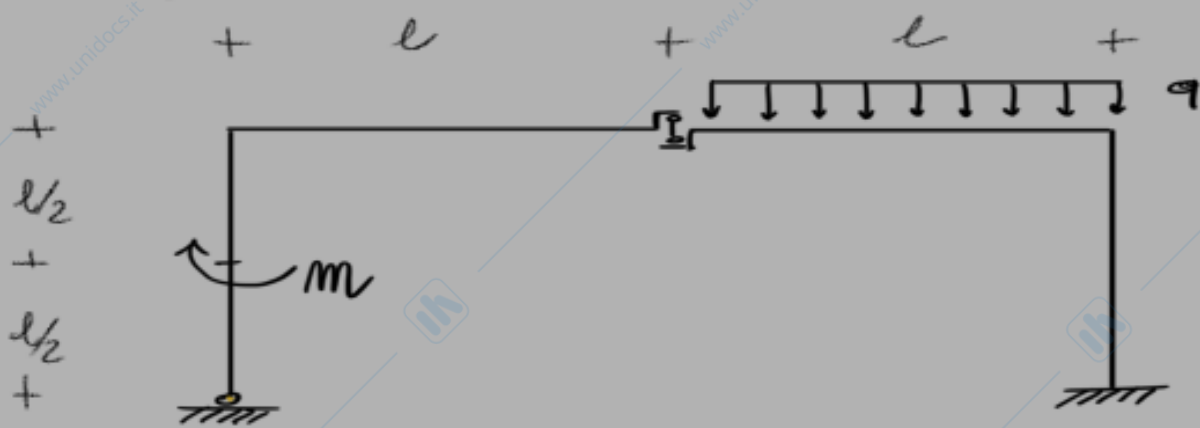


• **Esercizio 1**



$q =$
 $l =$
 $m =$

$$3r - s = l - 1$$

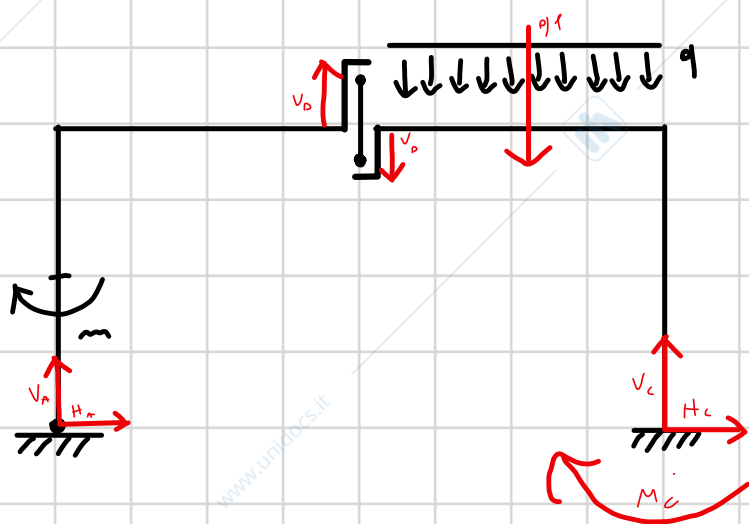
$$r = 2$$

$$s = 2 + 1 + 3 = 6$$

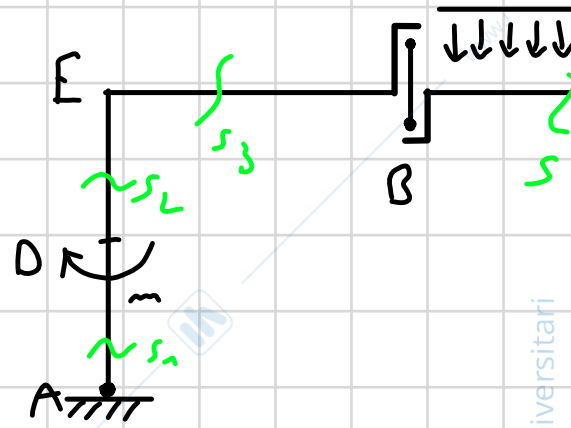
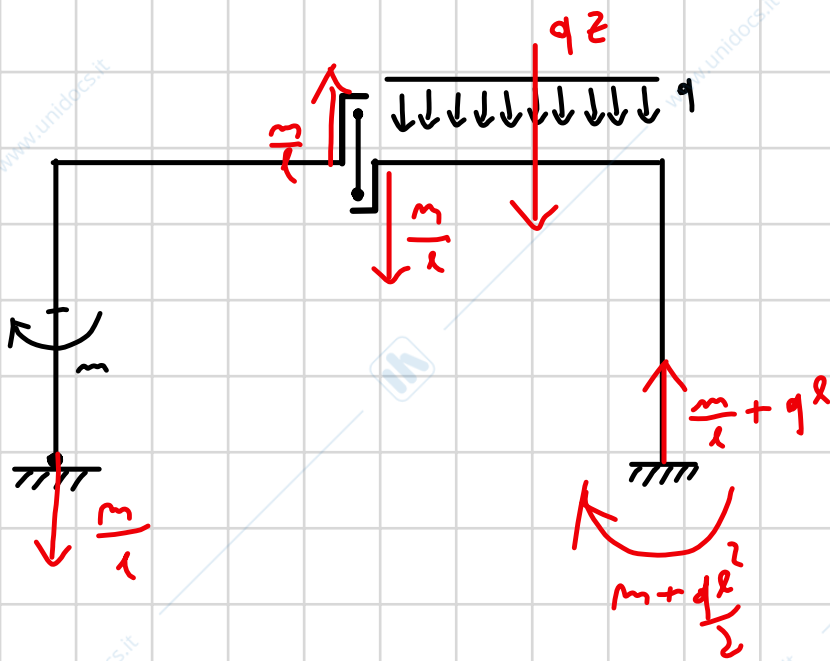
$$l - i = 0$$

C_1, C_2, C_3 NON AC

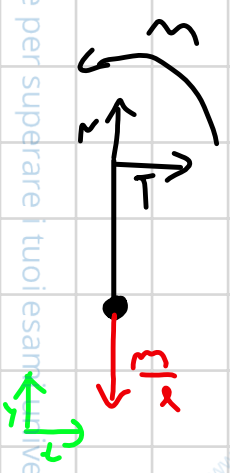
ISOSTATICA



$$\left\{ \begin{array}{l} H_A = 0 \\ V_A + V_B = 0 \\ -m + V_B l = 0 \\ H_C = 0 \\ V_C - V_B - ql = 0 \\ V_B l + \frac{q l^2}{2} - M_C = 0 \end{array} \right.$$



AD) $0 \leq z \leq \frac{l}{2}$

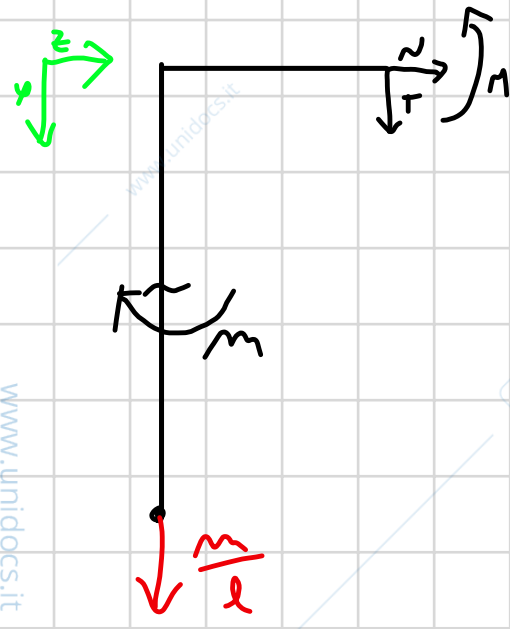


$$\begin{cases} -\frac{m}{l} + N = 0 \\ T = 0 \\ M = 0 \end{cases} \quad N = \frac{m}{l}$$

DF) $\frac{l}{2} \leq z \leq l$



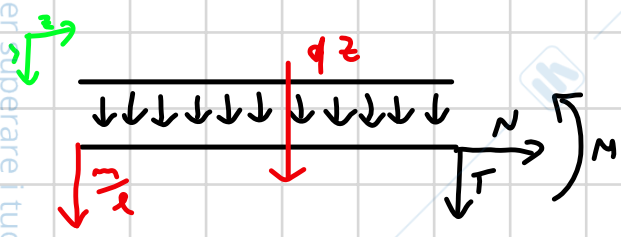
$$\begin{cases} N = \frac{m}{l} \\ T = 0 \\ -m + M = 0 \end{cases} \quad M = m$$



$$\begin{cases} N = 0 \\ \frac{m}{l} + T = 0 & T = -\frac{m}{l} \\ -m + \frac{m}{l}z + M = 0 & M = +m \end{cases}$$

$z=0 \quad M=m \quad z=l$

BF) $0 \leq z \leq l$



$$\begin{cases} N = 0 \\ \frac{m}{l} + qz + T = 0 & T = -\frac{m}{l} - qz \\ \frac{m}{l}z + qz \cdot \frac{z}{2} + M = 0 & M = -\frac{m}{l}z - \frac{qz^2}{2} \end{cases}$$

