	53,5634	419,37	0,000
day	1	59,985	59,9851	469,65	0,000
dummy	1	45,404	45,4043	355,49	0,000
Error	37	4,726	0,1277		
Total	39	111,853			

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
0,357384	95,78%	95,55%	95,25%

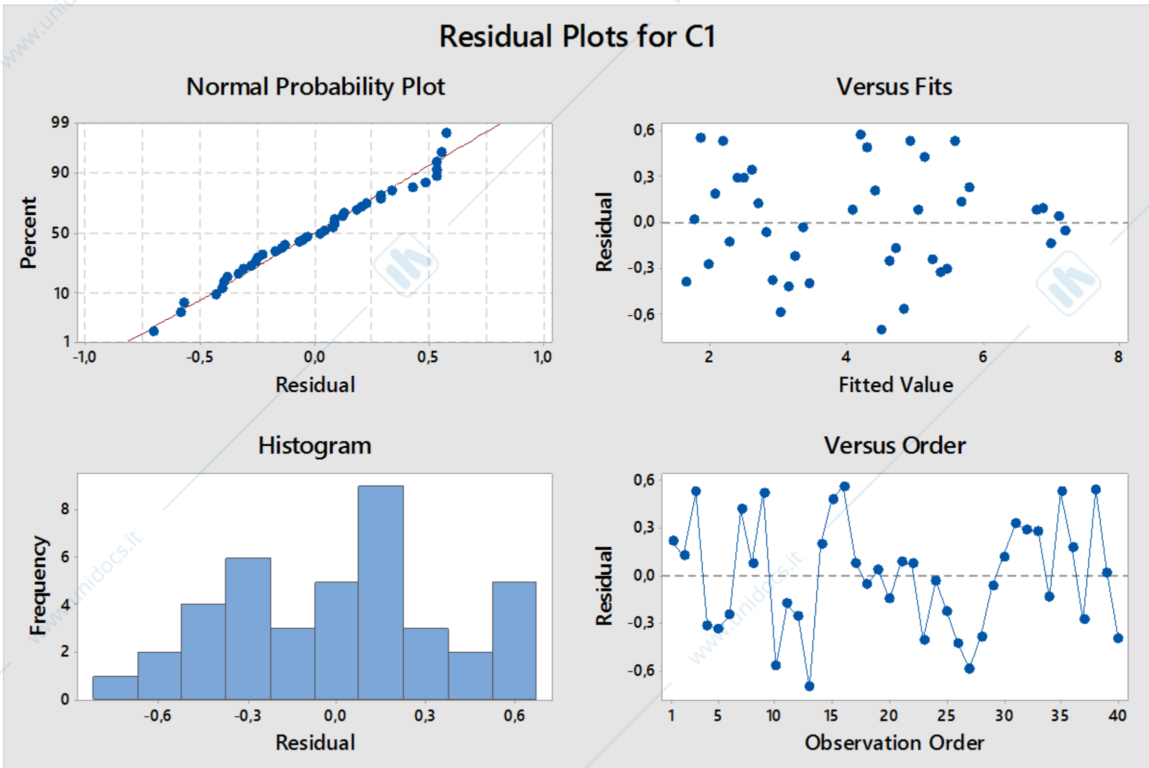
Coefficients

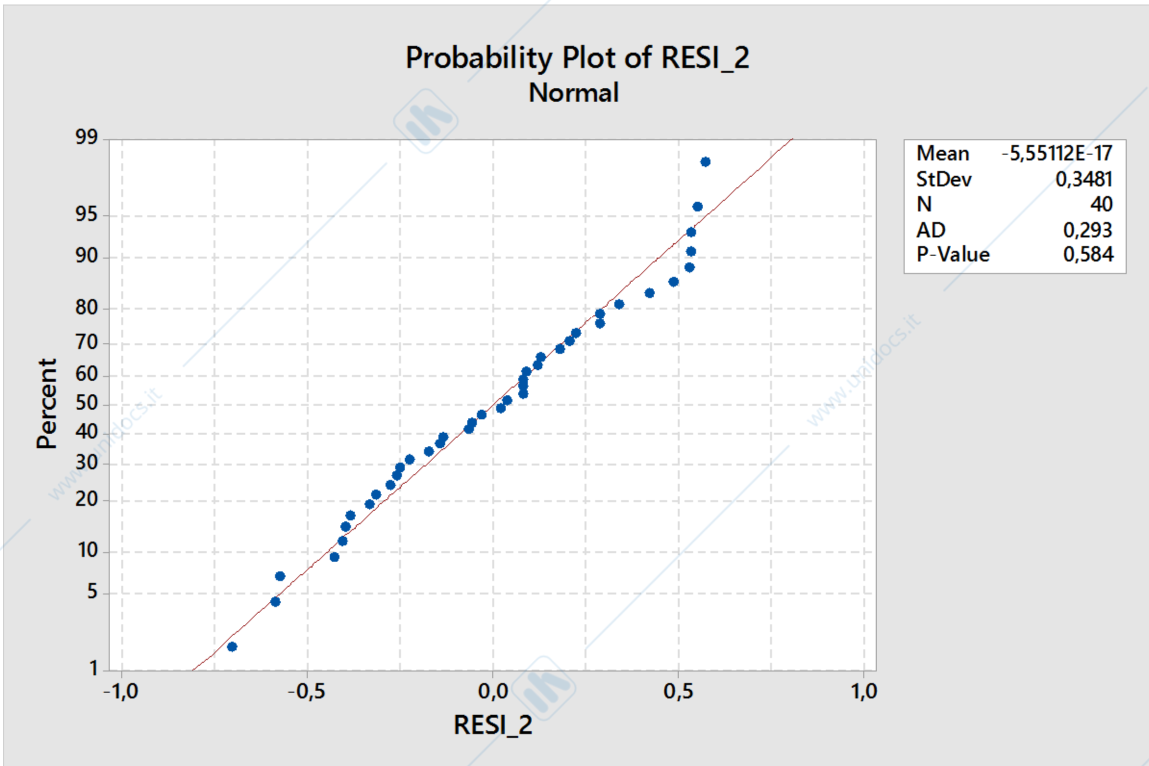
Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	5,885	0,117	50,11	0,000	
Day	-0,10610	0,00490	-21,67	0,000	1,00
dummy					
1	3,222	0,171	18,85	0,000	1,00

