THERMAL MACHINES - FORMULARY

– First law of Thermodynamics, closed system:
$$Q \pm L = \Delta U + \Delta E_c + \Delta E_g + \Delta E_\omega$$
$$= U_f - U_i + \frac{1}{2}\left(c_f^2 - c_i^2\right) + g(z_f - z_i) - \frac{1}{2}\omega^2\left(r_f^2 - r_i^2\right)$$

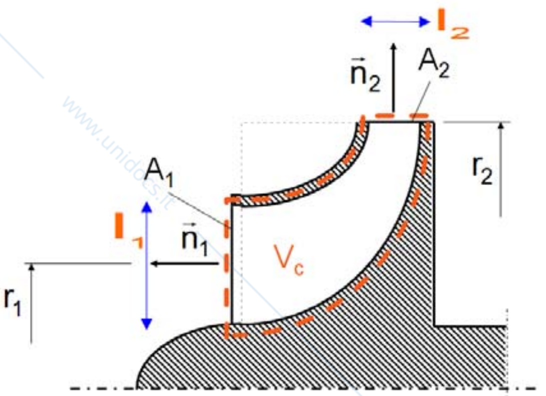
– Conservation of mechanical energy, closed system:
$$\pm L = -\int_i^f p \cdot dv + \frac{1}{2}\left(c_f^2 - c_i^2\right) + g(z_f - z_i) - \frac{1}{2}\omega^2\left(r_f^2 - r_i^2\right) + L_{w,i-f}$$

– First law of Thermodynamics, open system:
$$Q \pm L_i = \Delta h + \Delta E_c + \Delta E_g + \Delta E_\omega$$

– Conservation of mechanical energy, open system:
$$\pm L_i = \int_1^2 v \cdot dp + \Delta E_c + \Delta E_g + \Delta E_\omega + L_w$$

– Second law of Thermodynamics: $T \cdot dS = dQ + dL_w$

– Combined equations: $T \cdot dS = dU + p \cdot dv$
$$T \cdot dS = dh - v \cdot dp$$



– Example of mass flow rate calculation:
$$\dot{m} = \rho_2 c_{n2} A_2 = \rho_2 c_{r2} A_2\,;\ \dot{m} = \rho_1 c_{n1} A_1 = \rho_1 c_{a1} A_1$$
$$A_1 = \xi_1 2\pi r_1 l_1\,;\ A_2 = \xi_2 2\pi r_2 l_2$$

– Euler equation: $\pm L_i = \pm \frac{\dot{\mathcal{L}}_i}{\dot{m}} = u_2 c_{u,2} - u_1 c_{u,1}$

– Speed of sound: $c_s = \sqrt{\left(\frac{\partial p}{\partial \rho}\right)_{s=\text{const}}} = \sqrt{k\frac{p}{\rho}}$

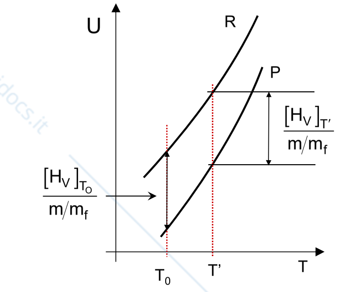
– Stagnation enthalpy: $h^0 = h + \frac{c^2}{2}$

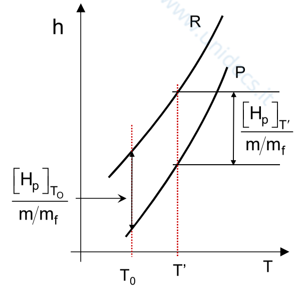
– For an ideal gas: $T^0 = T + \frac{c^2}{2 \cdot c_p}$
$$p^0 = p\left(\frac{T^0}{T}\right)^{\frac{k}{k-1}}$$

– Nozzle critical pressure ratio (which produces chocking): $\left(\frac{p}{p_1^0}\right)_c = \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}}$

– Values in the nozzle throat section under chocked conditions:
$$c_c = \sqrt{k\frac{p_c}{\rho_c}} = \sqrt{2\frac{k}{k+1}\frac{p_1^0}{\rho_1^0}}$$
$$\rho_c = \rho_1^o\left(\frac{p_c}{p_1^o}\right)^{\frac{1}{k}} = \rho_1^o\left(\frac{2}{k+1}\right)^{\frac{1}{k-1}}$$
$$T_c = T_1^o\left(\frac{p_c}{p_1^o}\right)^{\frac{k-1}{k}} = \frac{2}{k+1}T_1^o \text{ (ideal gas)}$$