$z=0$

$$T = -\frac{m}{l}$$

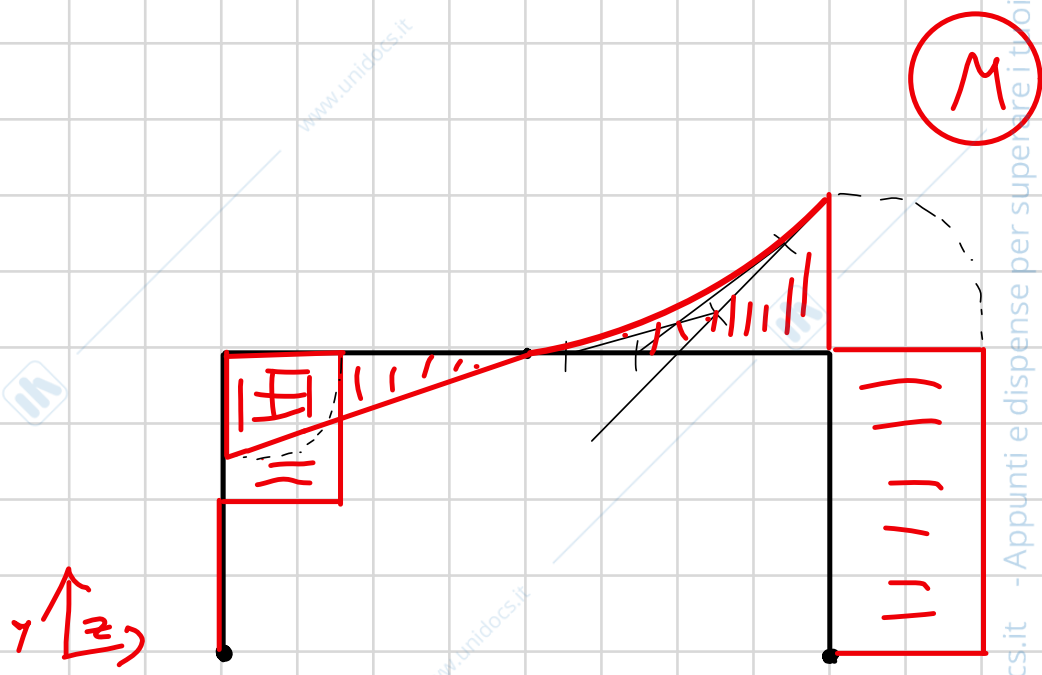
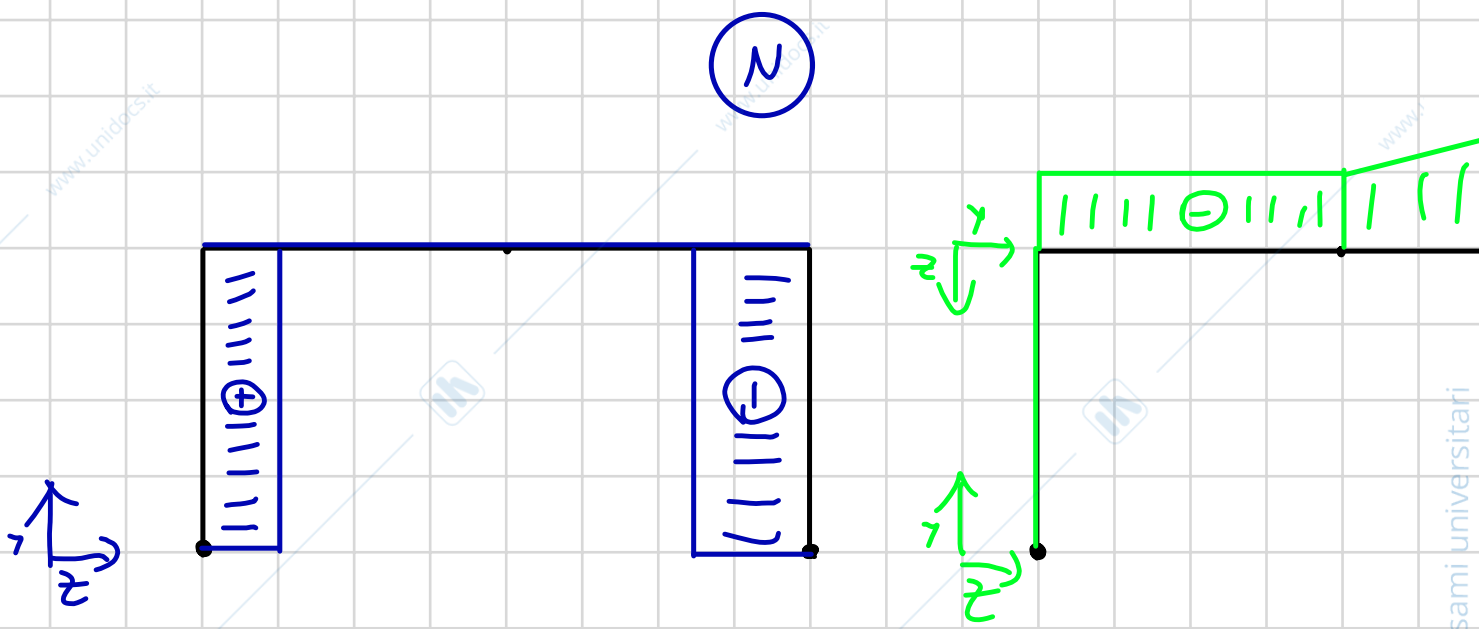
$$M = 0$$

$z=l$

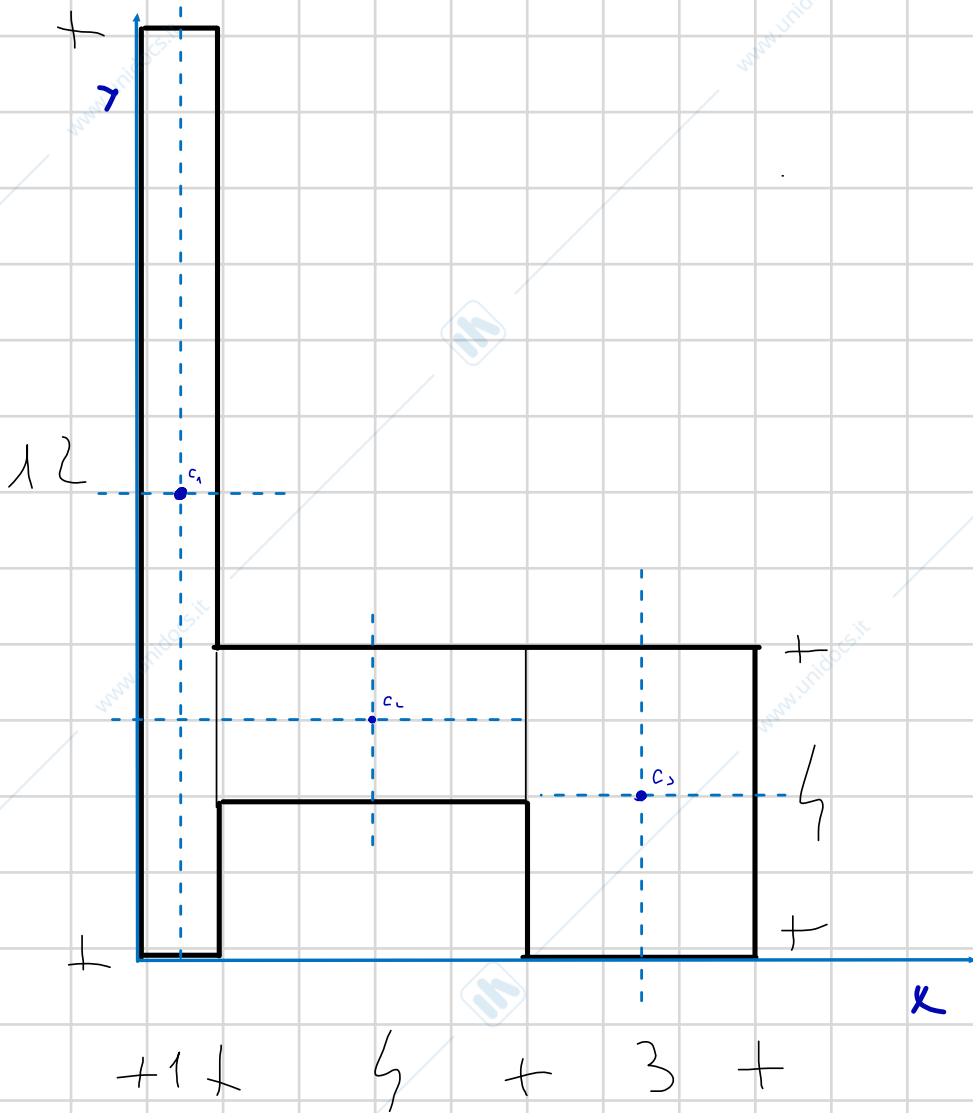
$$T = -\frac{m}{l} - ql$$

$$M = -\frac{m}{l}l - \frac{ql^2}{2}$$

CF) $0 \leq z \leq l$



ESERCIZIO 2



BARICENTRO

$$A = A_1 + A_2 + A_3$$

$$A_1 = 1 \cdot 12 = 12 \text{ cm}^2$$

$$A_2 = 4 \cdot 2 = 8 \text{ cm}^2$$

$$A_3 = 3 \cdot 2 = 6 \text{ cm}^2$$

$$G_1 \begin{cases} x_{c_1} = 0,5 \text{ cm} \\ y_{c_1} = 6 \text{ cm} \end{cases}$$

$$G_2 \begin{cases} x_{c_2} = 3 \text{ cm} \\ y_{c_2} = 3 \text{ cm} \end{cases}$$

$$G_3 \begin{cases} x_{c_3} = 6,5 \text{ cm} \\ y_{c_3} = 2 \text{ cm} \end{cases}$$

$$1) S_x^{(1)} = \int_{A_1} x \, dA = A_1 x_{c_1} = 12 \cdot 0,5 = 6 \text{ cm}^3$$

