UNIVERSITÀ CATTOLICA del Sacro Cuore

Derivative Securities Pricing

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Student Name	
Student ID #	

Question #1

Consider an option on a non-dividend-paying stock when the stock price is €10, the exercise price is €9, the risk-free interest rate is 5%, the volatility is 20% per annum, and the time to maturity is 6 months.

a. What is the price of the option if it is a European call?
b. What is the price of the option if it is an American call?
c. What is the price of the option if it is a European put?
d. Verify that put-call parity holds.

Question #2

Consider again the option described in the previous Question and assume now that the stock price is expected to pay a dividend equal to €0.50 in 2 months.

a. What is the price of the option if it is a European call?
b. What is the price of the option if it is a European put?
c. If the option is an American call, do you think that it might be optimal to exercise the option on the dividend date?

Question #3

a. Describe the Markov process.
b. Describe the Wiener process, its properties and applications.
c. Describe the Ito process and Ito's lemma.
d. Apply Ito's lemma to forward contract.

Question #4

A fund manager has a well-diversified portfolio that mirrors the performance of the S&P 500 and is worth $360 million. The value of the S&P 500 is 1,200, and the portfolio manager would like to buy insurance against a reduction of more than 5% in the value of the portfolio over the next six months. The risk-free interest rate is 6% per annum. The dividend yield on both the portfolio and the S&P 500 is 3%, and the volatility of the index is 30% per annum.

a. If the fund manager buys traded European put options, how much would the insurance cost?
b. Explain carefully alternative strategies open to the fund manager involving traded European call options, and show that they lead to the same result.
c. If the fund manager decides to provide insurance by keeping part of the portfolio in risk-free securities, what should the initial position be?
d. If the fund manager decides to provide insurance by using nine-month index futures, what should the initial position be? Each futures contract delivers 250 times the index.