# THERMAL MACHINES - FORMULARY

- First law of Thermodynamics, closed system:
$$Q \pm L = \Delta U + \Delta E_c + \Delta E_g + \Delta E_\omega$$
$$= U_f - U_i + \frac{1}{2}\left(c_f^2 - c_i^2\right) + g\left(z_f - z_i\right) - \frac{1}{2}\omega^2\left(r_f^2 - r_i^2\right)$$
- Conservation of mechanical energy, closed system:
$$\pm L = -\int_i^f p \cdot dv + \frac{1}{2}\left(c_f^2 - c_i^2\right) + g\left(z_f - z_i\right) - \frac{1}{2}\omega^2\left(r_f^2 - r_i^2\right) + L_{w,i-f}$$
- First law of Thermodynamics, open system:
$$Q \pm L_i = \Delta h + \Delta E_c + \Delta E_g + \Delta E_\omega$$
- Conservation of mechanical energy, open system:
$$\pm L_i = \int_1^2 v \cdot dp + \Delta E_c + \Delta E_g + \Delta E_\omega + L_w$$
- Second law of Thermodynamics: $T \cdot dS = dQ + dL_w$
- Combined equations: $T \cdot dS = dU + p \cdot dv$
$$T \cdot dS = dh - v \cdot dp$$

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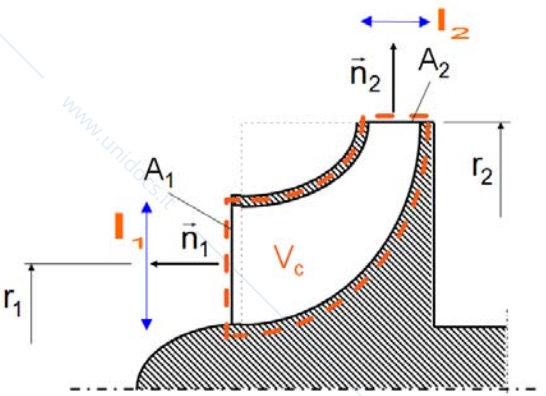


- Example of mass flow rate calculation:
$$\dot{m} = \rho_2 c_{n2} A_2 = \rho_2 c_{r2} A_2\,;\ \dot{m} = \rho_1 c_{n1} A_1 = \rho_1 c_{a1} A_1$$
$$A_1 = \xi_1 2\pi r_1 l_1\,;\ A_2 = \xi_2 2\pi r_2 l_2$$

- Euler equation: $\pm L_i = \pm \frac{\dot{\mathfrak{L}}_i}{\dot{m}} = u_2 c_{u,2} - u_1 c_{u,1}$

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- Speed of sound: $c_s = \sqrt{\left(\frac{\partial p}{\partial \rho}\right)_{s=const}} = \sqrt{k\frac{p}{\rho}}$
- Stagnation enthalpy: $h^0 = h + \frac{c^2}{2}$
- For an ideal gas: $T^0 = T + \frac{c^2}{2 \cdot c_p}$
$$p^0 = p\left(\frac{T^0}{T}\right)^{\frac{k}{k-1}}$$
- Nozzle critical pressure ratio (which produces chocking): $\left(\frac{p}{p_1^0}\right)_c = \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}}$
- Values in the nozzle throat section under chocked conditions:
$$c_c = \sqrt{k\frac{p_c}{\rho_c}} = \sqrt{2\frac{k}{k+1}\frac{p_1^0}{\rho_1^0}}$$
$$\rho_c = \rho_1{}^o\left(\frac{p_c}{p_1{}^o}\right)^{\frac{1}{k}} = \rho_1{}^o\left(\frac{2}{k+1}\right)^{\frac{1}{k-1}}$$
$$T_c = T_1^o\left(\frac{p_c}{p_1^o}\right)^{\frac{k-1}{k}} = \frac{2}{k+1}T_1^o \text{ (ideal gas)}$$

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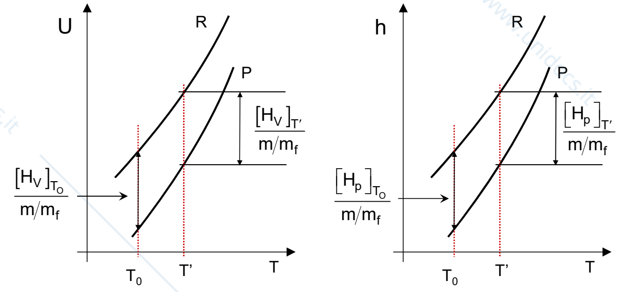
HEATING VALUES:




$$\left[H_V\right]_{T_0} = -\frac{(\Delta\mathcal{U})_{V,T_0}}{m_f}\,;\ \left[H_p\right]_{T_0} = -\frac{(\Delta\mathcal{H})_{p,T_0}}{m_f}$$