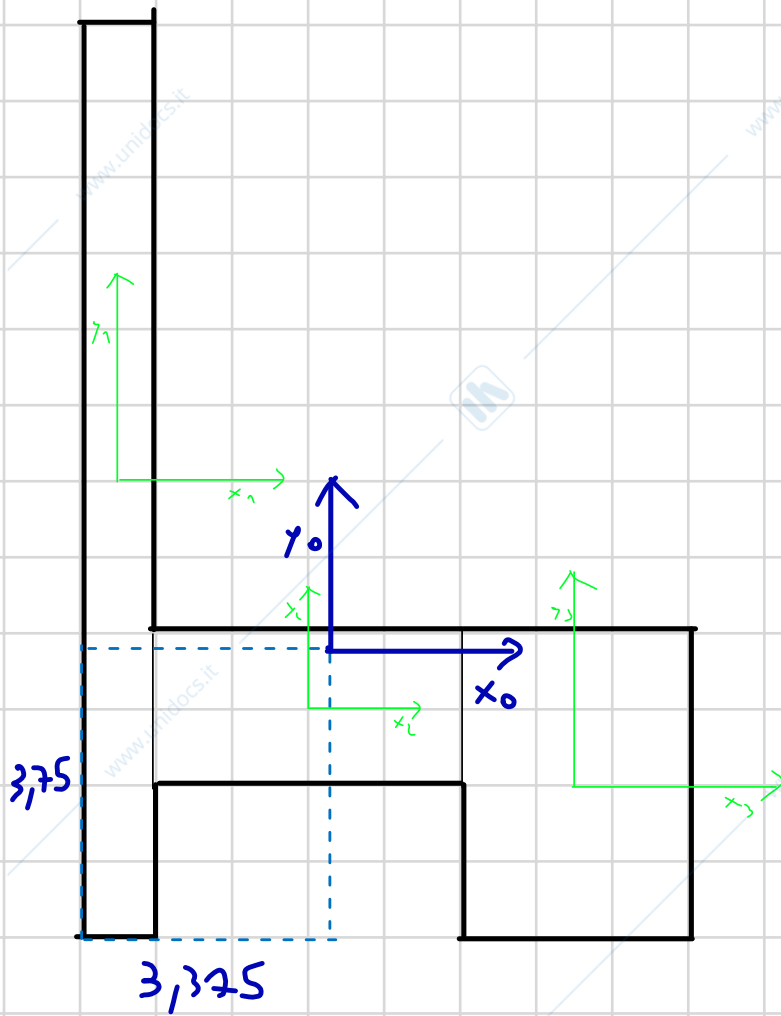
$$S_y^{(1)} = \int_{A_1} y \, dA = A_1 y_{c_1} = 12 \cdot 6 = 72 \text{ cm}^3$$

$$2) S_x^{(2)} = 8 \cdot 3 = 24 \text{ cm}^3$$

$$S_y^{(2)} = 8 \cdot 3 = 24 \text{ cm}^3$$

$$3) S_x^{(3)} = 6 \cdot 6,5 = 39 \text{ cm}^3$$

MOMENTO DI INER



$$x_{0c}^{(1)} = -(3,375 - 0,5) = -2,8$$

$$y_{0c}^{(1)} = 6 - 3,375 = 2,2$$

$$x_{0c}^{(2)} = -(3,375 - 3) = -0,375$$

$$y_{0c}^{(2)} = -(3,375 - 3) = -0,375$$

$$x_{0c}^{(3)} = 6,5 - 3,375 = 3,125$$

$$y_{0c}^{(3)} = -(3,375 - 2) = -1,375$$

$$I_{x_0} = \sum_{j=1}^3 I_{x_j} + \sum_{j=1}^3 A_j \cdot x_{0c}^{(j)2}$$

ASSIALE LUNGO

$$I_{x_1} = \frac{b_1^3 \cdot h_1}{12} = \frac{1^3 \cdot 12}{12} = 1 \text{ cm}^4$$

$$I_{x_2} = \frac{4^3 \cdot 2}{12} = 10,67 \text{ cm}^4$$

$$I_{x_3} = \frac{3^3 \cdot 4}{12} = 9 \text{ cm}^4$$

$$I_{x_0} = 20,67 + 218,$$

$$A_1 \cdot x_{0c}^{(1)2} = 99,53 \text{ cm}^4$$

$$I_{y_0} = \sum_{j=1}^3 I_{y_j} + \sum_{j=1}^3 A_j \cdot y_{j0}^{(j)2}$$

ASSIALE LUNGO Y

$$I_{y_1} = \frac{b_1 \cdot h_1^3}{12} = \frac{1 \cdot 12^3}{12} = 144 \text{ cm}^4$$

$$I_{y_2} = 2,67 \text{ cm}^4$$

$$I_{y_3} = 16 \text{ cm}^4$$

$$I_{y_0} = 162,67 + 16$$

$$A_1 \cdot y_{0c}^{(1)2} = 12 \cdot 2,25^2 = 60,75 \text{ cm}^4$$

$$A_2 \cdot y_{0c}^{(2)2} = 8 \cdot (-0,75)^2 = 4,5 \text{ cm}^4$$

$$A_3 \cdot y_{0c}^{(3)2} = 12 \cdot (-1,75)^2 = 36,75 \text{ cm}^4$$

$$I_{x_0 y_0} = \sum_{i=1}^3 A_i \cdot x_{0c}^{(i)} \cdot y_{0c}^{(i)}$$

CENTRIFUGO

$$I_{x_1 y_1} = A_1 \cdot x_{0c}^{(1)} \cdot y_{0c}^{(1)} = 12 \cdot (-2,88) \cdot 2,25 = -77,76$$

$$I_{x_2 y_2} = A_2 \cdot x_{0c}^{(2)} \cdot y_{0c}^{(2)} = 8 \cdot (-0,375) \cdot (-0,75) = 2,25$$

$$I_{x_3 y_3} = A_3 \cdot x_{0c}^{(3)} \cdot y_{0c}^{(3)} = 12 \cdot (3,13) \cdot (-1,75) = -65,7$$

DIREZIONI E MOMENTI PRINCIPALI INERZIA

$$\hat{I}_G = \begin{bmatrix} 239 & -141 \\ -141 & 264,67 \end{bmatrix}$$

$$\det(\hat{I}_G - I \lambda) = 0$$

$$\det \begin{bmatrix} 239 - \lambda & -141 \\ -141 & 264,67 - \lambda \end{bmatrix} = (239 - \lambda)(264,67 - \lambda)$$

$$\lambda^2 - 503,67\lambda + 43375,12 = 0$$

$$\lambda = \frac{503,67 \pm \sqrt{(503,67)^2 - 4 \cdot 43375,12}}{2} \begin{cases} 11 \\ 39 \end{cases}$$

$$I_{\eta} = 110 \text{ cm}^4$$

$$\rho_{\eta} = \sqrt{\frac{I_{\eta}}{A}} = \sqrt{\frac{110}{3}}$$

$$I_{\xi} = 393 \text{ cm}^4$$

$$\rho_{\xi} = \sqrt{\frac{I_{\xi}}{A}} = \sqrt{\frac{393}{3}}$$

$$\lambda = I \xi$$

$$\hat{I}_9 = \begin{bmatrix} 239 - 333 & -141 \\ -141 & 264,67 - 333 \end{bmatrix} \begin{pmatrix} d \xi^{(1)} \\ d \xi^{(2)} \end{pmatrix}$$

$$-154 d \xi^{(2)} - 141 d \xi^{(1)} = 0$$

$$d \xi^{(2)2} + d \xi^{(1)2} = 1$$

$$d \xi^{(2)} = -\frac{154}{141} d \xi^{(1)} = -1,1 d \xi^{(1)}$$

$$1 d \xi^{(2)2} + 1,20 d \xi^{(1)2} = 1 \Rightarrow 2,20 d \xi^{(1)2}$$

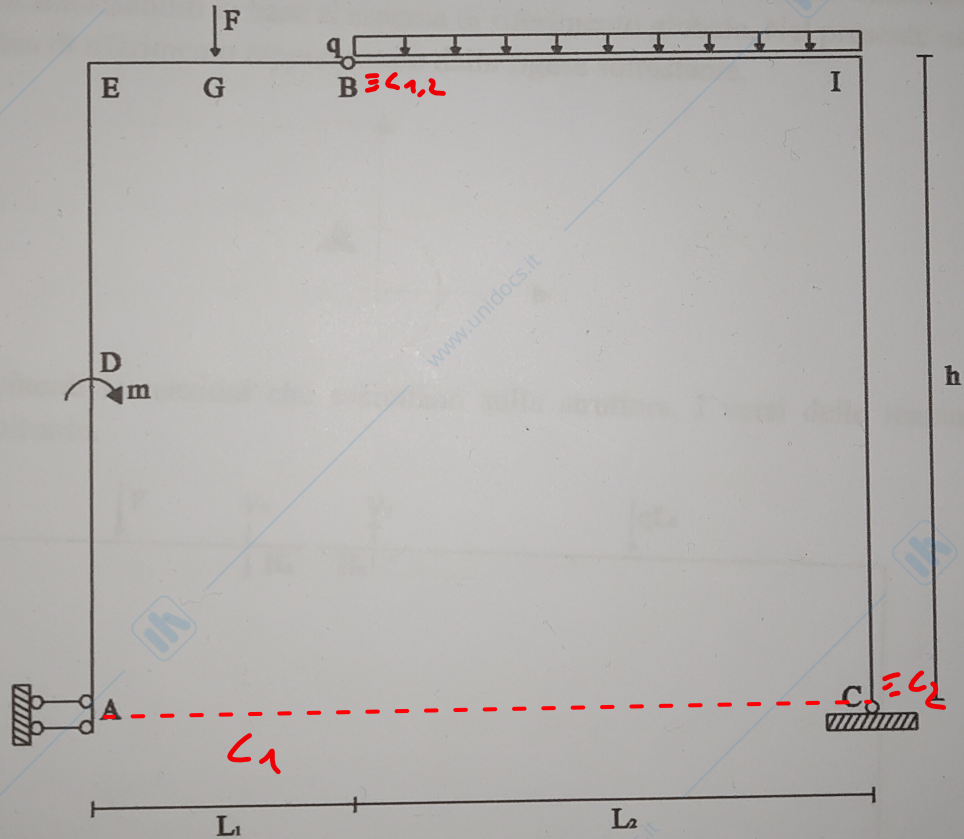
$$= d \xi^{(1)} = \sqrt{\frac{1}{2,20}} = 0,67 \text{ cm}$$

$$d \xi^{(2)} = -1,1 d \xi^{(1)} = -0,74 \text{ cm}$$

Telaio Isostatico - Esercizio

Risolvere lo schema a telaio e tracciare i diagrammi dello Sforzo Normale, del Taglio e del Momento Flettente indicandone i valori significativi.

