QUALITY ENGINEERING

10/01/2020

General recommendations:

- avoid (if not required) theoretical introductions or explanations covered during the course;
- always state the assumptions, formulas/expressions and the final results (when using hypothesis tests provide the numerical value of the test statistic and the test conclusion in terms of p-value);
- show (qualitatively) all the plots.
- Duration: 2h (+ 10' for manual data entry)

Instructions on how to access the software (Minitab and Excel) on the workstation:
Open the browser (Internet Explorer); Go on favourites and select "Polimi Virtual Desktop"; Enter your credentials. Do not use the link to Excel SW on the desktop.

Instructions on how to install the Solver in Excel:
Open Excel; go on "File\Options"; click Add-Ins, and then in the Manage box, select Excel Add-ins; Click Go; in the Add-Ins available box, select the Solver (Risolutore) and then click OK.

Exercise 1 (max score 15)

The following table shows data observed at time *t* from a continuous process.

t	data	*t*	data	*t*	data	*t*	data
1	30,94	11	33,29	21	31,94	31	34,85
2	32,03	12	32,88	22	31,47	32	34,41
3	32,19	13	33,51	23	31,41	33	33,59
4	32,64	14	34,18	24	32,75	34	34,36
5	32,92	15	33,91	25	33,77	35	33,14
6	34,1	16	32,41	26	34,57	36	34,75
7	34,23	17	32,24	27	35,49		
8	33,21	18	32,55	28	36,68		
9	34,43	19	32,35	29	36,05		
10	33,93	20	32,6	30	35,07		

a) Design an appropriate control chart to monitor the process
b) After the design phase is concluded, new data are collected.

t	data
37	33,49
38	31,68
39	30,63
40	31,33
41	32,55
42	32,27
43	32,98
44	33,16
45	32,12
46	33,68

We need to apply control charting in Phase 2. Show all the steps, formulas and final values used to perform the Phase 2 of control charting.

Exercise 2 (max score 15)

Holmes and Mergen (1993) gave 56 individual bivariate observations from a European plant producing grit, or gravel, consisting of the percent of particles (by weight) that were large and medium in size. Table 1 contains the data in the second and third columns.

time	%large	%med		*time*	%large	%med
1	5,4	93,6		*21*	3,8	92,7
2	3,2	92,6		*22*	2,8	91,5
3	5,2	91,7		*23*	2,9	91,8
4	3,5	86,9		*24*	3,3	90,6
5	2,9	90,4		*25*	7,2	87,3
6	4,6	92,1		*26*	7,3	79
7	4,4	91,5		*27*	7	82,6
8	5	90,3		*28*	6	83,5
9	8,4	85,1		*29*	7,4	83,6
10	4,2	89,7		*30*	6,8	84,8
11	3,8	92,5		*31*	6,3	87,1
12	4,3	91,8		*32*	6,1	87,2
13	3,7	91,7		*33*	6,6	87,3
14	3,8	90,3		*34*	6,2	84,8
15	2,6	94,5		*35*	6,5	87,4
16	2,7	94,5		*36*	6	86,8
17	7,9	88,7		*37*	4,8	88,8
18	6,6	84,6		*38*	4,9	89,8
19	4	90,7		*39*	5,8	86,9
20	2,5	90,2		*40*	7,2	83,8

a) FOR MANAGEMENT ENGINEERS ONLY: Design a multivariate control chart for monitoring the mean using ARL_0=200.
b) FOR MECHANICAL ENGINEERS ONLY: Design two univariate control charts for the process mean using ARL_0=200.
c) FOR ALL: compute the principal components showing all the steps. Compute a control chart on the first PC. What does it change with respect to the previous approach?

