June 17th, 2019 – Pb. 1

Pulse signal

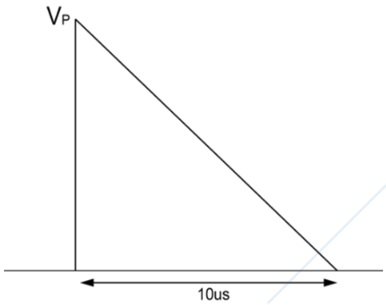
V_P variable pulse amplitude

A *sync* signal is provided for each pulse.

Preamplifier

$S_V^{1/2} = 10\ nV/Hz^{1/2}$ white noise powerdensity (unilateral)

$f_{pa} = 100\ MHz$ upper band-limit



A) Evaluate the minimum measurable amplitude $V_{P,MIN}$ without using any kind of filter. Consider now to employ a gated integrator, select its parameters for maximizing the Signal-to-Noise ratio (S/N) and evaluate the minimum measurable amplitude $V_{P,MIN}$.

Now consider the case in which a series of pulses with a random arrival time arrive at the preamplifier. The mean value of the arrival time is 50 µs. We want to measure the amplitude of each individual pulse. The measurement takes about 8 hours. Also 1/f noise component is present in the amplifier with a frequency corner of 10 kHz.

B) Discuss how much 1/f component has an impact on the final S/N and how to minimize this effect. Calculate the new final S/N.

C) Now consider the case in which the pulses arrive periodically with a period equal to 50µs. How does the answer change to the previous point?

D) The amplitude of the individual pulses changes slowly with a time scale around 1s. Assuming you are no longer interested in measuring the single pulse, how can you exploit this new information? How does the signal to noise ratio improve? Provide a quantitative evaluation.

Data framework

Signal:
V_P amplitude to measure
T_P = 10 μs time constant of the pulse signal

Noise:
$\sqrt{S_V}$ = 10 nV/Hz$^{1/2}$ white noise power density (unilateral)
$\sqrt{S_B} = \sqrt{\frac{S_V}{2}}$ (bilateral)
f_{pa} = 100 MHz upper band-limit

1/f Noise component:
f_c = 10 KHz

A) Calculation of $V_{P,MIN}$

A1) $V_{P,MIN}$ without filtering
Signal:
$S_{NF} = V_P$ because thanks to the *sync* signal we can take a specific value at a time t_m and consequently we choose the value V_P which also corresponds to the maximum value of the signal.
Noise:
Since it has no filtering, the noise is limited by the effect of the preamplifier only. Since the preamp has a pole inside it can be treated as a constant parameter filter. Let's consider the frequency domain. The transfer function of the preamplifier will be equal to $H(f) = \frac{1}{1+j2\pi f T_{pa}}$ and consequently $|H(f)|^2 = \left|\frac{1}{1+j2\pi f\frac{1}{2\pi f_{pa}}}\right|^2 = \frac{1}{1+\left(\frac{f}{f_{pa}}\right)^2}$ therefore the power spectral density at the output of the preamplifier will be $S_{out}(f) = S_B|H(f)|^2 = \frac{S_V}{2}\frac{1}{1+\left(\frac{f}{f_{pa}}\right)^2}$.

We can thus obtain the output noise from the preamplifier as:

$$n_{NF}^2 = \int_{-\infty}^{+\infty} S_{out}(f)df = \int_{-\infty}^{+\infty} \frac{S_V}{2}\frac{1}{1+\left(\frac{f}{f_{pa}}\right)^2}df = \frac{S_V}{2} f_{pa}\left[arctan\left(\frac{f}{f_{pa}}\right)\right]_{-\infty}^{+\infty} =$$

$$\frac{S_V}{2} f_{pa}\left[\frac{\pi}{2} - \left(-\frac{\pi}{2}\right)\right] = S_V \frac{\pi}{2} f_{pa}$$

$$\sqrt{n_{NF}^2} = \sqrt{S_V * \frac{\pi}{2} * f_{pa}} = \sqrt{S_V * \frac{\pi}{2} * \frac{1}{2\pi T_{pa}}} = \sqrt{S_V * \frac{1}{4T_{pa}}} = 125\ \mu V$$

We can, therefore, obtain the value of $V_{P,MIN,NF}$ by setting the signal-to-noise ratio equal to 1.

$$\left(\frac{S}{N}\right)_{NF} = \frac{V_P}{\sqrt{S_V * \frac{\pi}{2} * f_{pa}}} = 1 \;\rightarrow\; V_{P,MIN,NF} = \sqrt{S_V * \frac{\pi}{2} * f_{pa}} = 125\ \mu V$$

A2) Gated Integrator

The Gated Integrator is a switched-parameters RC network featuring a time constant of the RC network T_f much higher than T_G.