- $L_1 = 2 \text{ m}$
- $L_2 = 4 \text{ m}$
- $h = 5 \text{ m}$
- $q = 6,25 \text{ kN/m}$
- $F = 10 \text{ kN}$
- $M = 30 \text{ kNm}$



$$3\tau - s = n - 1$$

$$\tau = 2$$

$$s = 2 + 2 + 2 = 6$$

$$n - 1 = 0$$

$C_1, C_{1,2}, C_3$ NON ALLINEATI

ISOSTATICA



$$H_A - H_B = 0$$

$$V_B = F = 10 \text{ kN}$$

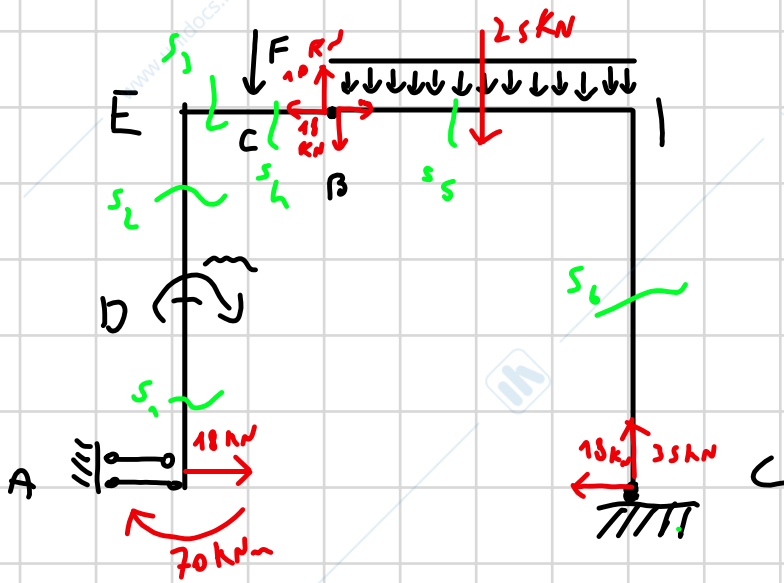
$$H_A = \frac{4F + 8q}{5} = 18 \text{ kN}$$

$$V_C = F + 4q = 35 \text{ kN}$$

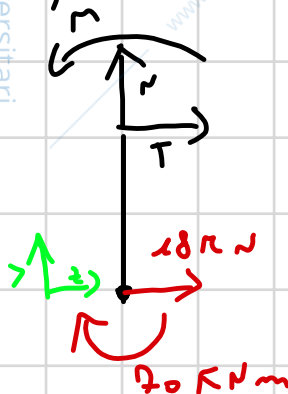
$$H_C = -\frac{4F + 8q}{5} = -18 \text{ kN}$$

$$H_B = \frac{4F + 8q}{5} = 18 \text{ kN}$$

$$M_A = -m - F + 2F + 4F + 8q = 70 \text{ kNm}$$



AD) $0 \leq z \leq 2,5$



$$\begin{cases} N = 0 \\ 18 + T = 0 & T = -18 \text{ kN} \\ -70 + 18z + M = 0 & M = 70 - 18z \end{cases}$$

$z = 0 \quad M = 70 \text{ kNm}$

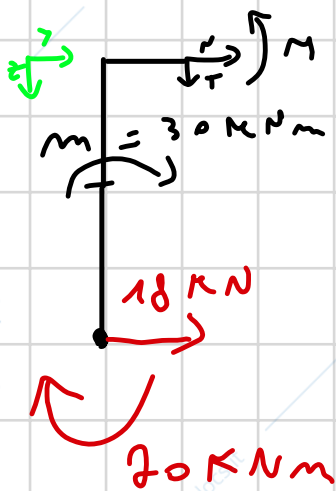
$z = 2,5 \quad M = 25 \text{ kNm}$

DE) $2,5 \leq z \leq 5$



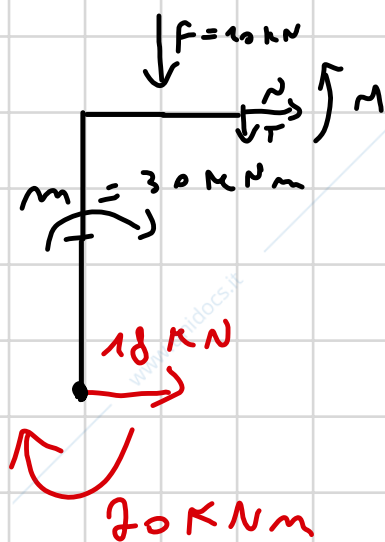
$N = 0$

EG) $0 \leq z \leq 1$



$$\begin{cases} 18 + N = 0 & N = -18 \text{ kN} \\ T = 0 \\ -70 - 30 + 5 \cdot 18 + M = 0 \end{cases}$$

GB) $1 < z < 2$

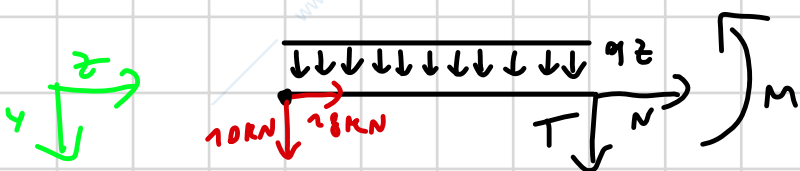


$$\begin{cases} N = -18 \text{ kN} \\ T = -10 \text{ kN} \\ -70 - 30 + 18 \cdot 5 + 10z = M \quad M = 10z - 10 \end{cases}$$

$z = 1 \quad M = 10 \text{ kNm}$

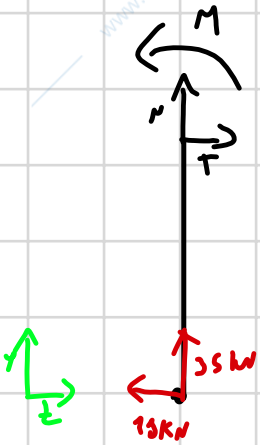
$z = 2 \quad M = 0$

BI) $0 \leq z \leq 4$



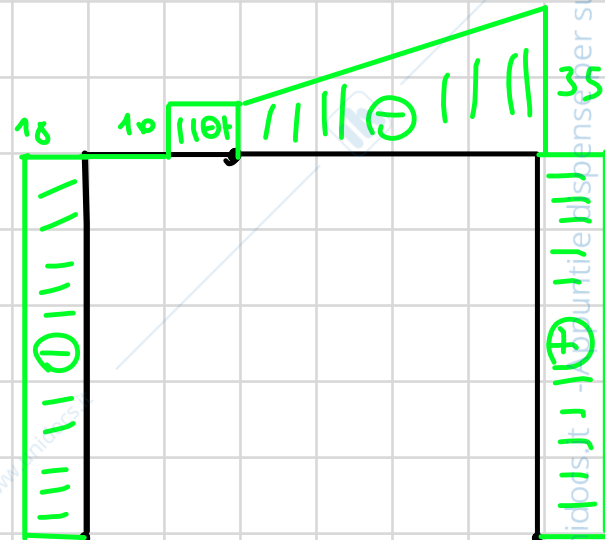
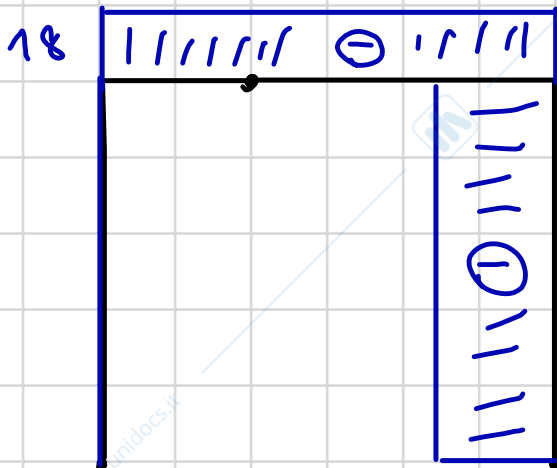
$$\begin{cases} 18 + N = 0 \Rightarrow N = -18 \text{ kN} \\ 10 + qz + T = 0 \\ 10z + qz^2 + M = 0 \end{cases}$$

c1) $0 \leq z \leq 5$

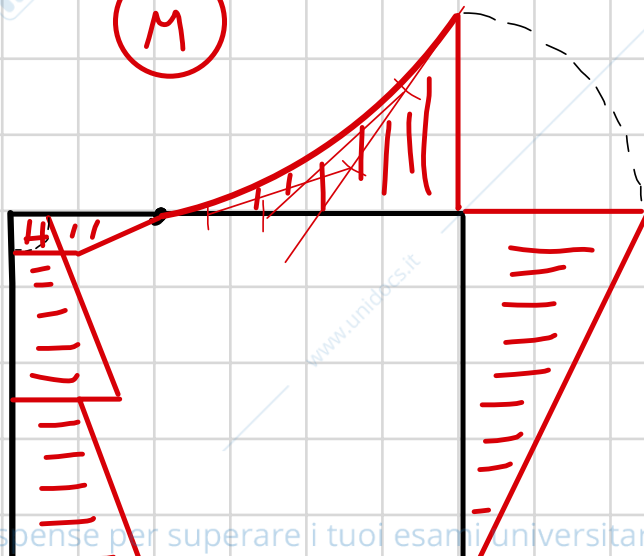


$$\begin{cases} 35 + N = 0 & N = -35 \\ -18 + T = 0 & T = +18 \\ -18z + M = 0 & M = 18z \end{cases} \quad \begin{matrix} z = \\ M = \end{matrix}$$

