Question 2

The illuminance E_V of a lamp is measured through a photodiode with the input/output relationship expressed as:

$$I_{\text{out}} = S \cdot E_{\text{V}} + I_{\text{OFF}}$$

S = 2 nA/lux, expanded uncertainty at the 95% confidence level $U(S)$ = 50 pA/lux; I_{OFF} uniformly distributed in the range (-100 ÷ 100) nA.

The output current of the photodiode has been measured through a multiple reading method, obtaining 10 readings characterized by an average value of 6.0 μA and an experimental standard deviation of 100 nA. The uncertainty specification of the ammeter, which is provided according to the deterministic approach, is δI_{out} = 100 nA. The distance between the lamp and the photodiode is d = 2.00 m with a standard uncertainty $u(d)$ = 8 mm.

Estimate value and standard uncertainty of the luminous intensity I_V of the lamp.

I/O relationship sensor $I_{OUT} = S \cdot E_V + I_{OFF}$

$S = 2\ mA/lx \Rightarrow U(S) = 50\ pA/lx$ at 95% conf. level

I_{OFF} uniformly distrib. $(-100 \div 100)\ mA$

10 multiple reading $I_{OUT} \Rightarrow \bar{I}_{OUT} = 6.0\ \mu A$, $s(I_{OUT}) = 100\ mA$

ammeter $\Rightarrow \delta I_{OUT} = 100\ mA$ (det. approach)

$d = 2.00\ m$, $u(d) = 8\ mm$

I_V, $u(I_V)$?

measuring model

$$I_V = \frac{E_V}{d^2} \Rightarrow E_V = \frac{I_{OUT} - I_{OFF}}{S}$$

$$I_V = \frac{I_{OUT} - I_{OFF}}{S} d^2$$

nominal value of $I_V \Rightarrow I_{V_0} = \dfrac{\bar{I}_{OUT} - 0}{S_0} d^2 = 12\,000\ lx$

no correlation is expected between the quantities

standard uncertainties

$u(d) = 8\ mm$

$$u(S) = \frac{U(S)}{K_p}, \quad K_p \approx 2 \Rightarrow u(S) = 25 \cdot 10^{-12}\ \frac{A}{lx}$$

$$u_A(\bar{I}_{OUT}) = \frac{s(I_{OUT})}{\sqrt{N}} = 3.16 \cdot 10^{-8}\ A$$

$\delta I_{OUT} = 100\ mA \Rightarrow$ assuming uniform distribution of det. app $\to$ prob. app. $\Rightarrow u_B(I_{OUT}) = \dfrac{\delta I_{OUT}}{\sqrt{3}} = 5.7 \cdot 10^{-8}\ A$

$$u(I_{OUT}) = \sqrt{u_A^2(\bar{I}_{OUT}) + u_B^2(I_{OUT})} = 6.51 \cdot 10^{-8}\ A$$

$\delta I_{OFF} = 100\ mA \Rightarrow$ prob. approach $\Rightarrow u(I_{OFF}) = 5.7 \cdot 10^{-8}\ A$

sensitivity coeff.

$$\frac{\partial I_V}{\partial I_{OUT}} = \frac{d_0^2}{S_0} \triangleq c_1$$

$$\frac{\partial I_V}{\partial I_{OFF}} = -\frac{d_0^2}{S_0} \triangleq c_2$$

$$\frac{\partial I_V}{\partial S} = \frac{I_{OFF_0} - \bar{I}_{OUT}}{S^2} d_0^2 \triangleq c_3$$

$$\frac{\partial I_V}{\partial d} = 2d_0 \frac{\bar{I}_{OUT} - I_{OFF_0}}{S_0} \triangleq c_4$$

$$u^2(I_V) = c_1^2 u^2(I_{OUT}) + c_2^2 u^2(I_{OFF}) + c_3^2 u^2(S) + c_4^2 u^2(d) =$$

$$\cong 16952 + 12996 + 22500 + 9216 \simeq 248\ cd$$

$\Rightarrow I_{V_0} = 12.00\ Kcd \quad u(I_V) \simeq 0.25\ Kcd$