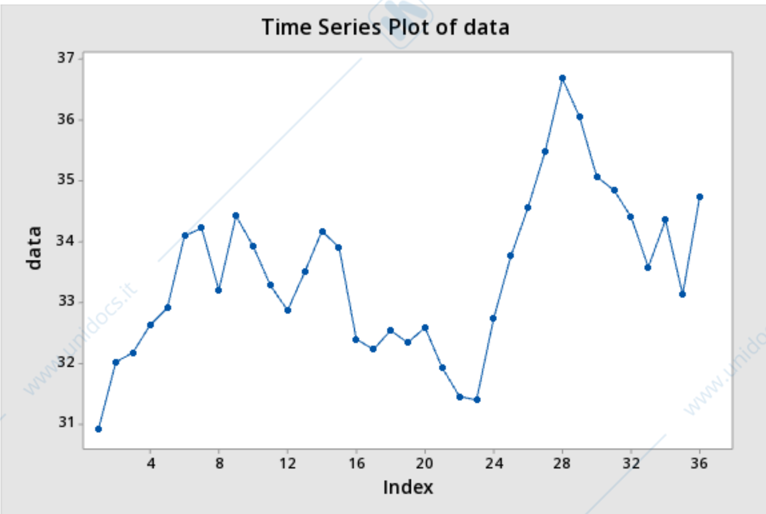
Excercise 3 (max score 5)

Show the expression of the Operating characteristic of a Range control chart with n=2. Assume $LCL = 0{,}0024$, $UCL = 4{,}5327$ and $\sigma_R = 1$ compute the OC curve for λ equal 1 and 1,5 where λ is the ratio of the new to the old standard deviations..
(Suggestion: let X be a random variable with continuous distribution and cumulative distribution F. The absolute value of X, $|X|$, has the cumulative distribution function $G(x) = F(x) - F(-x)$, for $x > 0$)

Solution

Exercise 1

a) Design the appropriate control chart

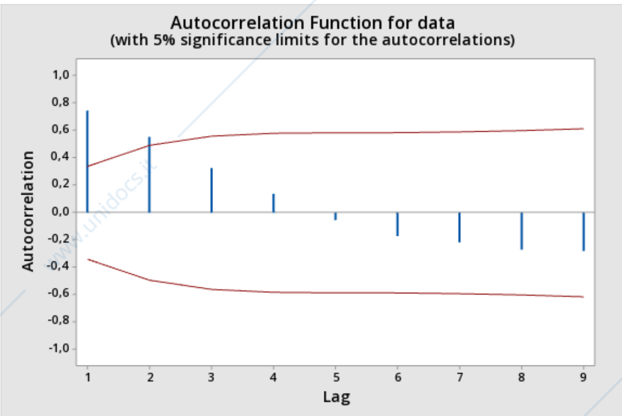


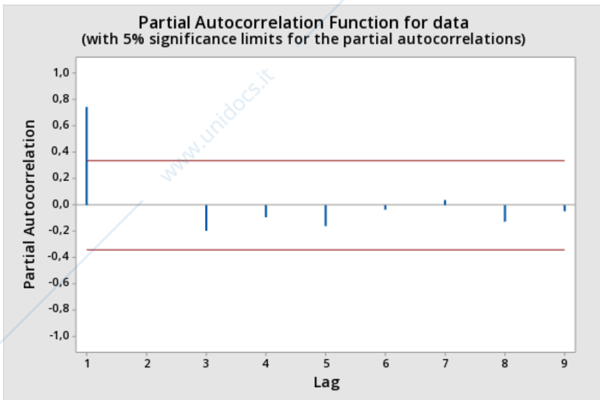
Runs test:

Test

Null hypothesis H_0: The order of the data is random
Alternative hypothesis H_1: The order of the data is not random

Number of Runs		
Observed	Expected	P-Value
10	19,00	0,002





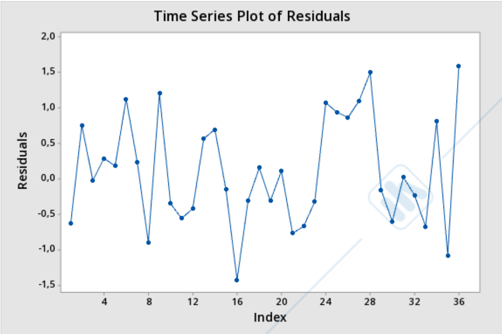
It is deemed to be a AR(1) process. Let's estimate the coefficient and check the assumptions