Regression Equation

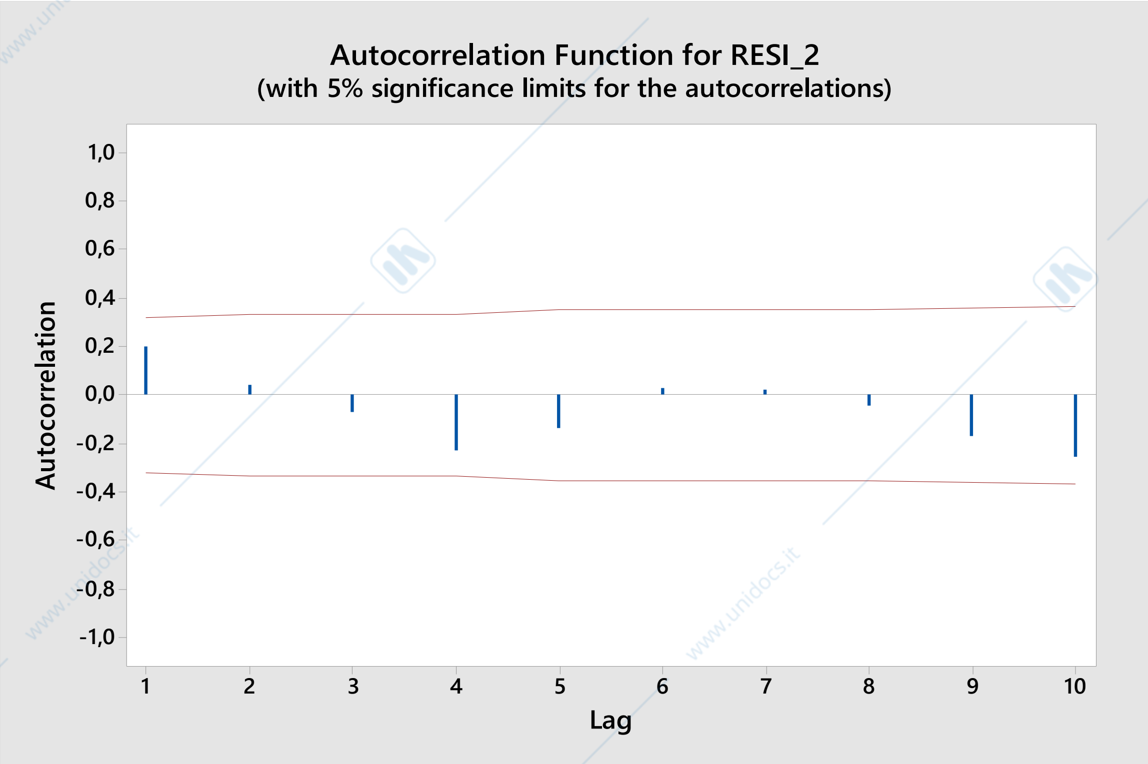
C4			
0	C1	=	5,885 - 0,10610 C3
1	C1	=	9,107 - 0,10610 C3

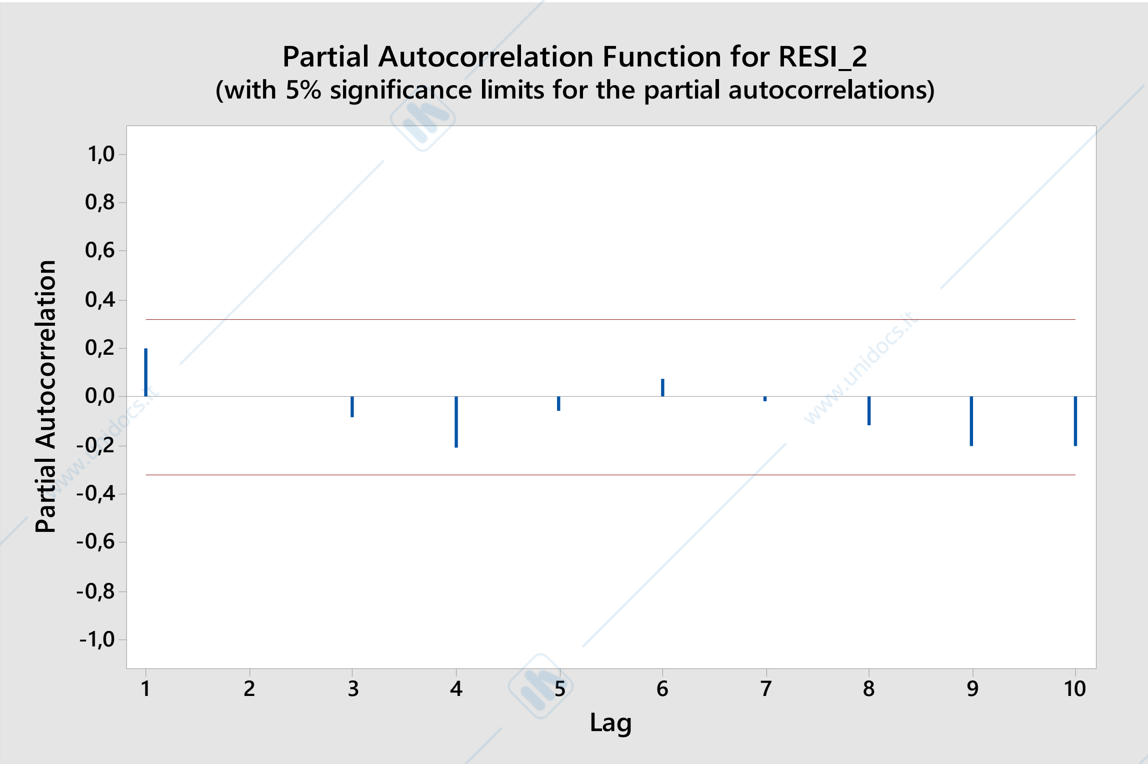
Check of residuals:





The residuals are normal.





Test

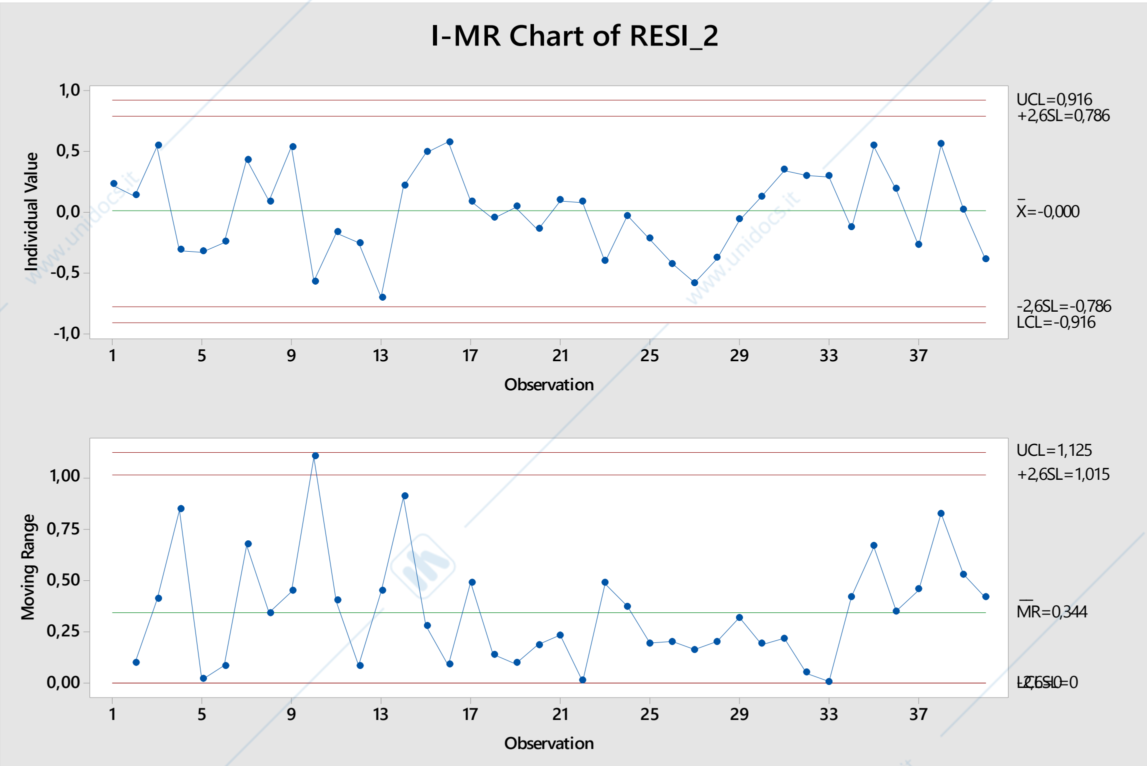
Null hypothesis	H_0: The order of the data is random
Alternative hypothesis	H_1: The order of the data is not random