(N)

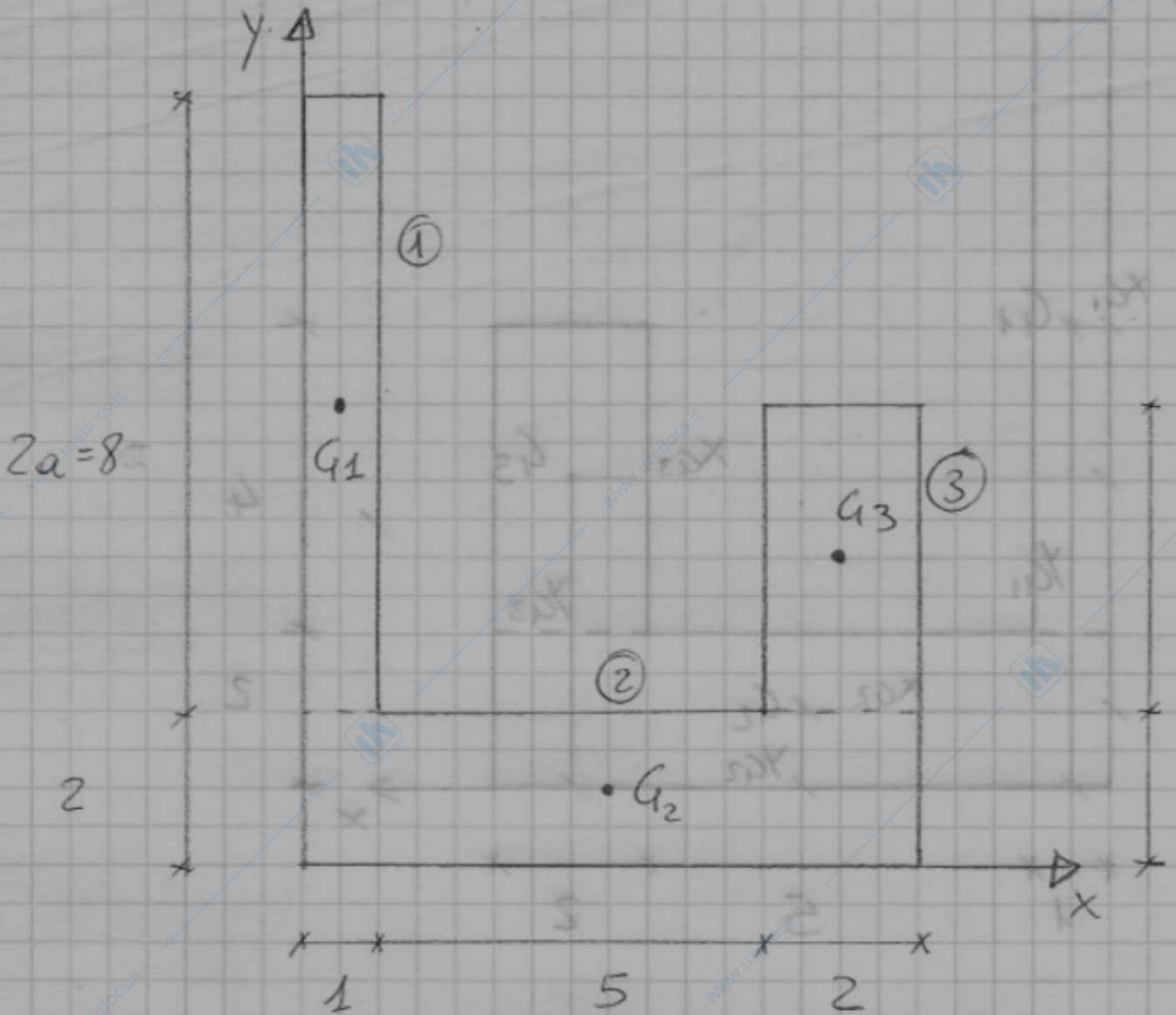


(M)



GEOMETRIA DELLE AREE - ESERCIZI

$a = 4 \text{ cm}$



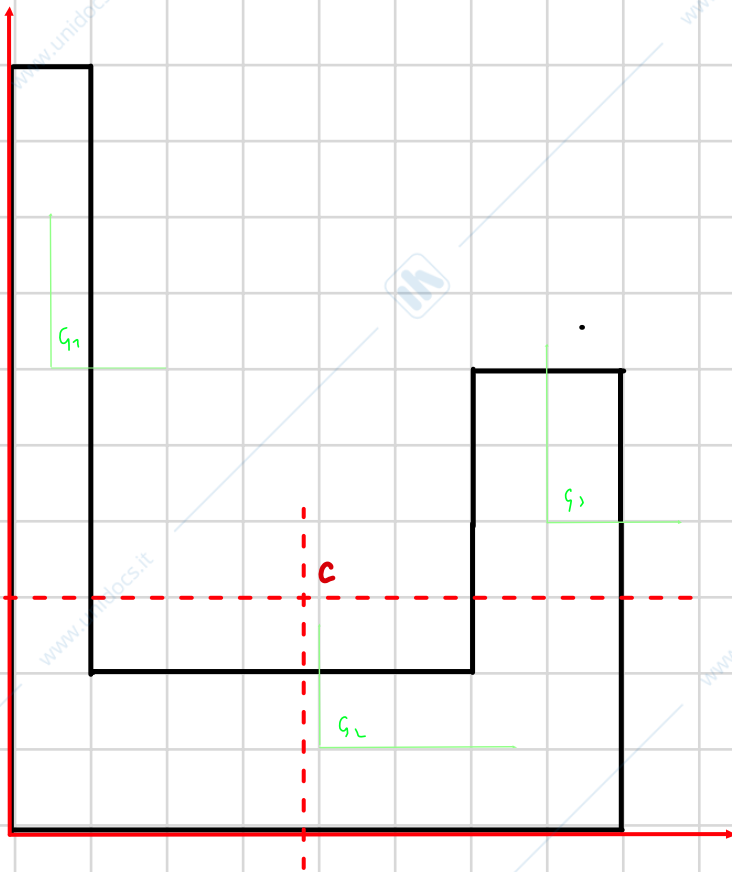
$$A = A_1 + A_2 + A_3 = 32 \text{ cm}^2$$

$$A_1 = 1 \cdot 8 = 8 \text{ cm}^2 \quad A_2 = 8 \cdot 2 = 16 \text{ cm}^2 \quad A_3 = 2 \cdot 4 = 8 \text{ cm}^2$$

$$G_1 \begin{cases} x_{G_1} = 0,5 \text{ cm} \\ y_{G_1} = 6 \text{ cm} \end{cases} \quad G_2 \begin{cases} x_{G_2} = 4 \text{ cm} \\ y_{G_2} = 1 \text{ cm} \end{cases} \quad G_3 \begin{cases} x_{G_3} = 7 \text{ cm} \\ y_{G_3} = 4 \text{ cm} \end{cases}$$

$$1) S_x^{(1)} = \int_A x_{G_1} dA = A_1 \cdot x_{G_1} = 8 \cdot 0,5 = 4 \text{ cm}^2 \quad S_y^{(1)} = \int_A y_{G_1} dA = y_{G_1} \cdot A_1 =$$

$$2) S_x^{(2)} = \int_A x_{G_2} dA = A_2 \cdot x_{G_2} = 16 \cdot 4 = 64 \text{ cm}^2 \quad S_y^{(2)} = \int_A y_{G_2} dA = y_{G_2} \cdot A_2 =$$



$$x_{oq}^{(1)} = -(3,88 - 0,5) = -3,38$$

$$y_{oq}^{(1)} = 6 - 3 = 3 \text{ cm}$$

$$x_{oq}^{(2)} = 5 - 3,88 = 1,12 \text{ cm}$$

$$y_{oq}^{(2)} = -(3 - 1) = -2 \text{ cm}$$

$$x_{oq}^{(3)} = 7 - 3,88 = 3,12 \text{ cm}$$

$$y_{oq}^{(3)} = 4 - 3 = 1$$

$$I_{x_o} = \sum_{i=1}^3 I_{x_i} + \sum_{i=1}^3 A_i x_{oq}^{(i)2}$$

$$I_{x_1} = 10,7 \text{ cm}^4 \quad I_{x_2} = 85,33 \text{ cm}^4 \quad I_{x_3} = 2,7 \text{ cm}^4$$