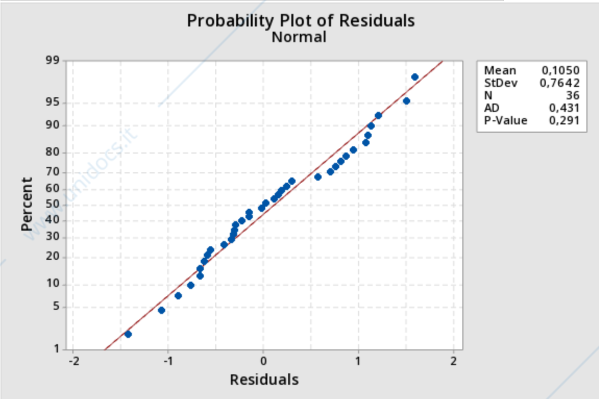
AR(1)

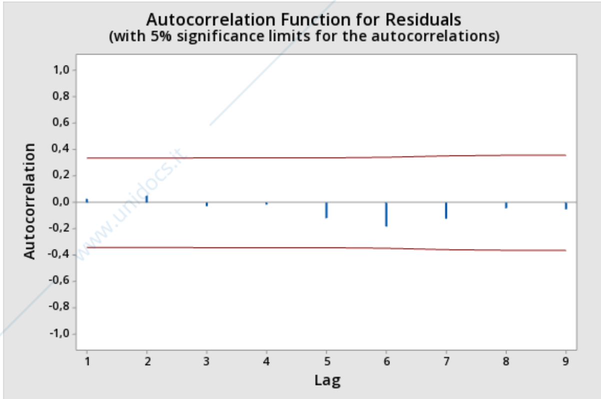
Final Estimates of Parameters

Type	Coef	SE Coef	T-Value	P-Value
AR 1	0,8591	0,0978	8,78	0,000
Constant	4,692	0,131	35,93	0,000
Mean	33,311	0,927		

Residuals are independent and normally distributed







Runs test

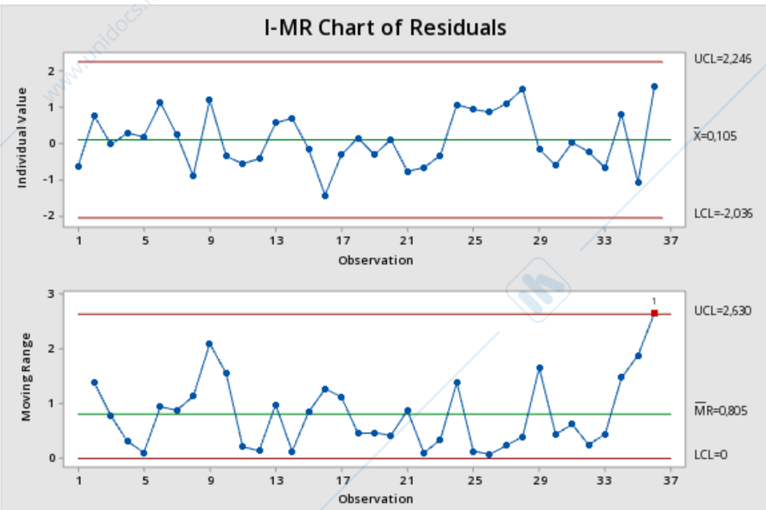
Test

Null hypothesis	H_0: The order of the data is random
Alternative hypothesis	H_1: The order of the data is not random

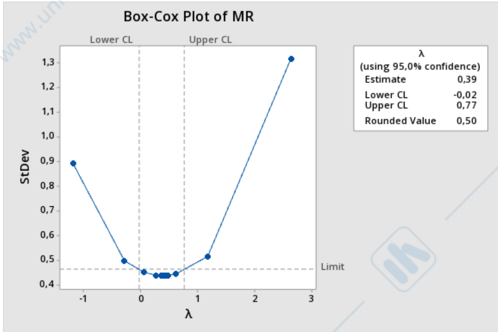
Number of Runs

Observed	Expected	P-Value
18	18,94	0,749

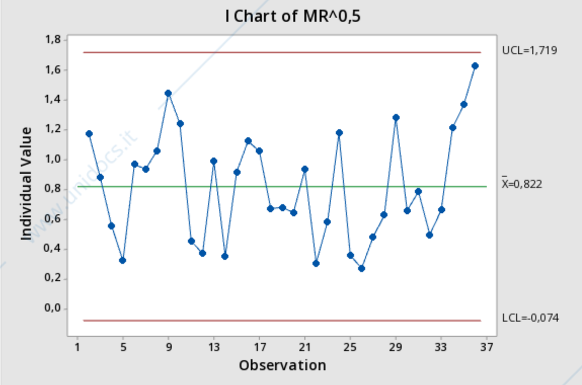
We cannot reject the assumption that residuals are iid normal.