In this case, the circuit performs a true integration of the input signal. This filter is particularly useful when a relatively slow signal (so that it is constant over the integration window) must be recovered in presence of strong wide band noise, on noise with autocorrelation width much shorter than the gate duration T_G.

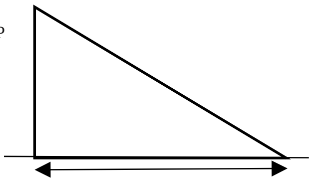
The result depends on the gate time T_G which therefore must be chosen in order to obtain the best possible result with this type of filtering.

<u>Position of the T_G gate with respect to the signal</u>: it must be chosen in order to collect the signal with the best efficiency. In this case, the signal has its maximum at its start, so the T_G gate must start when the signal starts.

<u>T_G gate duration</u>: must be chosen in order to maximize the signal-to-noise ratio taking into account the dependence of signal and noise on T_G.

Consequently, we choose to consider a GI with an amplitude $A = \frac{1}{T_G}$ and duration $0 < T_G < T_P$.

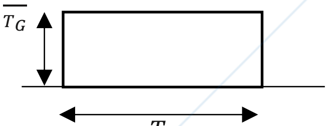
V_P



$T_P = 10\ \mu s$

Signal: $S_{IN} = V_P\left(1 - \frac{t}{T_P}\right)$ for $0 \le t \le T_P$

$\frac{1}{T_G}$



T_G

Weighting function of GI: $w_{GI} = \frac{1}{T_G} rect[0, T_G]$

that is to say $w_{GI} = \begin{cases} \frac{1}{T_G}\ for\ 0 < t < T_G \\ 0\ somewhere\ else \end{cases}$

- Output signal of the GI:

$$S_{GI} = \int_{-\infty}^{+\infty} S_{IN}(t) w_{GI}(t) dt = \int_{-\infty}^{+\infty} V_P\left(1 - \frac{t}{T_P}\right) \times \frac{1}{T_G} rect[0, T_G] dt =$$

$$\frac{V_P}{T_G}\int_0^{T_G}\left(1 - \frac{t}{T_P}\right) dt = \frac{V_P}{T_G}\left[t - \frac{t^2}{2T_P}\right]_0^{T_G} = V_P\left(1 - \frac{T_G}{2T_P}\right)$$

- Output noise of the GI:

$n_{GI}^2 = \int_{-\infty}^{+\infty} S_B k_{ww,GI}(t)dt = S_B k_{ww,GI}(0) = \frac{S_V}{2} k_{ww,GI}(0)$

$k_{ww,GI} = \int_{-\infty}^{+\infty} w_{GI}^2(t)dt = \int_{-\infty}^{\infty} \frac{1}{T_G^2} rect^2[0,T_G]dt = \int_0^{T_G} \frac{1}{T_G^2} dt = \frac{1}{T_G^2}[t]_0^{T_G} = \frac{1}{T_G}$

So $n_{GI}^2 = S_B k_{ww,GI}(0) = S_V \frac{1}{2T_G}$

- So the signal-to-noise ratio will be given by:

$$\left(\frac{S}{N}\right)_{GI} = \frac{S_{GI}}{\sqrt{n_{GI}^2}} = \frac{V_P\left(1-\frac{T_G}{2T_P}\right)}{\sqrt{S_V \frac{1}{2T_G}}} = \frac{V_P}{\sqrt{\frac{S_V}{2}}}\sqrt{T_G}\left(1-\frac{T_G}{2T_P}\right) = \frac{V_P}{\sqrt{\frac{S_V}{2}}}\sqrt{T_G}\frac{\sqrt{T_P}}{\sqrt{T_P}}\left(1-\frac{T_G}{2T_P}\right) =$$

$$\frac{V_P}{\sqrt{\frac{S_V}{2T_P}}}\frac{\sqrt{T_G}}{\sqrt{T_P}}\left(1-\frac{T_G}{2T_P}\right) = \frac{V_P}{\sqrt{\frac{S_V}{2T_P}}}\sqrt{x}\left(1-\frac{x}{2}\right) \quad with \ x = \frac{T_G}{T_P}$$