HEATING VALUES:





$$[H_V]_{T_0} = -\frac{(\Delta\mathcal{U})_{V,T_0}}{m_f}\ ;\ [H_p]_{T_0} = -\frac{(\Delta\mathcal{H})_{p,T_0}}{m_f}$$

– Constant-volume combustion:
$$\frac{H_{L,v}}{m/m_f} = c_v' \cdot (T_f - T_i) - Q \mp L \quad \left(+D(T_f - 1850)^2\right)$$

– Constant-pressure combustion:
$$\frac{H_{L,p}}{m/m_f} = c_p' \cdot (T_f - T_i) - Q \quad \left(+D(T_f - 1850)^2\right)$$

– Steady-state combustion in open systems:
$$\frac{H_{L,p}}{\dot{m}/\dot{m}_f} = c_p' \cdot (T_f - T_i) - Q \mp L_i \quad \left(+D(T_f - 1850)^2\right)$$

In the case of incomplete combustion the left hand side might be multiplied by a lower-than-unity coefficient.

- Velocity coefficients for turbine blades:
  $$\varphi = c_1 / c_{1,is} \;;\quad \psi = w_2 / w_{2',is}$$
- Total-to-static efficiency of turbines: $\eta_\theta = \dfrac{h_0^O - h_2^O}{h_0^O - h_{2,is}}$
- Total-to-total efficiency of turbines:
  $$\eta_\Theta = \frac{h_0^O - h_2^O}{h_0^O - h_{2,is} - c_2^2/2}$$
- Total-to-total polytropic efficiency of a turbine stage (perfect gas):
  $$\eta_{pol} = \frac{\frac{k}{k-1}RT_0\left[1-\left(\frac{p_2}{p_0}\right)^{\frac{m-1}{m}}\right]+\frac{c_0^2-c_2^2}{2}}{\frac{m}{m-1}RT_0\left[1-\left(\frac{p_2}{p_0}\right)^{\frac{m-1}{m}}\right]+\frac{c_0^2-c_2^2}{2}}$$
- Degree of reaction: $\chi = \dfrac{\Delta h_{is,rotor}}{\Delta h_{is,rotor}+\Delta h_{is,stator}}$
  $$\chi = \frac{w_2^2/\psi^2 - w_1^2 - u_2^2 + u_1^2}{w_2^2/\psi^2 - w_1^2 - u_2^2 + u_1^2 + c_1^2/\varphi^2 - c_0^2}$$
- Kinematic degree of reaction: $R = \dfrac{\Delta h_{rotor}}{\Delta h^O_{stage}}$
  $$R = \frac{w_2^2 - w_1^2 - u_2^2 + u_1^2}{w_2^2 - w_1^2 - u_2^2 + u_1^2 + c_1^2 - c_2^2}$$

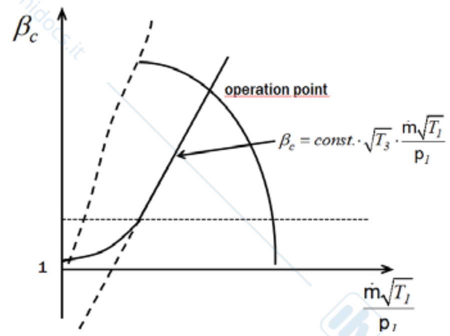
- Multistage compressor:
  $$L_{c,is} = c_p(T_{2,is} - T_1) = \frac{k}{k-1}RT_1\left[\left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}} - 1\right]$$
  $$L_c = c_p(T_2 - T_1) = \frac{k}{k-1}RT_1\left[\left(\frac{p_2}{p_1}\right)^{\frac{m-1}{m}} - 1\right] = \frac{1}{\eta_c}L_{c,is}$$
  $$L_{c,pol} = \int_1^2 v\cdot dp = \frac{m}{m-1}RT_1\left[\left(\frac{p_2}{p_1}\right)^{\frac{m-1}{m}} - 1\right] = L_c - L_w$$
  $$\eta_c = \frac{L_{c,is}}{L_c} \;;\quad \eta_{y,c} = \frac{L_{c,pol}}{L_c} = \frac{m}{m-1}\cdot\frac{k-1}{k}$$