For the last point, which is out of control in the MR chart, we can check if this is due to the assumption behind the MR cc (that from the theory are not respected, as it is known that the MR data are not NID by construction)
Let's try a probabilistic cc by transforming the MR series to achieve normality. By using a Box-Cox transformation:



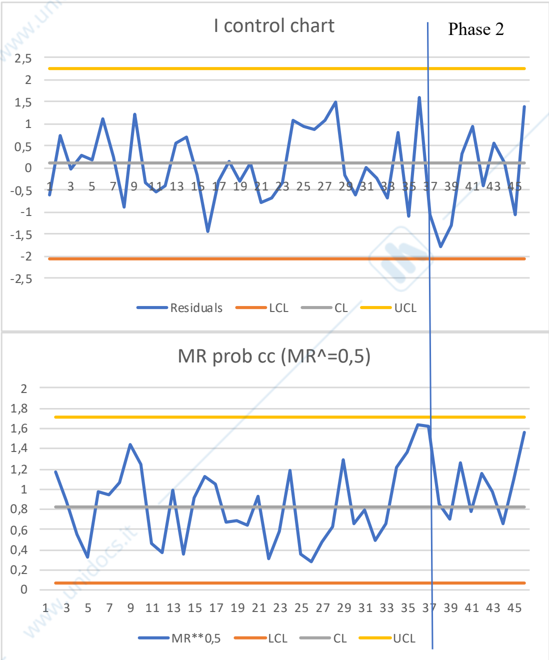
By applying the control chart on the transformed MR, the out-of-control point disappears:



b) In Phase 2- we need to keep the same AR model identified, while keeping the same coefficients, in order to estimate the residuals. The control charts' limits will not change as well (as the dsign phase is completed)

In order to estimate the residuals, we can use excel (as minitab does not allow to compute predicted value by imposing fixed coefficients), using:

			Coefficients	
		fi	0,85914767	
		cost	4,69200222	
	t	data	predicted	residuals
PH1	36	34,75	33,1641558	1,58584416
PH2	37	33,49	34,5473836	-1,0573836
PH2	38	31,68	33,4648575	-1,7848575
PH2	39	30,63	31,9098002	-1,2798002
PH2	40	31,33	31,0076952	0,3223048
PH2	41	32,55	31,6090986	0,94090143
PH2	42	32,27	32,6572587	-0,3872587
PH2	43	32,98	32,4166974	0,56330263
PH2	44	33,16	33,0266922	0,13330779
PH2	45	32,12	33,1813388	-1,0613388



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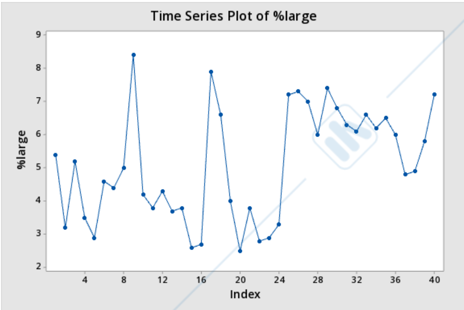
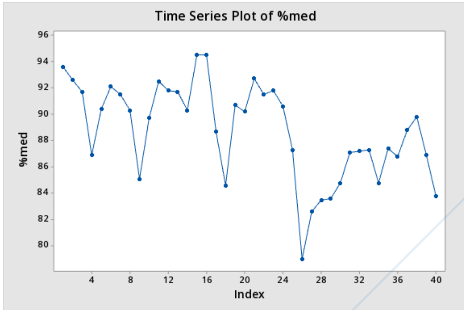
Exercise 2

a) Data snooping





The two plots show some unnatural paths, starting at observation 24. It is clear a level shift in the % of Large