We look for the maximum of $f(x) = \left(1-\frac{x}{2}\right)\sqrt{x}$ which is easily obtained from $\frac{df(x)}{dx} = \frac{1}{2\sqrt{x}} - \frac{3\sqrt{x}}{4} = 0$ for which we obtain $x = \frac{T_G}{T_P} = \frac{2}{3}$ which corresponds to the maximum (because using the second derivative, thanks to which we can determine the intervals in which the curve is concave or convex, we obtain that

$$\frac{d^2 f(x)}{dx^2} = -\frac{1}{4}\frac{1}{\sqrt{x^3}} - \frac{3}{4}\frac{1}{2\sqrt{x}}$$

is < 0 then the concavity will be facing downwards so the point is of maximum and not of minimum) and thanks to which we obtain the best possible Signal-to-noise ratio.

So $\left(\frac{S}{N}\right)_{GI} = \frac{V_P}{\sqrt{\frac{S_V}{2T_P}}}\sqrt{\frac{2}{3}}\left(1-\frac{2}{6}\right) = \frac{V_P}{\sqrt{\frac{S_V}{2T_P}}}\sqrt{\frac{2}{3}}\left(\frac{2}{3}\right) = 1$ from which we derive

$$V_{P,MIN,GI} = \frac{\sqrt{\frac{S_V}{2T_P}}}{\frac{2}{3}\sqrt{\frac{2}{3}}} = 4{,}1\ \mu V$$

<u>Data framework</u>

Signal:

T_R = 50 µs interval between one pulse and the next

8 hours = duration of the measure

Sync signal is provided for each pulse

<u>B) 1/f Noise component of the pulses arrives at the preamplifier with a random arrival time.</u>

Without any filter, 1/f noise contribution would lead to an infinite noise. One possibility is to add an initial zero setting with a duration much higher than the GI to avoid noise doubling.

We proceed in this way as the basic sampling is intended to acquire the contributions of the low frequency components that we want to subtract from the measurement. However instant sampling (delta-like) acquires all the frequency components at low and high frequency and by subtracting them we double the noise passed above the CDS cutoff. An intuitive remedy is to modify the baseline sampling in such a way that it only acquires low frequency components.
We can do this by sampling the baseline with a low-pass weighting function $w_F(t)$ with band limit f_{Fn} which includes only the frequencies to be subtracted. In this case the baseline acquisition takes place by means of a gated integrator with a narrower filter band $f_{Fn} \ll f_s$, where f_S is the upper noise band limit. In CDS-FB the noise-doubling effect is strongly reduced with respect to the simple CDS as it occurs only in the range from the low-frequency cutoff to the GI filtering band-limit. So in cases where the GI band-limit is much smaller than the noise band-limit ($f_s \gg f_{Fn}$) the effect of noise doubling is practically negligible:
$n_{1/f}^2 = S_V f_c \ln\left(\frac{f_s}{f_i}\right)$ and $n_w^2 = \frac{S_V}{2} f_s$

Equivalent High Pass Bandwidth, in the worst case (since we have a very large pole, consequently its frequency will be very small compared to the corner frequency and therefore the contribution of the 1/f noise will be high because we are eliminating only a small part of it), is:
$f_i = \frac{1}{2\pi T_i} = 5{,}5\ \mu Hz$ with $T_i = 8h = 8 \times 60^2\ s$.
White noise is the same of point A) (we use the same GI):
$\sqrt{n_w^2} = \sqrt{\frac{S_V}{2T_G}} = \sqrt{S_V \frac{3}{4T_P}} = 2{,}7\ \mu V$ while the 1 / f noise will also be obtained thanks to the use of the low pass component equal to: $f_s = \frac{1}{2T_G} = \frac{3}{4T_P} = 75\ kHz$, so

$$\sqrt{n_{1/f}^2} = \sqrt{S_V f_c \ln\left(\frac{f_s}{f_i}\right)} = \sqrt{S_V f_c \ln\left(\frac{\frac{1}{2T_G}}{\frac{1}{2\pi T_i}}\right)} = 4{,}8\ \mu V.$$

However, we can see that the 1/f noise component is greater than that of white noise and consequently a different type of high pass filtering is required. We must take into account that, in this case, since the repetition frequency of the pulses is random, it will not be possible to use constant parameters filters. So the only option available to us is the Baseline Restorer.