$$A_1 x_{oq}^{(1)2} = 91,4 \text{ cm}^4 \quad A_2 x_{oq}^{(2)2} = 1,23 \text{ cm}^4 \quad A_3 x_{oq}^{(3)2} = 77,88$$

$$I_{x_o} = 258,2 \text{ cm}^4$$

$$I_{y_o} = \sum_{i=1}^3 I_{y_i} + \sum_{i=1}^3 A_i y_{oq}^{(i)2}$$

$$I_{y_1} = \frac{b_1 h_1^3}{12} = 12,7 \text{ cm}^4 \quad I_{y_2} = 5,3 \text{ cm}^4 \quad I_{y_3} = 10,7 \text{ cm}^4$$

$$A_1 y_{oq}^{(1)2} = 72 \text{ cm}^4 \quad A_2 y_{oq}^{(2)2} = 64 \text{ cm}^4 \quad A_3 y_{oq}^{(3)2} = 8 \text{ cm}^4$$

$$I_{y_o} = 202,7 \text{ cm}^4$$

$$\hat{I}_G = \begin{vmatrix} 258,2 & -60 \\ -60 & 202,7 \end{vmatrix}$$

$$(\hat{I}_G + \lambda I) d = 0$$

$$\det \begin{vmatrix} 258,2 - \lambda & -60 \\ -60 & 202,7 - \lambda \end{vmatrix} = (258,2 - \lambda)(202,7 - \lambda) - 60^2 =$$

$$= \lambda^2 + 52337 - 3600 - 460,9\lambda =$$

$$\lambda^2 - 460,9\lambda + 48737$$

$$\lambda = \frac{460,9 \pm \sqrt{460,9^2 - 4 \cdot 48737}}{2} = \frac{460,9 \pm 132,2}{2} = \begin{matrix} 296, \\ 164, \end{matrix}$$

$$g_y = \sqrt{\frac{I_y}{A}} = 3,04 \text{ cm}$$

$$g_z = \sqrt{\frac{I_z}{A}} = 2,27$$

$$\det \begin{vmatrix} 258,2 - 296,53 & -60 \\ -60 & 202,7 - 296,53 \end{vmatrix} \begin{vmatrix} d y^{(1)} \\ d y^{(2)} \end{vmatrix} = \begin{vmatrix} 0 \\ 0 \end{vmatrix}$$

$$(-38,33 d y^{(1)} - 60 d y^{(2)}) = 0$$

$$\begin{cases} -38,33 d \xi^{(1)} - 60 d \xi^{(2)} = 0 \\ d \xi^{(1)2} + d \xi^{(2)2} = 1 \end{cases}$$

$$\begin{cases} d \xi^{(1)} = -\frac{60}{38,33} d \xi^{(2)} \\ (-1,57 d \xi^{(2)})^2 + d \xi^{(2)2} = 1 \Rightarrow (2,46 + 1) d \xi^{(2)2} = 1 \end{cases}$$

$$\Rightarrow d \xi^{(2)} = \sqrt{\frac{1}{3,46}} = 0,53$$

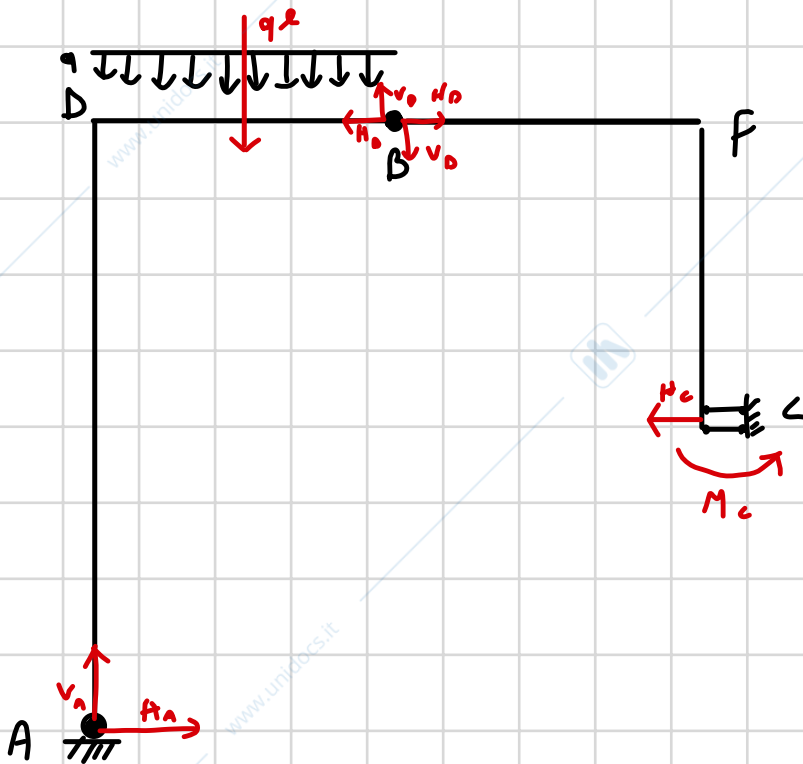
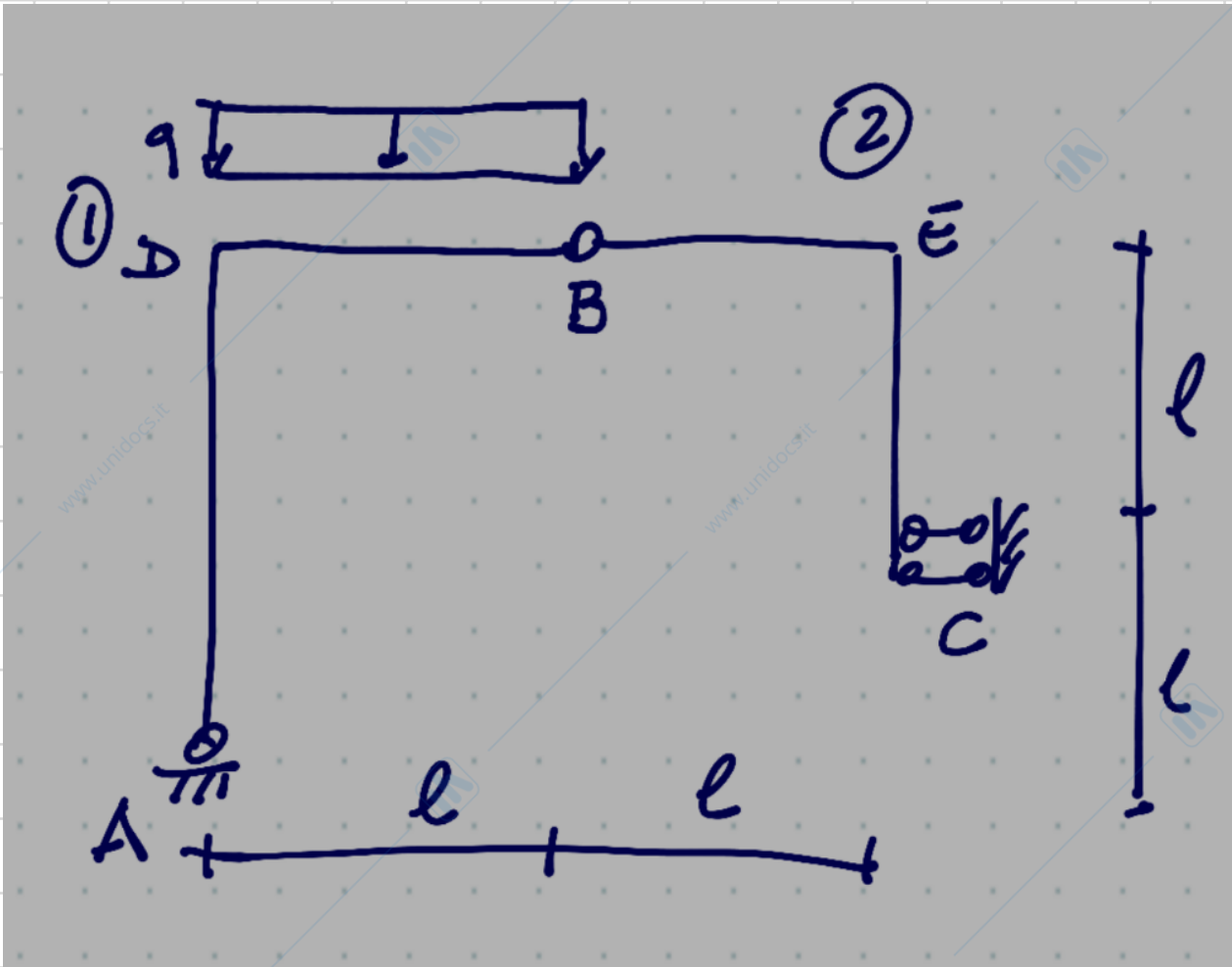
$$d \xi^{(1)} = -0,84$$

$$\det \begin{vmatrix} 258,2 & -164,32 & -60 \\ -60 & 202,7 & -164,32 \end{vmatrix} \begin{vmatrix} d m^{(1)} \\ d m^{(2)} \end{vmatrix} = \begin{vmatrix} 0 \\ 0 \end{vmatrix}$$