- Constant-volume combustion:
$$\frac{H_{L,v}}{m/m_f} = c_v{}' \cdot \left(T_f - T_i\right) - Q \mp L \quad \left(+D\left(T_f - 1850\right)^2\right)$$
- Constant-pressure combustion:
$$\frac{H_{L,p}}{m/m_f} = c_p{}' \cdot \left(T_f - T_i\right) - Q \quad \left(+D\left(T_f - 1850\right)^2\right)$$
- Steady-state combustion in open systems:
$$\frac{H_{L,p}}{\dot{m}/\dot{m}_f} = c_p{}' \cdot \left(T_f - T_i\right) - Q \mp L_i \quad \left(+D\left(T_f - 1850\right)^2\right)$$

In the case of incomplete combustion the left hand side might be multiplied by a lower-than-unity coefficient.

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- Velocity coefficients for turbine blades:

$$\varphi = {c_1}/{c_{1,is}} \; ; \quad \psi = {w_2}/{w_{2',is}}$$

- Total-to-static efficiency of turbines: $\eta_\theta = \dfrac{h_0^O - h_2^O}{h_0^O - h_{2,is}}$

- Total-to-total efficiency of turbines:

$$\eta_\Theta = \frac{h_0^O - h_2^O}{h_0^O - h_{2,is} - {c_2^2}/{2}}$$

- Total-to-total polytropic efficiency of a turbine stage (perfect gas): $\eta_{pol} = \dfrac{\frac{k}{k-1}RT_0\left[1-\left(\frac{p_2}{p_0}\right)^{\frac{m-1}{m}}\right]+\frac{c_0^2-c_2^2}{2}}{\frac{m}{m-1}RT_0\left[1-\left(\frac{p_2}{p_0}\right)^{\frac{m-1}{m}}\right]+\frac{c_0^2-c_2^2}{2}}$

- Degree of reaction: $\chi = \dfrac{\Delta h_{is,rotor}}{\Delta h_{is,rotor}+\Delta h_{is,stator}}$

$$\chi = \frac{w_2^2/\psi^2 - w_1^2 - u_2^2 + u_1^2}{w_2^2/\psi^2 - w_1^2 - u_2^2 + u_1^2 + c_1^2/\varphi^2 - c_0^2}$$

- Kinematic degree of reaction: $R = \dfrac{\Delta h_{rotor}}{\Delta h^O_{stage}}$

$$R = \frac{w_2^2 - w_1^2 - u_2^2 + u_1^2}{w_2^2 - w_1^2 - u_2^2 + u_1^2 + c_1^2 - c_2^2}$$

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- Multistage compressor:

$$L_{c,is} = c_p\left(T_{2,is} - T_1\right) = \frac{k}{k-1}RT_1\left[\left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}} - 1\right]$$

$$L_c = c_p\left(T_2 - T_1\right) = \frac{k}{k-1}RT_1\left[\left(\frac{p_2}{p_1}\right)^{\frac{m-1}{m}} - 1\right] = \frac{1}{\eta_c}L_{c,is}$$

$$L_{c,pol} = \int_1^2 v \cdot dp = \frac{m}{m-1}RT_1\left[\left(\frac{p_2}{p_1}\right)^{\frac{m-1}{m}} - 1\right] = L_c - L_w$$

$$\eta_c = \frac{L_{c,is}}{L_c} \; ; \quad \eta_{y,c} = \frac{L_{c,pol}}{L_c} = \frac{m}{m-1}\cdot\frac{k-1}{k}$$