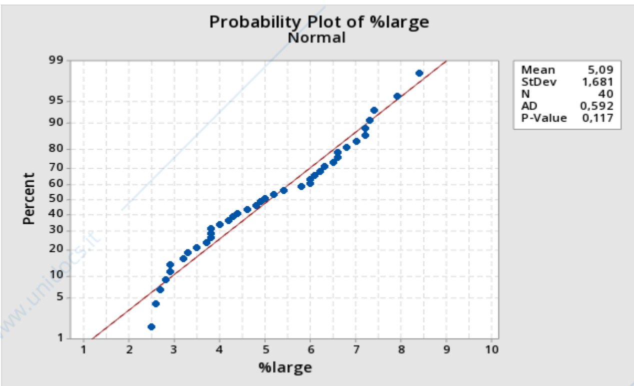
The runs test confirm the unusual pattern in both the time series

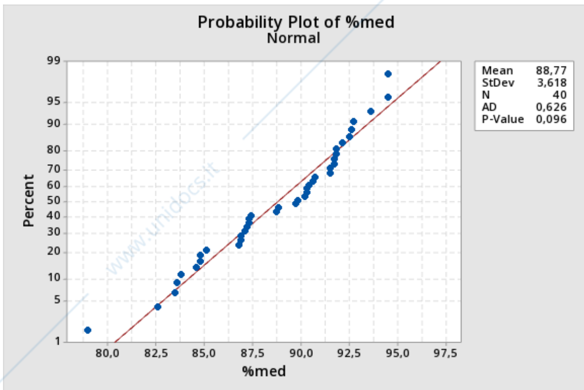
Null hypothesis H_0: The order of the data is random
Alternative hypothesis H_1: The order of the data is not random

Number of Runs

Variable	Observed	Expected	P-Value
%large	11	20,95	0,001
%med	10	20,80	0,000

Normality check on the marginal distributions (necessary condition)





a) MANAGEMENT ENGINEERING

ARL0=200

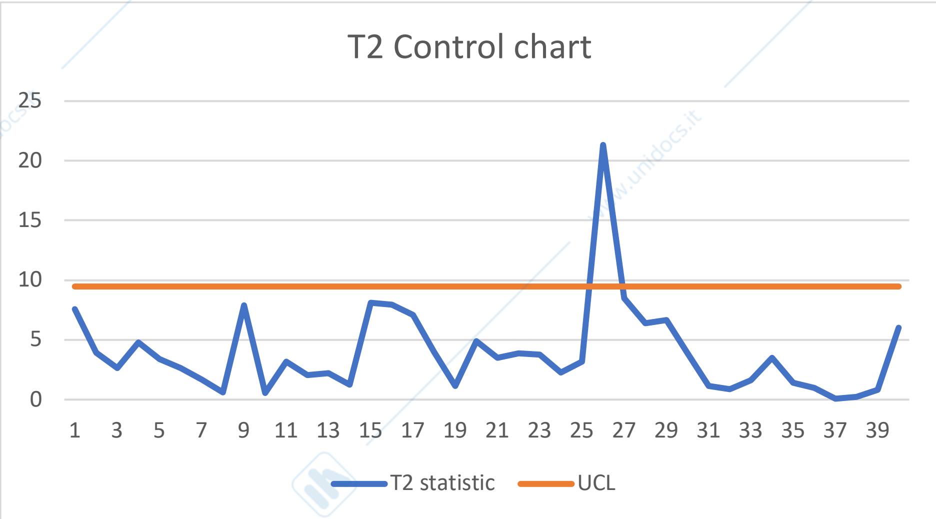
<u>T^2 Control Chart (n=1)</u>

Design phase	$UCL = \frac{(m-1)^2}{m} \beta_{\alpha, p/2, (m-p-1)/2}$ $LCL = 0$
Phase II	$UCL = \frac{p(m+1)(m-1)}{m^2 - mp} F_{\alpha, p, m-p}$ $LCL = 0$