The BLR can thus be employed for avoiding the limitations and drawbacks of the constant-parameter CR differentiator because the effect of the CR on the signal is disadvantageous:
- it decreases the signal amplitude by cutting off the low frequencies of the signal, hence f_i must be kept low ($f_i << f_s$ of the pulse) in order to limit the signal loss. However, this limits also the reduction of 1/f noise;

- it generates slow tails after the pulses, which down shifts the baseline and thus it can cause an error in the amplitude measurement of a following pulse;
- With random-repetition pulses the pulses occur randomly in time. Hence the random superposition of tails produces a randomly fluctuating baseline shift. The resulting amplitude error is random: in this case the effect is equivalent to that of an additional noise source.

To solve this trade off problem of the constant-parameter CR differentiator we introduce now a switched-parameter filter. The idea is to work in a different way on noise and signal using the possibility to switch the parameters. The circuit is quite the same of the standard constant-parameter filter, but we introduced a switch to stop the differentiator effect when the signal is present. This new circuit is called BASELINE RESTORER.
In the BLR the differentiator action is not applied to the pulses, but only to the noise.
BLR principle is alike filtered zero-setting, but with a basic advantage:
the high-frequency band limit f_i (high-pass) can be very high providing a very short T_B, which can be achieved with a fast electronically-controlled switch.
The BLR does not affect the pulse amplitude and establishes for the 1/f noise a high pass cutoff at frequency $f_B = \frac{1}{2\pi T_B}$.
This cutoff frequency can be much lower than that allowed for a constant-parameter CR filter. However, it can be shown that it must not be very short, in order to avoid the noise enhancement effect typical of a simple CDS.
So we can say that avoiding any enhancement of the white noise requires a fairly slow BLR differentiation, i.e. a quite long T_B

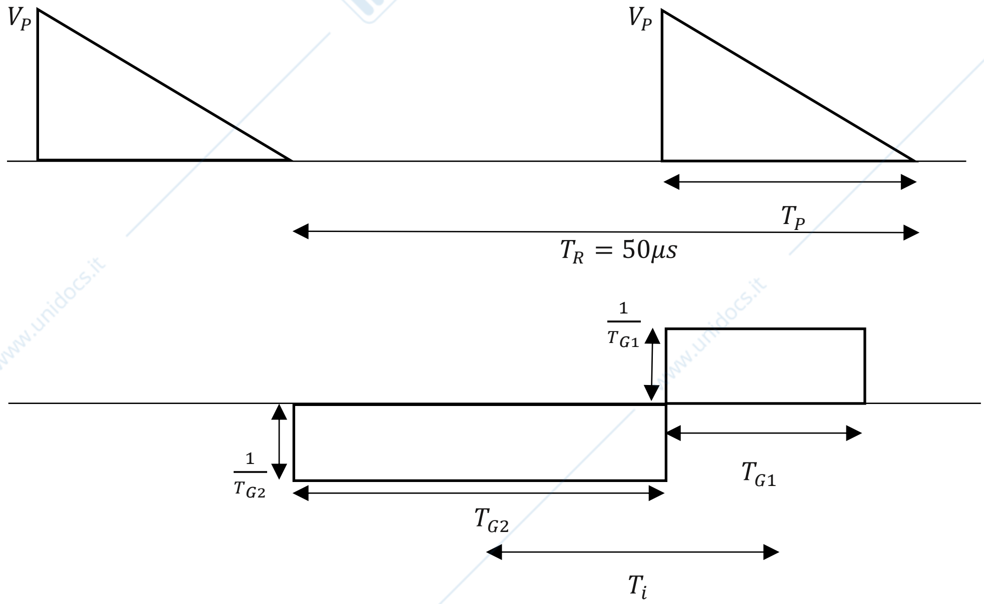
$$T_B \geq 5T_P$$

This approach is suitable also for 1/f noise filtering, notwithstanding that making T_F longer than T_P shifts down the BLR cut-off frequency, hence reduces the attenuation of 1/f noise. This is counterbalanced by the fact that the enhancement of 1/f noise at frequencies above the cutoff is limited by the low-pass filtering in the baseline subtraction, whereas with short T_B it is remarkable.

C) 1/f Noise component of the pulses arrives at the preamplifier with a periodic arrival time.