Number of Runs

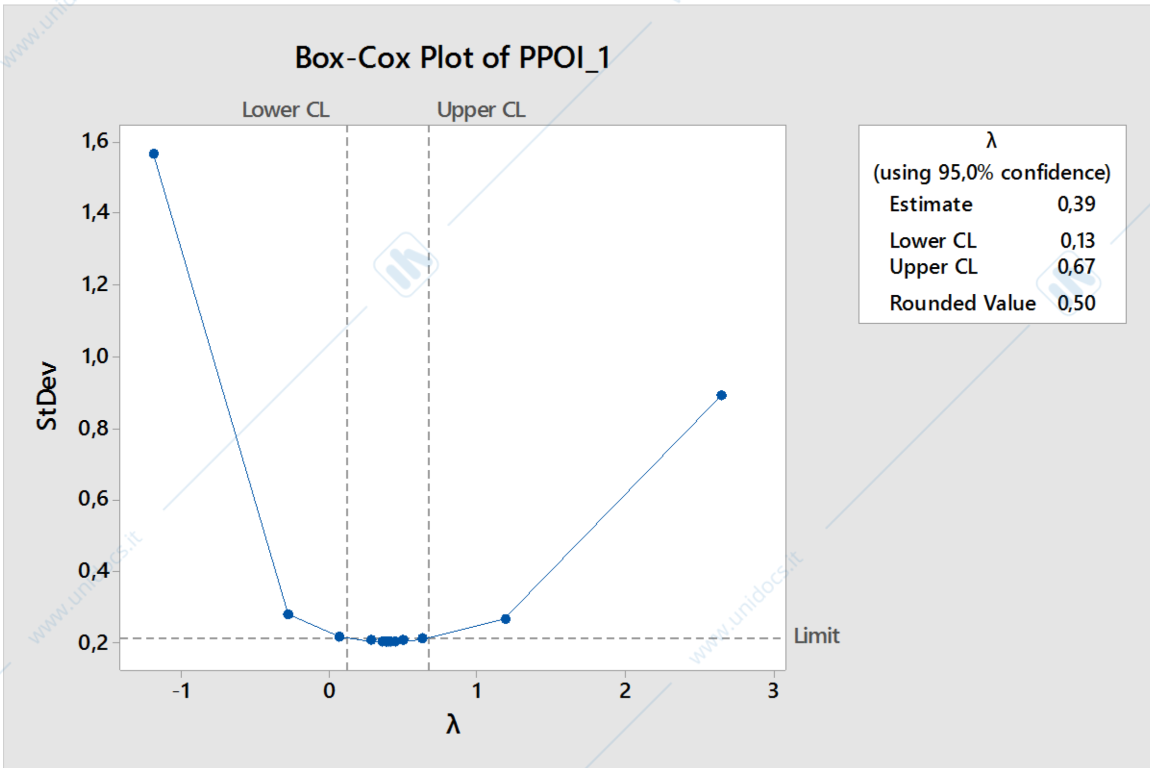
Observed	Expected	P-Value
16	20,95	0,112

The residuals are normal and independent.

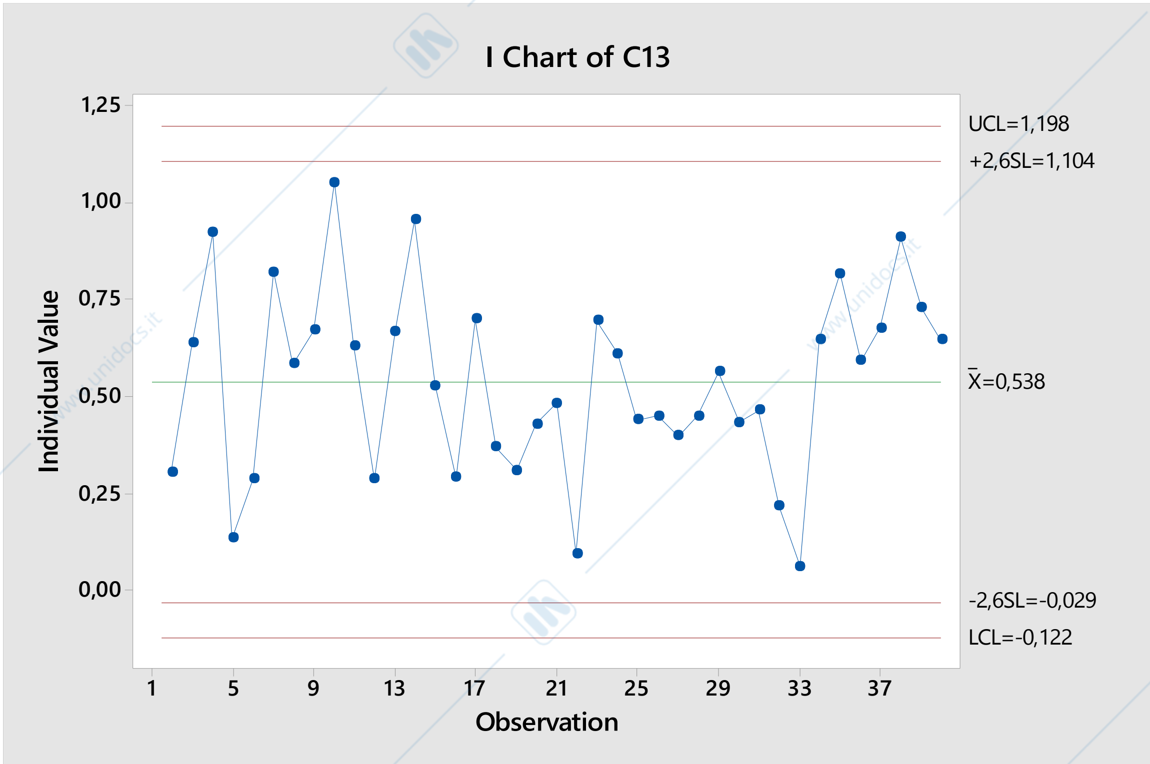
With ARL=100, α=0.01 the following control charts can be designed:



There is a violation of the limit in the MR control chart at day 10. It could be caused by the violation of assumptions in the design of the MR chart itself.
It is possible to transform the MR statistic with Box-Cox and design a control chart for the transformed statistic.