- Multistage gas turbine:

$$L_{t,is} = c'_p\left(T_3 - T_{4,is}\right) = \frac{k'}{k'-1}R'T_3\left[1 - \frac{1}{\left(\frac{p_3}{p_4}\right)^{\frac{k'-1}{k'}}}\right]$$

$$L_t = c'_p\left(T_3 - T_4\right) = \frac{k'}{k'-1}R'T_3\left[1 - \frac{1}{\left(\frac{p_3}{p_4}\right)^{\frac{m'-1}{m'}}}\right] = \eta_t \cdot L_{t,is}$$

$$L_{t,pol} = \int_3^4 v \cdot dp = \frac{m'}{m'-1}R'T_3\left[1 - \frac{1}{\left(\frac{p_3}{p_4}\right)^{\frac{m'-1}{m'}}}\right] = L_t + L_w$$

$$\eta_t = \frac{L_t}{L_{t,is}} \; ; \quad \eta_{y,t} = \frac{L_t}{L_{t,pol}} = \frac{k'}{k'-1}\cdot\frac{m'-1}{m'}$$

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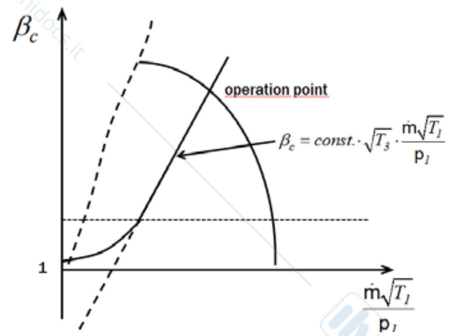
Turbine characteristic:

$$\frac{\dot m\sqrt{p_O^0 \cdot v_O^0}}{p_O^0} = K \qquad \text{(chocked)}$$

The gas turbine as a compressor 'user'

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- Combustion equation applied to TG plants:

$$\eta_b \cdot H_{L,p} = \left(1+\alpha\right)c'_p\left(T_3 - T_2\right)$$

- Overall efficiency of a power plant:

$$\eta_{plant} = \frac{\dot{\mathfrak{L}}_{plant}}{\dot m_f H_{Lp}} = \eta_b \cdot \frac{\dot{\mathfrak{L}}_{plant}}{\dot Q_1}$$

For turbogas plants:

$$\eta_{plant} = \eta_b \cdot \frac{\alpha \cdot L_{plant}}{(1+\alpha)\cdot c'_p\left(T_3 - T_2\right)}$$

$$\left(L_{plant} = \frac{\dot{\mathfrak{L}}_{plant}}{\dot m_a}\right)$$

- Overall efficiency of a power plant with cogeneration: $\eta_{plant} = \dfrac{\dot{\mathfrak{L}}_{plant}}{\dot m_f H_{Lp} - {\dot Q_u}/{\eta_{b,REF}}}$.

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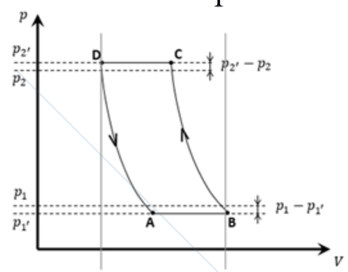
Volumetric compressors:




$$\mathcal{L}_c = \frac{m^*}{m^*-1}p_B V_B\left[\beta_i^{\frac{m^*-1}{m^*}} - 1\right] - \frac{m^{*\prime}}{m^{*\prime}-1}p_D V_D\left[1 - {1}/{\beta_i^{\frac{m^{*\prime}-1}{m^{*\prime}}}}\right]$$

$$L_i = \frac{\mathcal{L}_c}{M} \approx C_p \cdot (T_2 - T_1)$$

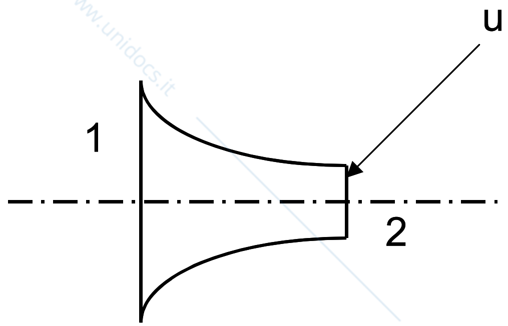
$$M = \lambda_v \rho_1 V_0 = M_C - M_D - M_{leak,\,CD}$$