- Multistage gas turbine:
  $$L_{t,is} = c'_p(T_3 - T_{4,is}) = \frac{k'}{k'-1}R'T_3\left[1 - \frac{1}{\left(\frac{p_3}{p_4}\right)^{\frac{k'-1}{k'}}}\right]$$
  $$L_t = c'_p(T_3 - T_4) = \frac{k'}{k'-1}R'T_3\left[1 - \frac{1}{\left(\frac{p_3}{p_4}\right)^{\frac{m'-1}{m'}}}\right] = \eta_t \cdot L_{t,is}$$
  $$L_{t,pol} = \int_3^4 v\cdot dp = \frac{m'}{m'-1}R'T_3\left[1 - \frac{1}{\left(\frac{p_3}{p_4}\right)^{\frac{m'-1}{m'}}}\right] = L_t + L_w$$
  $$\eta_t = \frac{L_t}{L_{t,is}} \;;\quad \eta_{y,t} = \frac{L_t}{L_{t,pol}} = \frac{k'}{k'-1}\cdot\frac{m'-1}{m'}$$

Turbine characteristic:

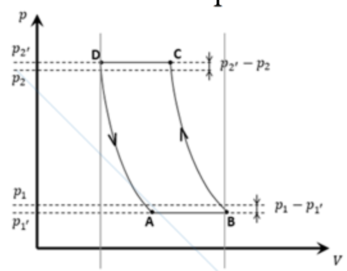
$$\frac{\dot m\sqrt{p_O^0 \cdot v_O^0}}{p_O^0} = K \qquad \text{(chocked)}$$

The gas turbine as a compressor 'user'



- Combustion equation applied to TG plants:
  $$\eta_b \cdot H_{L,p} = (1+\alpha)c'_p(T_3 - T_2)$$
- Overall efficiency of a power plant:
  $$\eta_{plant} = \frac{\dot{\mathcal{L}}_{plant}}{\dot m_f H_{Lp}} = \eta_b \cdot \frac{\dot{\mathcal{L}}_{plant}}{\dot Q_I}$$
  For turbogas plants:
  $$\eta_{plant} = \eta_b \cdot \frac{\alpha \cdot L_{plant}}{(1+\alpha)\cdot c'_p(T_3 - T_2)}$$
  $$\left(L_{plant} = \frac{\dot{\mathcal{L}}_{plant}}{\dot m_a}\right)$$
- Overall efficiency of a power plant with cogeneration: $\eta_{plant} = \dfrac{\dot{\mathcal{L}}_{plant}}{\dot m_f H_{Lp} - \dot Q_u / \eta_{b,REF.}}$

Volumetric compressors:



$$\mathcal{L}_c = \frac{m^*}{m^*-1}p_B V_B\left[\beta_i^{\frac{m^*-1}{m^*}} - 1\right] - \frac{m^{*\prime}}{m^{*\prime}-1}p_D V_D\left[1 - 1/\beta_i^{\frac{m^{*\prime}-1}{m^{*\prime}}}\right]$$

$L_i = \dfrac{\mathcal{L}_c}{M} \approx C_p \cdot (T_2 - T_1)$

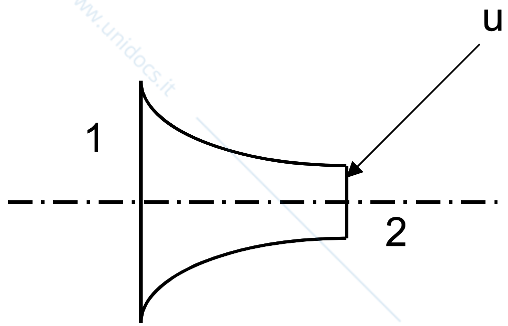
$M = \lambda_v \rho_1 V_0 = M_C - M_D - M_{leak,CD}$