p	2
m	40
alpha	0,005
(m-p-1)/2	18,5

Beta with first shape parameter = 1 and second = 18,5

P(X ≤ x)	x
0,995	0,249034
UCL (alpha 0,005)	9,47

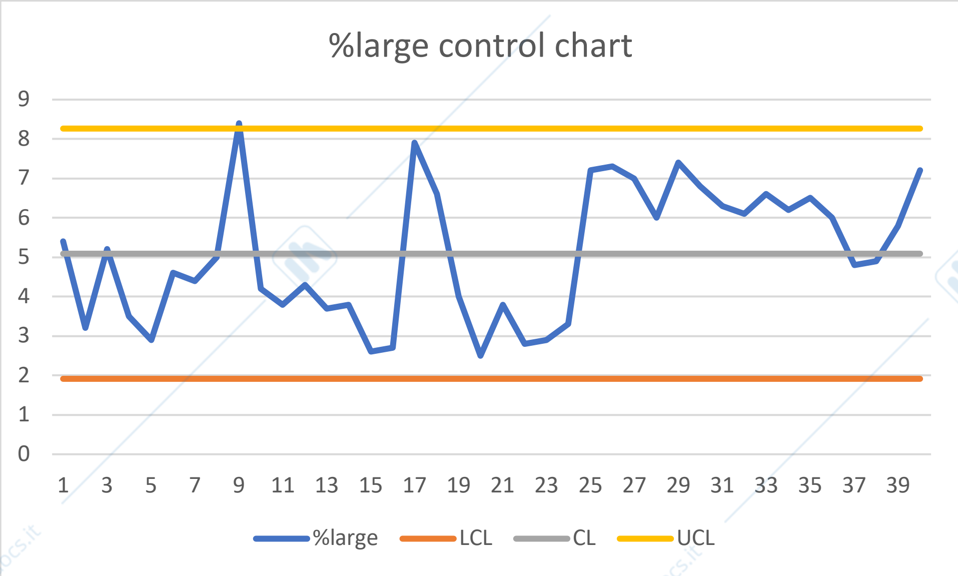


There is a clear OOC at point 26 but no clear indication of the small shift (after time 25) is given.

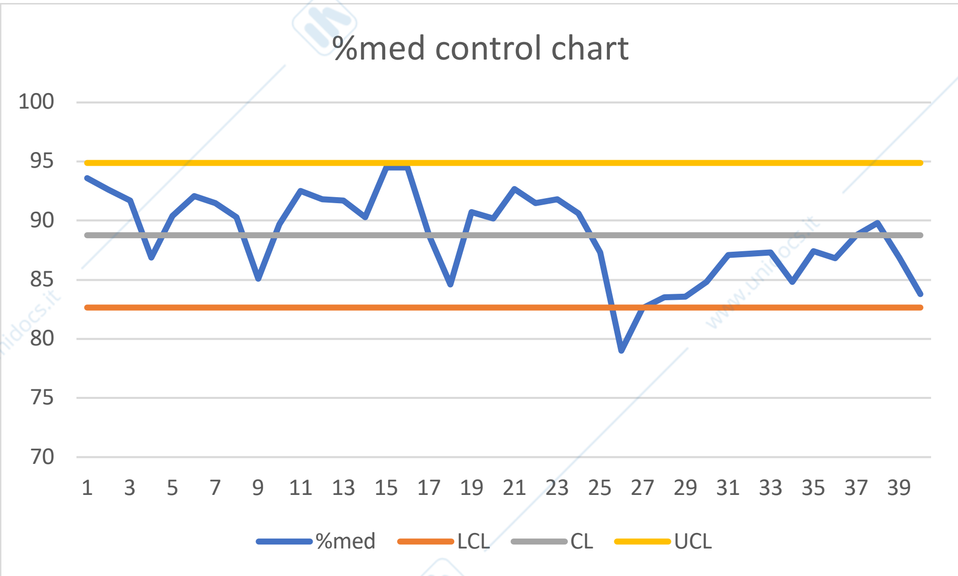
b) MECHANICAL ENGINEERING

%large	data_mean	MR_mean
	5,09	1,18461538
d2	1,128	
LCL	CL	UCL
1,9149142	5,09	8,2650858

It is clear a (small) shift from sample 25 on



%med		
data_mean	MR_mean	
88,7675	2,28205128	
d2	1,128	
LCL	CL	UCL
82,65099272	88,7675	94,8840073