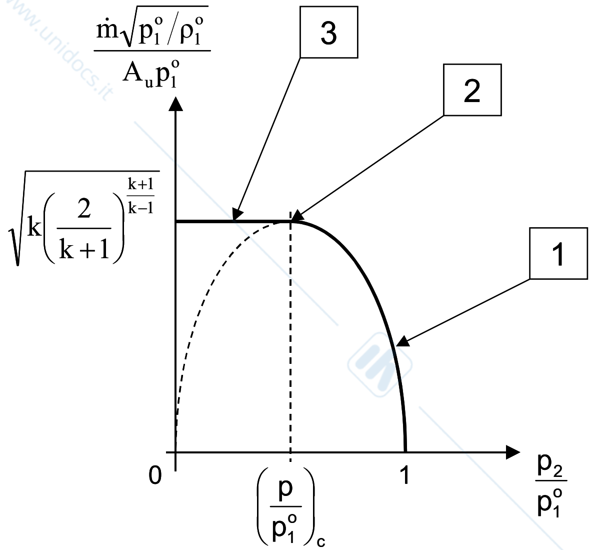
$$\lambda_v = \eta_\varphi(1 - \delta_1)\frac{(\eta_\tau V_B - V_A)}{V_0}$$

Delivery temperature (general): $T_2 = T_1 + \frac{L_i + |Q|}{c_p}$

(roots): $\frac{T_2}{T_1} = 1 + \frac{k-1}{k}\cdot\frac{1}{\lambda_v}(\beta - 1)$

# SIMPLE CONVERGENT NOZZLE



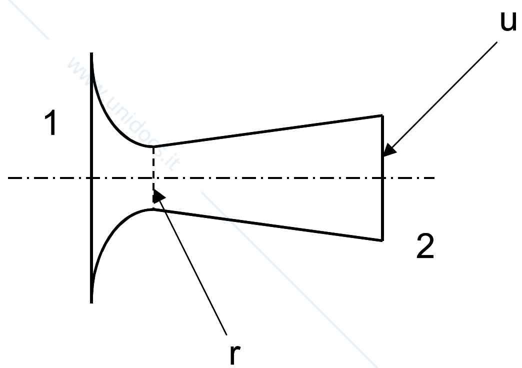

$$\dot{m} = A_{out}\frac{p_1^o}{\sqrt{p_1^o/\rho_1^o}}\sqrt{2\frac{k}{k-1}\left[\left(\frac{p}{p_1^o}\right)^{\frac{2}{k}}-\left(\frac{p}{p_1^o}\right)^{\frac{k+1}{k}}\right]} \qquad (1)$$

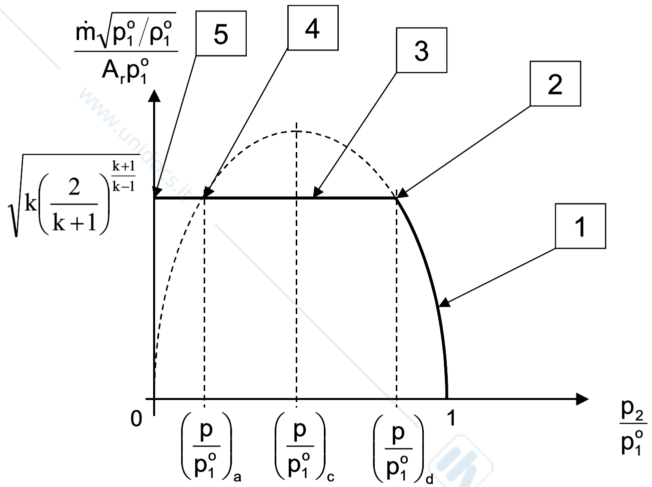
$$\dot{m} = A_{out}\frac{p_1^o}{\sqrt{p_1^o/\rho_1^o}}\sqrt{k\left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}}$$

$A_{out} = A_u$

(2,3)

# CONVERGENT-DIVERGENT (DE LAVAL) NOZZLE




$$\dot{m} = A_{out}\frac{p_1^o}{\sqrt{p_1^o/\rho_1^o}}\sqrt{2\frac{k}{k-1}\left[\left(\frac{p}{p_1^o}\right)^{\frac{2}{k}}-\left(\frac{p}{p_1^o}\right)^{\frac{k+1}{k}}\right]} \qquad (1)$$

$$\dot{m} = A_{th}\frac{p_1^o}{\sqrt{p_1^o/\rho_1^o}}\sqrt{k\left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}}$$

$$= A_{out}\frac{p_1^o}{\sqrt{p_1^o/\rho_1^o}}\sqrt{2\frac{k}{k-1}\left[\left(\frac{p}{p_1^o}\right)_a^{\frac{2}{k}}-\left(\frac{p}{p_1^o}\right)_a^{\frac{k+1}{k}}\right]} \qquad (2\text{-}5)$$

$A_{out} = A_u$

$A_{th} = A_r$