Now we know the distance between the pulses and consequently we can use a CDF in order to avoid signal loss. This type of filtering is easier to carry out as a GI is already used. We will then proceed by measuring the amplitude of the triangular input signal through two GI acquisitions. More precisely: a) the acquisition already considered that it begins when the impulse begins and b) another (baseline acquisition) which precedes the signal by an interval T_i greater than T_G because we want to have an integration window for noise that is larger than the window on the signal to limit its impact.

Compared to the previous case in which the BLR has no signal loss since a switch has been introduced in the same circuit as the constant-parameter CR differentiator a switch to stop the differentiator effect when the signal is present, the CDF avoids signal losses thanks to the fact that only the first integration window (T_{G1}) acts on it, that is the GI treated in point 1, while the second integration window has no effect on the signal as its only task is to act on the noise.



The values of this integration window are easy to calculate as we know that the pulses are at a distance of 50 µs from each other while the duration of a stack is 10 µs. Consequently we take a second integration window of duration $T_{G2} = 50\ \mu s - T_P = 40\ \mu s$ as it will have to be $T_{G2} > T_{G1}$ to limit the impact of the white noise and ensure that it does not double.

We therefore know that T_{G1} is $T_{G1} = T_G = \frac{2}{3} T_P = 6{,}66\ \mu s$ so we can derive $T_{i,CDF} = \frac{T_{G1}+T_{G2}}{2} = 23{,}34\ \mu s$ and $f_{i,CDF} = \frac{1}{2\pi T_{i,CDF}} = 6{,}8\ kHz.$

It was easy to carry out the correct synchronization of the two acquisitions since the pulses are periodic with a known repetition frequency equal to $f_r = \frac{1}{T_R} = 20kHz$ and an auxiliary *sync* signal is also available which indicates the start of each pulse. Furthermore, we also had to consider that the area of the two integration windows must be equal.

$$\sqrt{n^2_{1/f,CDF}} = \sqrt{S_V f_c\, ln\left(\frac{f_s}{f_{i,CDF}}\right)} = \sqrt{S_V f_c\, ln\left(\frac{\frac{1}{2T_G}}{\frac{1}{2\pi T_{i,CDF}}}\right)} = 1{,}54\ \mu V.$$

As for the white noise we have to take into consideration the effect of both integration windows:

$\sqrt{n^2_{w,CDF}} = \sqrt{\frac{S_V}{2T_{G1}} + \frac{S_V}{2T_{G2}}} = 2{,}97\ \mu V$ from which we derive

$\sqrt{n^2_{TOT,CDF}} = \sqrt{n^2_{1/f,CDF} + n^2_{w,CDF}} = 3{,}3\ \mu V.$

The 1/f noise contribution is acceptable as it is less than the noise contribution of the free spectrum of 1/f components. In fact, we can note that the 1/f noise is no longer the dominant contribution as it has been reduced thanks to the high-pass filtering. We can thus calculate the $V_{P,MIN,CDF}$:

$$\left(\frac{S}{N}\right)_{CDF} = \frac{V_P\left(1-\frac{T_G}{2T_P}\right)}{\sqrt{n^2_{TOT,CDF}}} = 1 \rightarrow V_{P,MIN,CDF} = \frac{\sqrt{n^2_{TOT,CDF}}}{\left(1-\frac{T_G}{2T_P}\right)} = 4{,}95\ \mu V.$$

D) Repetitive Signals

In this case, to have an improvement in the signal-to-noise ratio, we can use the Boxcae Integrator or Ratemeter Integrator. In both filters the switch acts as gate on the input source, but in the RI it doesn't affect the RC integrator. RI does not have any hold state because there is a buffer to decouple, so the average is done with constant parameters for a given time. On the contrary in the BI the switch acts also on the RC integrator: for this reason, the time constant of the integrator switches between $T_F = RC$, during the sampling phase (when the switch is close) and infinite in the hold phase. The sample average is done on a given number of samples, defined by T_F/T_G. Also in the Boxcar it is independent of the T_R time as opposed to the Ratemter.

We choose to use a Boxcar circuit to calculate the number of distant pulses $T_R = 50\ \mu s$ between them in a time of 1 s. Operating with a fairly long temp constant T_F, the filter performs on the sequence:

1) GI of the single pulses
2) weighted sum of the GI measurements of the pulses of the sequence.