It is clear a shift starting rom sample 25

c) All

It is better to use a correlation matrix approach

Eigenanalysis of the Correlation Matrix

Eigenvalue	1,7732	0,2268
Proportion	0,887	0,113
Cumulative	0,887	1,000

Eigenvectors

Variable	PC1	PC2
%large	-0,707	-0,707
%med	0,707	-0,707

PCA	coef	
with corr	-0,7071	0,7071

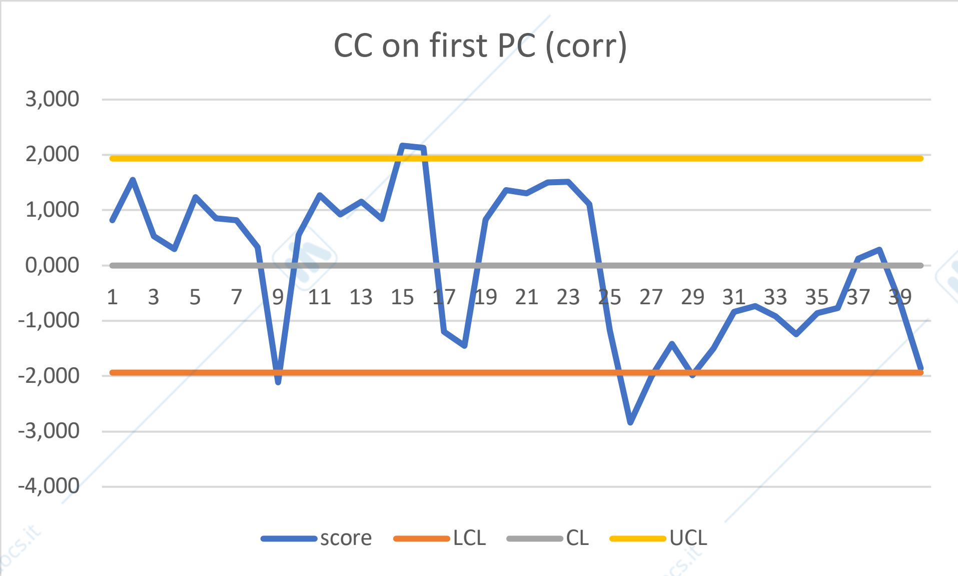
This is the first score:

score

1	0,814	11	1,272	21	1,311	31	-0,835		
2	1,544	12	0,925	22	1,497	32	-0,731		
3	0,527	13	1,158	23	1,514	33	-0,922		
4	0,304	14	0,842	24	1,111	34	-1,242		
5	1,240	15	2,168	25	-1,174	35	-0,860		
6	0,858	16	2,126	26	-2,839	36	-0,767		
7	0,824	17	-1,195	27	-2,009	37	0,128		
8	0,337	18	-1,450	28	-1,412	38	0,282		
9	-2,109	19	0,836	29	-1,982	39	-0,664		
10	0,557	20	1,369	30	-1,495	40	-1,859		

In order to design the CC on the first PC we need to compute the control limits:

alpha	0,005	
k=z_alpha/2	2,807	
%large		
data_mean	MR_mean	
0,000	0,7777	
d2	1,128	
LCL	CL	UCL
-1,935	0,000	1,935



In this case, there are lot of out of controls. This cc is revealing many OOC in Phase 1 data

Excercise 3

Range: R = max(X1 − X2) = |X1 − X2|.

The suggestion:
let X be a random variable with continuous distribution and cumulative distribution F. The absolute value of X, $|X|$, has the cumulative distribution function $G(x) = F(x) - F(-x)$, for $x > 0$
It can be rewritten as:
$P(|X| \leq x) = P(X \leq x) - P(X \leq -x) = P(-x \leq X \leq x)$.

Therefore
Let LCL e UCL be the control limits of the Range control chart, if data are NID with variance $\sigma_1 = \lambda\sigma_0$)

$$\beta = P\left(LCI \leq R \leq LCS \middle| \sigma_1^2\right) = P\left(LCI \leq |X_1 - X_2| \leq LCS \middle| \sigma_1^2\right)$$

$$= P\left(|X_1 - X_2| \leq LCS \middle| \sigma_1^2\right) - P\left(|X_1 - X_2| \leq LCI \middle| \sigma_1^2\right)$$

$$= P\left(-LCS \leq X_1 - X_2 \leq LCS \middle| \sigma_1^2\right) - P\left(-LCI \leq X_1 - X_2 \leq LCI \middle| \sigma_1^2\right)$$

dove

$$X_1 - X_2 \sim N\left(0, 2 \cdot \sigma_1^2\right)$$

λ	β
1	0,9973
1,5	0,9665