$$23,08 d m^{(1)} - 60 d m^{(2)} = 0$$

$$-60 d m^{(1)} + 38,38 d m^{(2)} = 0$$

$$d m^{11} = 0,54$$



$$3r - s = l - i$$

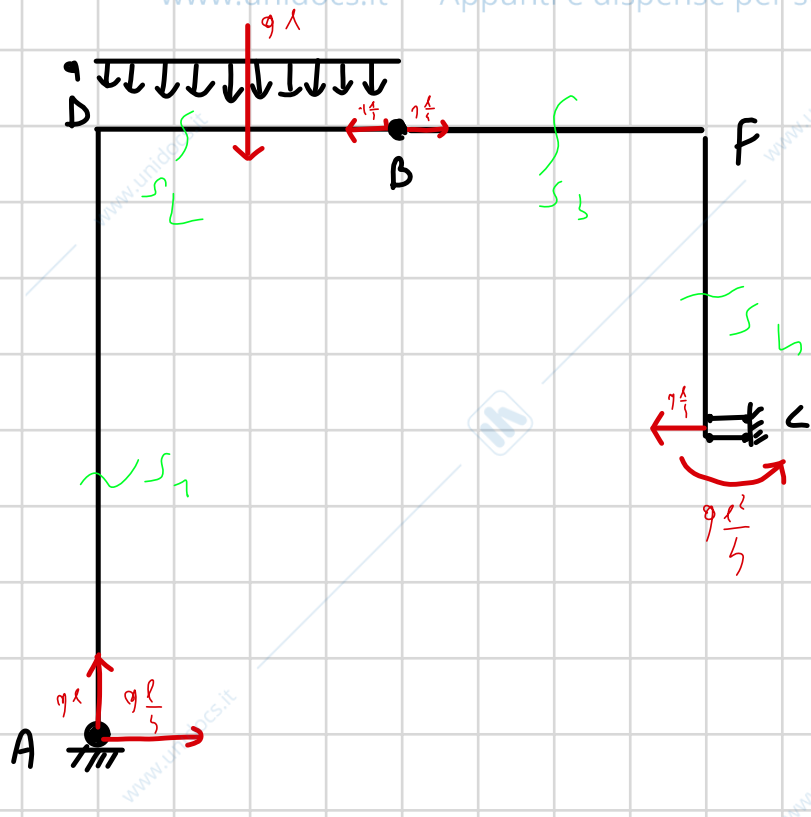
$$r = 2$$

$$s = 2 + 2 + 2 = 6$$

$$l - i = 0$$

C_1, C_2, C_3 NON ALLINEATI

$$l = 0 \quad i = 0$$



AD) $0 \leq z \leq 2l$

$z=0 \quad M=0$

$z=2l$

$M = -q \frac{l^2}{5}$



$$q l + N = 0$$

$$q \frac{l}{5} + T = 0$$

$$M = -q \frac{l}{5} z$$

$$N = -q l$$

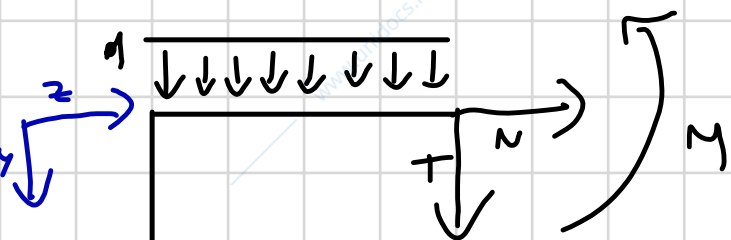
$$T = -q \frac{l}{5}$$

$$M = -q \frac{l}{5} z$$

DB) $0 \leq z \leq l$

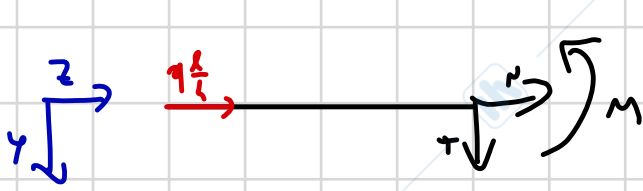
$z=0 \quad T=q l \quad M = -q \frac{l^2}{2}$

$z=l \quad T=0 \quad N=0$



$$q \frac{l}{5} + N = 0 \quad N = -q \frac{l}{5}$$

BF) $0 \leq t \leq l$



$$\begin{cases} N = -q \frac{l}{4} \\ T = 0 \\ M = 0 \end{cases}$$

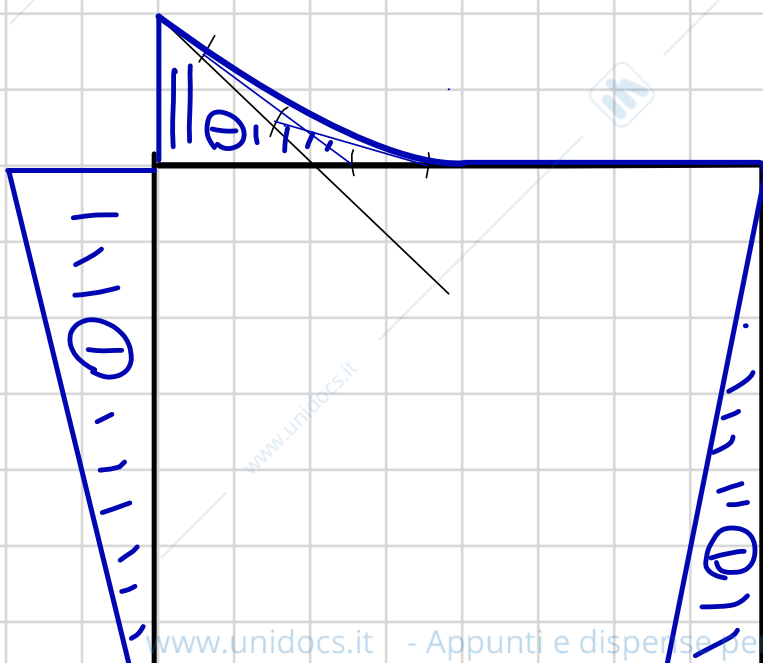
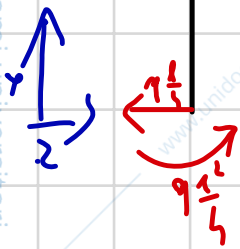
CF) $0 \leq z \leq l$

$z = 0 \quad M = q \frac{l^2}{4} \quad z = l \quad M = 0$



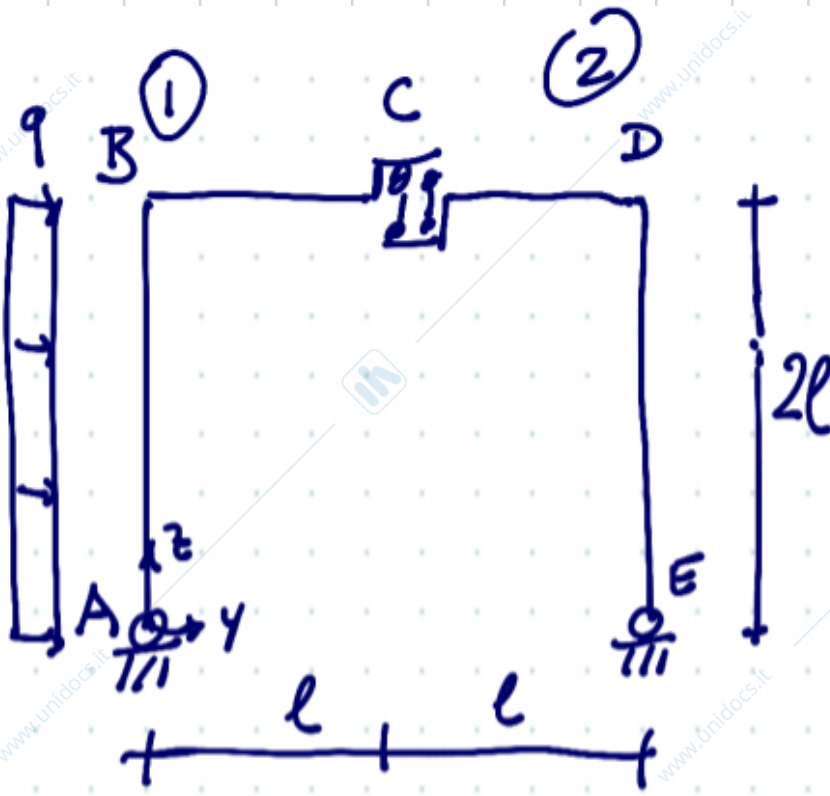
$$\begin{cases} N = 0 \\ -q \frac{l}{4} + T = 0 \\ q \frac{l^2}{4} - q \frac{l}{4} z + M = 0 \end{cases}$$

$$\begin{cases} N = 0 \\ T = -q \frac{l}{4} \\ M = -q \frac{l^2}{4} + q \frac{l}{4} z \end{cases}$$