With λ=0.5, the resulting MR control chart is:



No out-of-control points are present. The design phase is over.

c)
For days 41 to 50, the following fits are obtained:

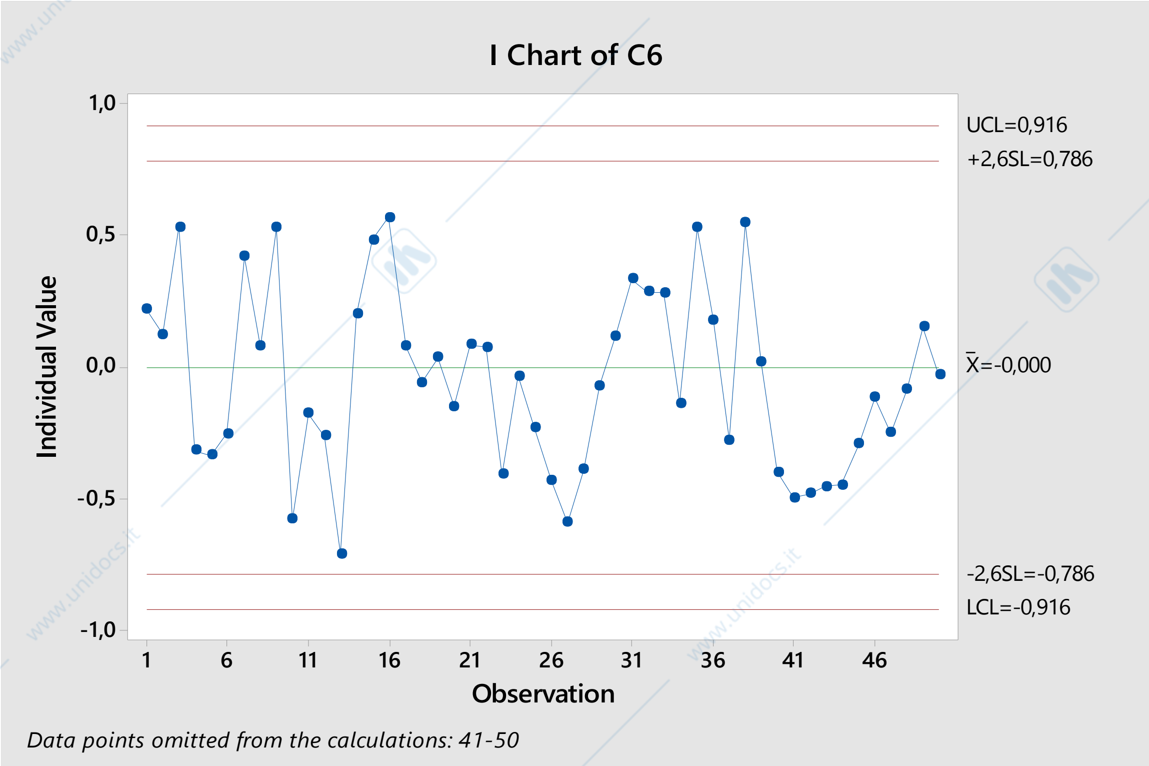
Fitted value (°C)	Day
1,5349	41
1,4288	42
1,3227	43
1,2166	44
1,1105	45
1,0044	46
0,8983	47
0,7922	48
0,6861	49
0,58	50

Leading to the following residuals:

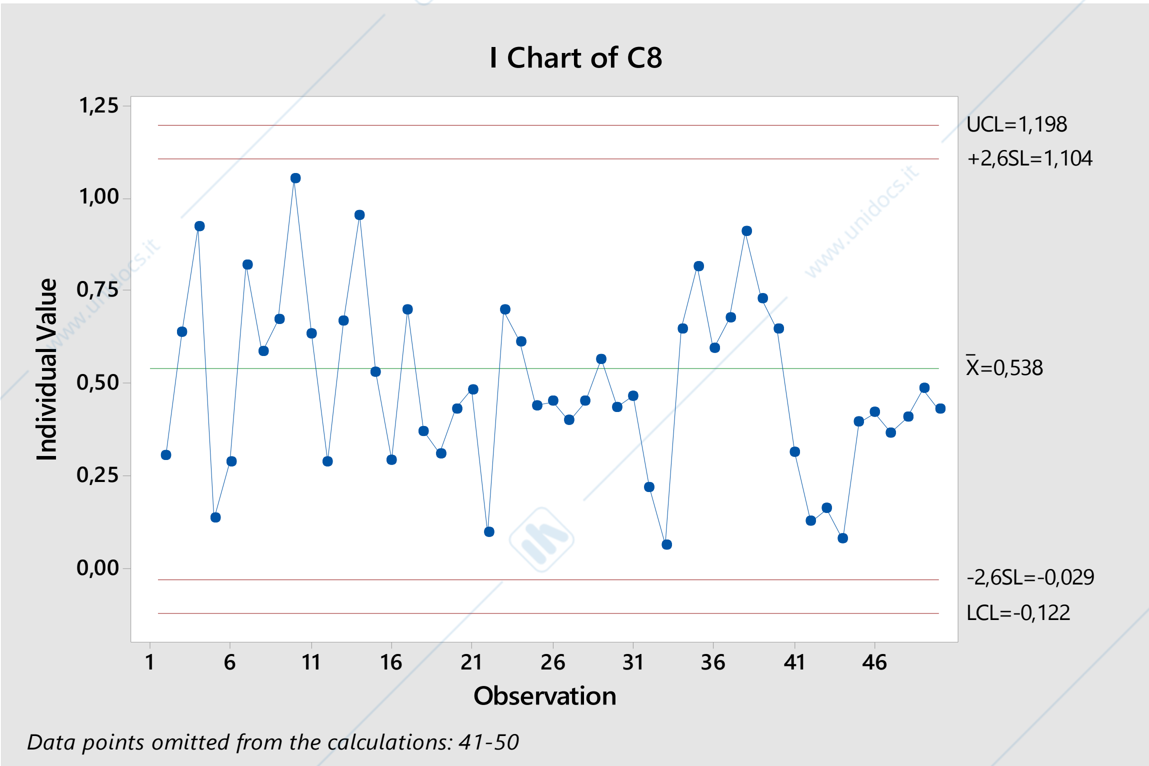
Day	Residuals
41	-0,4949
42	-0,4788
43	-0,4527
44	-0,4466
45	-0,2905
46	-0,1144
47	-0,2483
48	-0,0822
49	0,1539
50	-0,03