To this aim, a basic hypothesis must hold, that is $T_F = RC \gg T_G$. In this scenario, we can derive the weighting function of this filter, starting from a key difference with a GI: in a classical GI, the capacitor is always reset between two consecutive acquisitions. In other words, the amount of charge on the capacitor of a GI at the beginning of the measure, that is when a pulse arrives, is zero. On the contrary, when the switch of a BI is open the capacitor can't discharge and there is a buffer that copies the value at the output. As a result, when the switch is closed (time interval TG) not only the incoming signal is acquired, but also the capacitor can discharge, meaning that the previously stored value is reduced by a factor r equal to: $r = e^{-\frac{T_G}{T_F}}$.

The weight w_i given by the i-th pulse preceding the measurement acquisition instant is: $w_i = r^i$ with value of r depending on the value of the time constant T_F of the filter, but in any case $r < 1$ (to avoid averaging on pulses with different amplitude).

In one second there is a number of pulses equal to $N = \frac{1s}{50\ \mu s} = 20000$.
Furthermore, T_F must be chosen so that: $r^N \leq 10^{-2}$ (since the weight we give to a sample after 20000 acquisitions I want it to be equal to 1% of the weight we had at the beginning) that is $e^{-\frac{T_G}{T_F}} \leq \frac{1}{100}$

$-N\frac{T_G}{T_F} \leq ln\left(\frac{1}{100}\right) \rightarrow N\frac{T_G}{T_F} \geq ln(100) = 4{,}6 \rightarrow T_F \leq \frac{NT_G}{4{,}6} = 29\ ms.$

NB: being $T_F \gg T_G$ it is confirmed that on a single impulse the Boxcar gives a filtering equal to that of a GI. We can therefore approximate $e^{-\frac{T_G}{T_F}} \approx 1 - \frac{T_G}{T_F}$.

- Through the use of the Boxcar the signal will be increased by a factor:

$S_{BC} = S_{CDF}(1 + r + r^2 + r^3 + \cdots) = \sum_{k=0}^{20000} S_{CDF} r^k = S_{CDF}\frac{1}{1-r}$

- While the noise is equal to:

$n_{BC}^2 = n_{CDF}^2(1 + r^2 + r^4 + r^6 + \cdots) = \sum_{k=0}^{20000} n_{CDF}^2 r^{2k} = n_{CDF}^2\frac{1}{1-r^2}$

Consequently, the signal to noise ratio of the Boxcar will be equal to:

$$\left(\frac{S}{N}\right)_{BC} = \frac{S_{CDF}\frac{1}{1-r}}{\sqrt{n_{CDF}^2\frac{1}{1-r^2}}} = \left(\frac{S}{N}\right)_{CDF}\sqrt{\frac{1-r^2}{(1-r)^2}} = \left(\frac{S}{N}\right)_{CDF}\sqrt{\frac{(1-r)(1+r)}{(1-r)^2}} = \left(\frac{S}{N}\right)_{CDF}\sqrt{\frac{1+r}{1-r}}$$

If you use r just below 1, the improvement is considerable: $1 - r \ll 1$ therefore $\left(\frac{S}{N}\right)_{BC} \gg \left(\frac{S}{N}\right)_{CDF}$.

Noting, then, that proceeding in this way $1 + r \approx 2$ so

$\left(\frac{S}{N}\right)_{BC} \approx \left(\frac{S}{N}\right)_{CDF}\sqrt{\frac{2}{1-r}}$.

Also remembering that $r = e^{-\frac{T_G}{T_F}} = 1 - \frac{T_G}{T_F}$ we can rewrite the sensitivity as:

$$\left(\frac{S}{N}\right)_{BC} = \left(\frac{S}{N}\right)_{CDF}\sqrt{\frac{2T_F}{T_G}} = \left(\frac{S}{N}\right)_{CDF} 93{,}27 = \frac{V_P\left(1-\frac{T_G}{2T_P}\right)}{\frac{\sqrt{n_{TOT,CDF}^2}}{93{,}27}}$$

$$\sqrt{n_{TOT,BC}^2} = \frac{\sqrt{n_{TOT,CDF}^2}}{93{,}27} = 35{,}38\ nV$$

In this case, the sensitivity of the Boxcar is better than that of the CDF as with the Boxcar the measurement exploits the redundancy of information carried by the various pulses in the sequence, carrying out a weighted average. It is logical and intuitive that by exploiting more information than those carried by a single impulse, the result is better.