X TAGLIO E SFORZO

VEDI LIVELLO 4 E 5



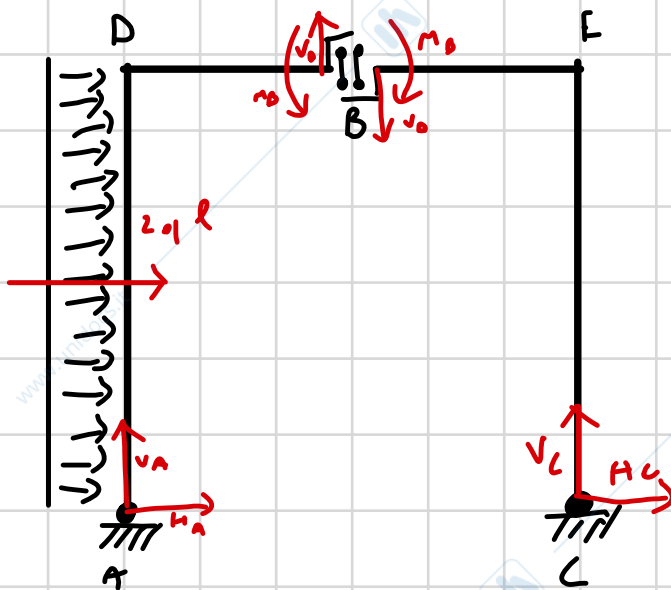
$$3\mathcal{L} - S = \mathcal{L} - i$$

$$\mathcal{L} = 2$$

$$S = 2 + 2 + 2 = 6$$

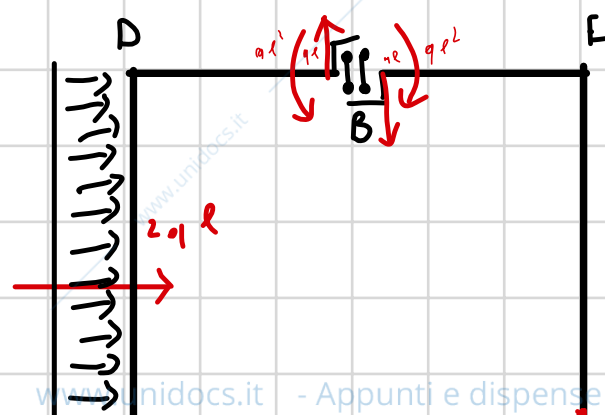
$$\mathcal{L} = 0$$

$$C_1, C_{1,2}, C_2 \text{ NON A}$$

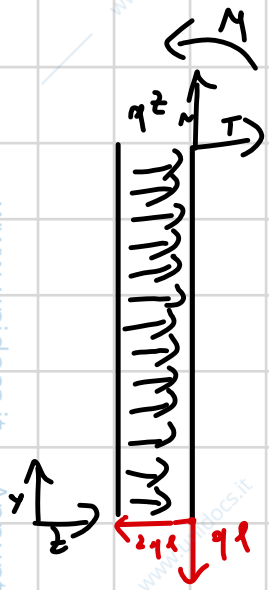


$$\begin{cases} H_A + 2ql = 0 \\ V_A + V_B = 0 \\ -2ql^2 + M_B + V_B l = 0 \\ H_C = 0 \\ V_C - V_B = 0 \\ V_B l - M_B = 0 \end{cases}$$

$$\begin{aligned} H_A &= -2ql & V_B &= +ql \\ M_B &= V_B l & V_C &= +ql \\ 2M_B &= +ql^2 & V_A &= -ql \end{aligned}$$

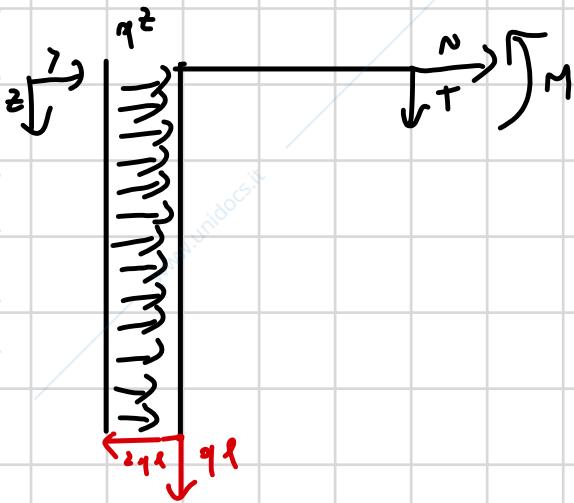


A) $0 \leq z \leq 2l$



$$\begin{cases} -ql + N = 0 & N = ql \\ -2ql + qz + T = 0 & T = 2ql - qz \\ q\frac{z^2}{2} - 2qlz + M = 0 & M = -q\frac{z^2}{2} + 2qlz \end{cases}$$

B) $0 \leq z \leq l$



$$\begin{cases} -2ql + 2ql + N = 0 \\ ql + T = 0 \\ qlz - 4ql^2 + 2ql^2 + M = 0 \end{cases}$$

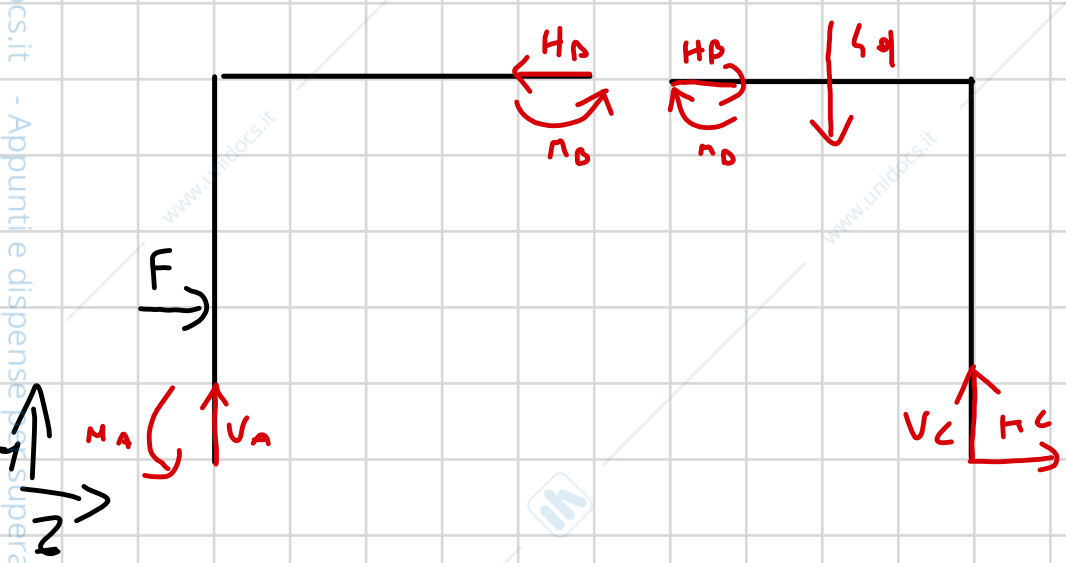
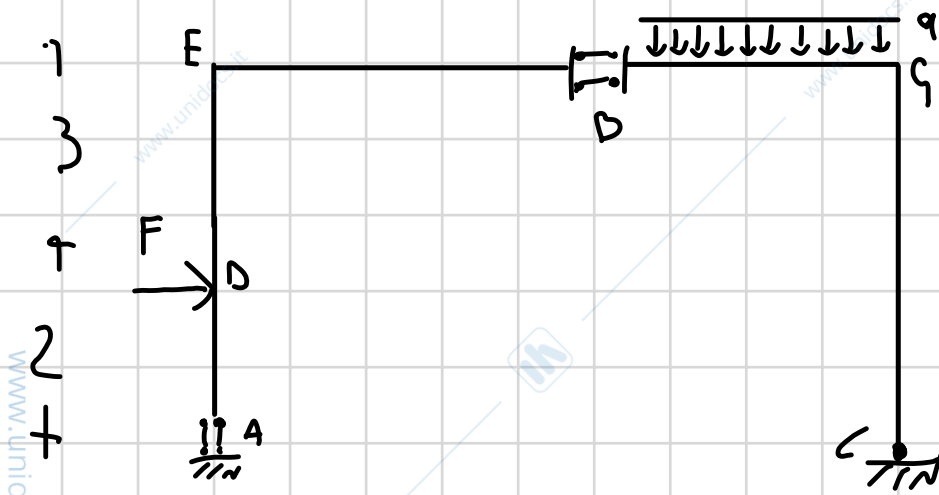
$z = 0$

$$N = 0$$

$$T = -ql$$

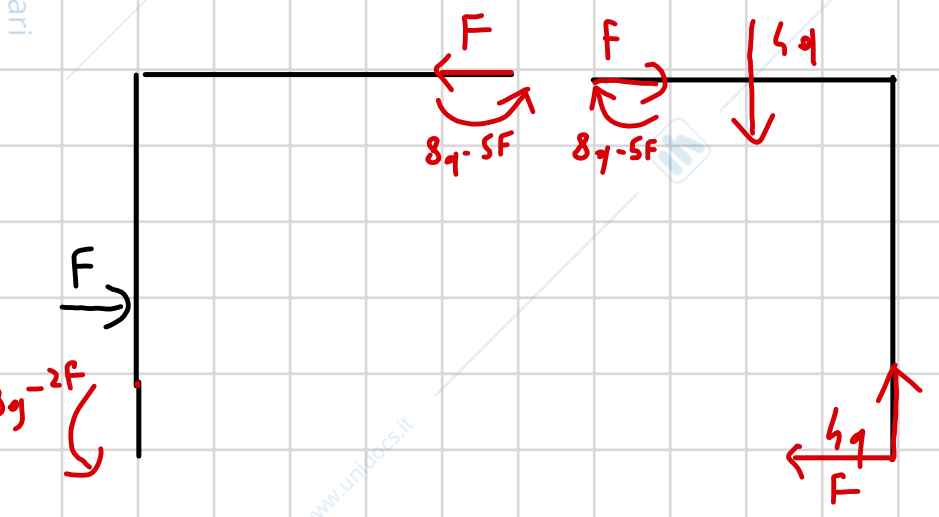
$$M = 2ql^2 - qlz \quad M = 2ql^2$$

S T h



$$\begin{cases} F - H_B = 0 \\ V_A = 0 \\ -2F + 5H_B + 8q = 0 \\ H_C + H_B = 0 \\ -4q + V_C = 0 \end{cases}$$

$$\begin{aligned} M_A &= 8q - 2F & M_B &= 8q - 5F \\ M_D &= 8q - 5F & M_B &= 2F - 5H_B \\ H_C &= -F & H_D &= F \\ V_C &= 4q \end{aligned}$$



$$\begin{aligned} 8q - 5F &= 2F \\ -M_B + 4V_C & \end{aligned}$$