The resulting controls charts are:

Individual control chart (I):

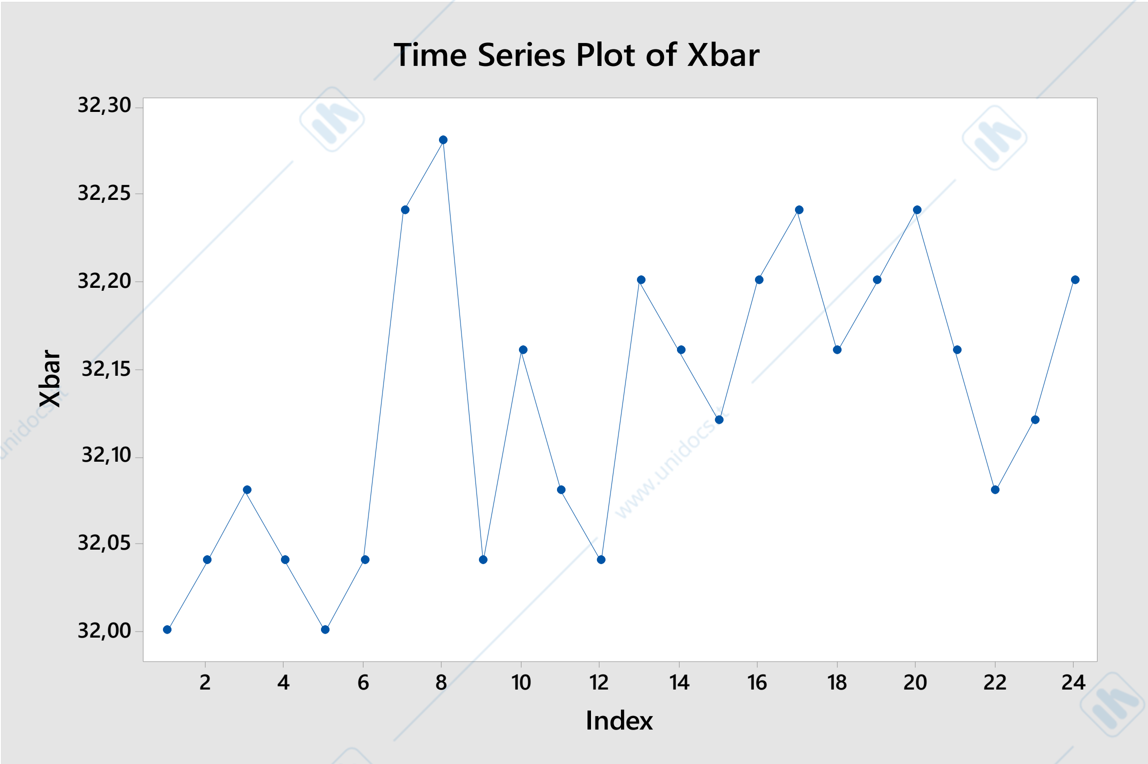


MR control chart (after Box Cox transformation):



Exercise 2

Data snooping:



Data snooping reveals a possible change from time 6 on.

Randomness and normality check:

Runs Test: Xbar

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Runs test for Xbar

Runs above and below K = 32,13

The observed number of runs = 10
The expected number of runs = 13
12 observations above K; 12 below
P-value = 0,210
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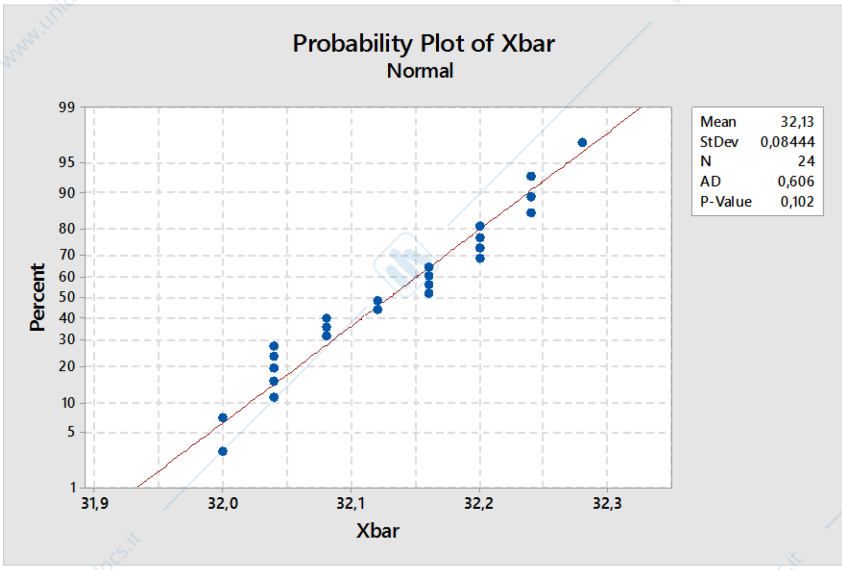
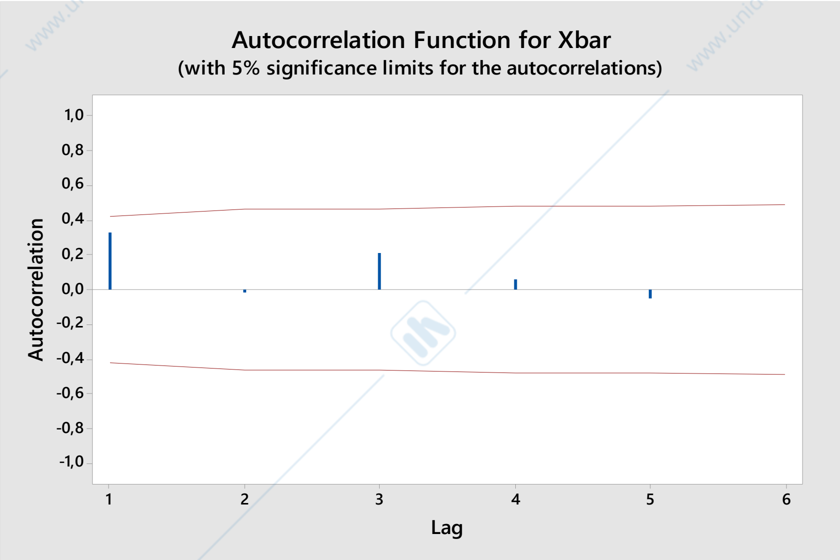
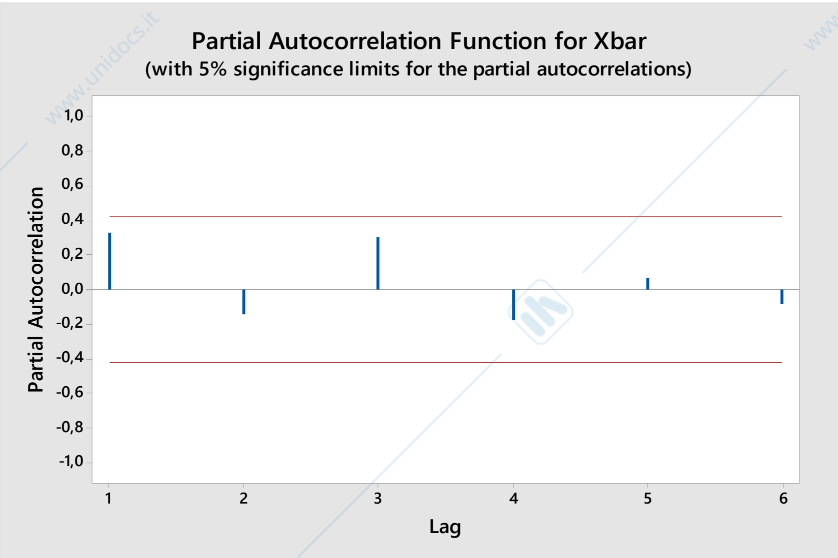


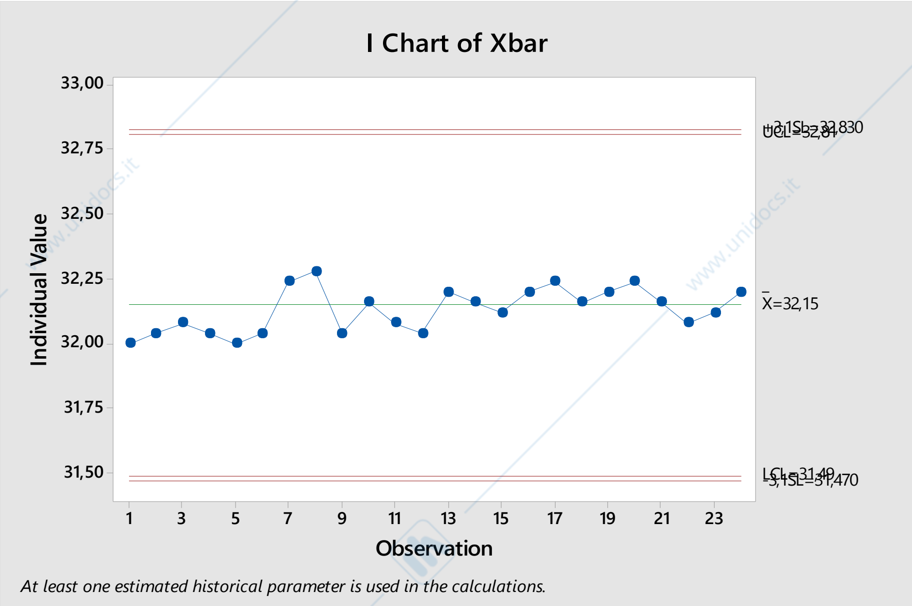




No violation of assumptions. No outlier.

Shewhart chart design

ARL0	500
alpha	0,002
z_alpha/2	3,090232
Sigma of X-bar	$\frac{0,49}{\sqrt{5}} = 0,22$
Mean (known)	32,15



Hugging is present. Probably, the assumed standard deviation is not an appropriate estimate.

b) ONLY FOR MECHANICAL ENGINEERING STUDENTS

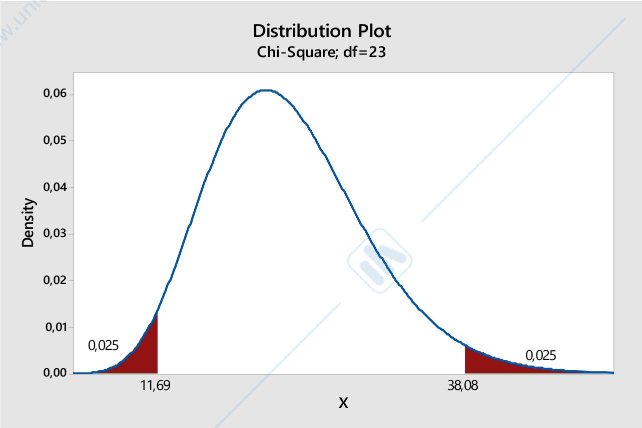
Hypothesis testing for the variance.