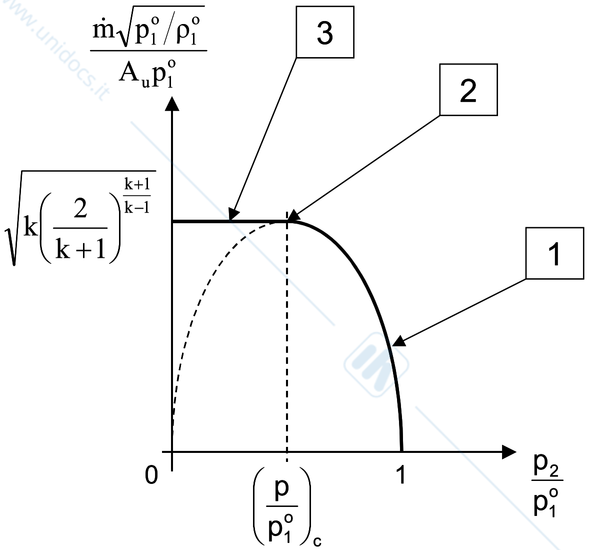
$\lambda_V = \eta_\varphi(1-\delta_1)\dfrac{(\eta_\tau V_B - V_A)}{V_0}$

Delivery temperature (general): $T_2 = T_1 + \frac{L_i + |Q|}{c_p}$

(roots): $\frac{T_2}{T_1} = 1 + \frac{k-1}{k}\cdot\frac{1}{\lambda_V}(\beta - 1)$

SIMPLE CONVERGENT NOZZLE



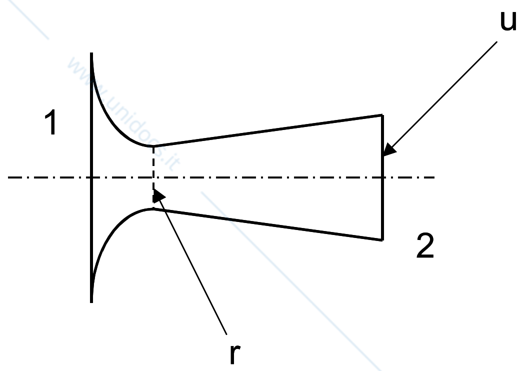


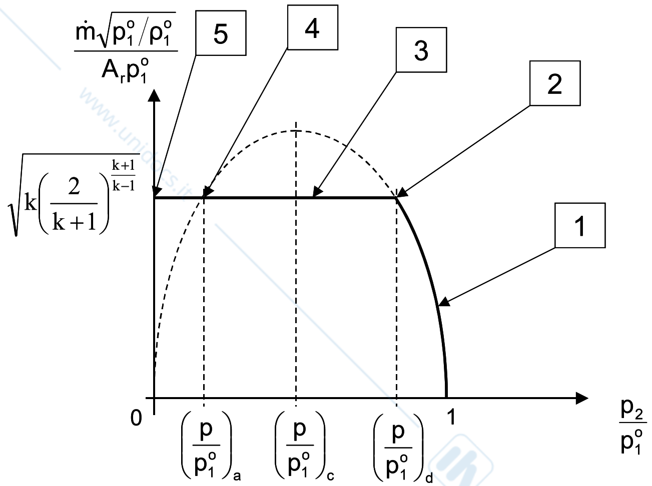
$$\dot{m} = A_{out}\frac{p_1^o}{\sqrt{p_1^o/\rho_1^o}}\sqrt{2\frac{k}{k-1}\left[\left(\frac{p}{p_1^o}\right)^{\frac{2}{k}} - \left(\frac{p}{p_1^o}\right)^{\frac{k+1}{k}}\right]} \quad (1)$$

$$\dot{m} = A_{out}\frac{p_1^o}{\sqrt{p_1^o/\rho_1^o}}\sqrt{k\left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}}$$

$$A_{out} = A_u \quad (2,3)$$

CONVERGENT-DIVERGENT (DE LAVAL) NOZZLE





$$\dot{m} = A_{out}\frac{p_1^o}{\sqrt{p_1^o/\rho_1^o}}\sqrt{2\frac{k}{k-1}\left[\left(\frac{p}{p_1^o}\right)^{\frac{2}{k}} - \left(\frac{p}{p_1^o}\right)^{\frac{k+1}{k}}\right]} \quad (1)$$

$$\dot{m} = A_{th}\frac{p_1^o}{\sqrt{p_1^o/\rho_1^o}}\sqrt{k\left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}}$$

$$= A_{out}\frac{p_1^o}{\sqrt{p_1^o/\rho_1^o}}\sqrt{2\frac{k}{k-1}\left[\left(\frac{p}{p_1^o}\right)_a^{\frac{2}{k}} - \left(\frac{p}{p_1^o}\right)_a^{\frac{k+1}{k}}\right]} \quad (2\text{-}5)$$

$A_{out} = A_u$

$A_{th} = A_r$