$\sigma^2_{xbar} = 0,0484$ (known)
$s^2 = 0,0071$ (estimated from data)

The test statistic is:

$$\chi^2_0 = \frac{(n-1)s^2}{\sigma^2_{xbar}} = \frac{23 * 0,0071}{0,0484} = 3,373967$$

$$\chi^2_{1-\frac{\alpha}{2};n-1} = \chi^2_{0,975;23} = 11,69$$



$p - value = 0.000$

Thus we can reject the null hypothesis at 5%. A bad estimate of the standard deviation caused the hugging effect observed in the designed chart.

c) ONLY FOR MANAGEMENT ENGINEERING STUDENTS

Design a CUSUM chart by using the Siegmund's approach. We have:

$\mu_0 = 32{,}15$

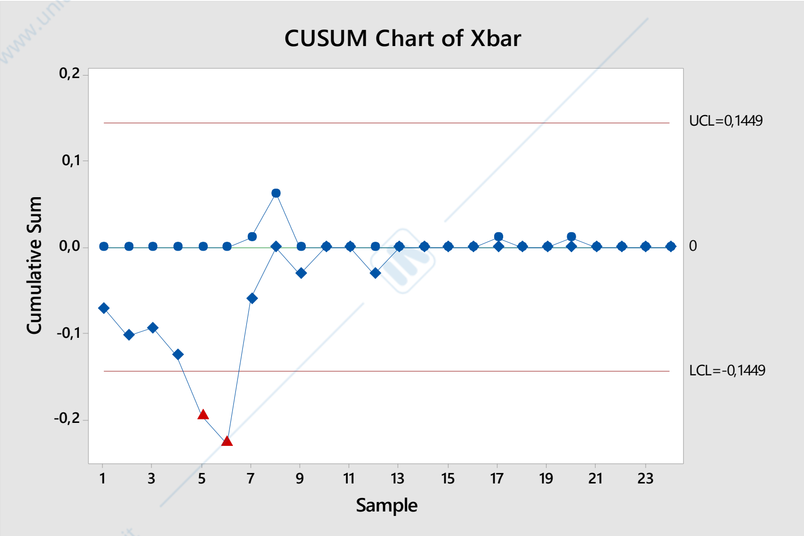
$\mu_1 = 32{,}35$

$\delta^* = \frac{\mu_1 - \mu_0}{s} = 2{,}37$ (we use s, i.e., the standard deviation estimated from the data)

We have to impose $ARL(0) = 500$ and minimize ARL(delta=2,37); we got:

k	1,185015						
h	2,185684						
deltastar	b	Delta+	Delta-	ARL+	ARL-	1/ARL	ARL
0	3,351684	-1,18502	-1,18502	1000	1000	0,002	500
0,25	3,351684	-0,93502	-1,43502	297,4094	3652,735	0,003636	275,0172
0,5	3,351684	-0,68502	-1,68502	99,19184	14163,57	0,010152	98,502
0,75	3,351684	-0,43502	-1,93502	38,45005	57394,83	0,026025	38,42431
1	3,351684	-0,18502	-2,18502	17,76429	240520,9	0,056297	17,76298
1,25	3,351684	0,064985	-2,43502	9,765801	1034832	0,102399	9,765708
1,5	3,351684	0,314985	-2,68502	6,211333	4547672	0,160996	6,211325
1,75	3,351684	0,564985	-2,93502	4,401455	20336257	0,227198	4,401454
2	3,351684	0,814985	-3,18502	3,362979	92273661	0,297355	3,362979
2,25	3,351684	1,064985	-3,43502	2,706674	4,24E+08	0,369457	2,706674
2,37	3,351684	1,184985	-3,55502	2,472511	8,85E+08	0,404447	2,472511

The corresponding CUSUM chart is:



The chart signals an out-of-control at samples 5 and 6. In the absence of assignable causes, the control chart design is over.

Exercise 3

The S control chart with known parameters is:

$$LCS = \mu_S + k\sigma_S = c_4\sigma + k\sqrt{1-c_4{}^2}\,\sigma$$
$$LC = \mu_S = c_4\sigma$$
$$LCI = \mu_S - k\sigma_S = c_4\sigma - k\sqrt{1-c_4{}^2}\,\sigma$$

Alpha	0,01
Alpha/2	0,005
K=z_alpha/2	2,576
n	6
c4(6)	0,9515

S-chart		
UCL	CL	LCL
0,05232	0,02855	0,00477

a) The alternative hypothesis is

H_1: $\mu_{new} = \mu_0 + \delta$, where $\delta = 0.4$

The Type II error for the S chart can be computed as follows:

$$\beta_S = P(LCL_S \le S \le UCL_S | H_1) = P\left(\frac{(n-1)}{\sigma^2} LCL_S{}^2 \le \frac{(n-1)}{\sigma^2} S^2 \le \frac{(n-1)}{\sigma^2} UCL_S{}^2\right)$$

This is constant (it is not a function the process mean). The result is β_S =0,99.

a) The alternative hypothesis is

H_1: $\sigma_{new} = \sigma + \delta$, where $\delta = 0.04$

The Type II error for the S chart can be computed as follows:

$$\beta_S = P(LCL_S \leq S \leq UCL_S | H_1) = P\left(\frac{(n-1)}{\sigma_{new}{}^2} LCL_S{}^2 \leq \frac{(n-1)}{\sigma_{new}{}^2} S^2 \leq \frac{(n-1)}{\sigma_{new}{}^2} UCL_S{}^2\right)$$

The result is β_S =0.2682.