Introduction Basics concepts of Thermodynamics and Fluid-dynamics

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Outline

- Introduction and machines classification
- The first law of thermodynamics and the mechanical energy balance for closed systems
- The second law of thermodynamics for closed systems
- Conservation laws for open systems

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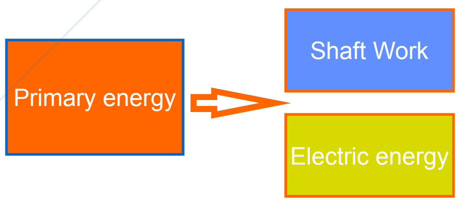
Introduction

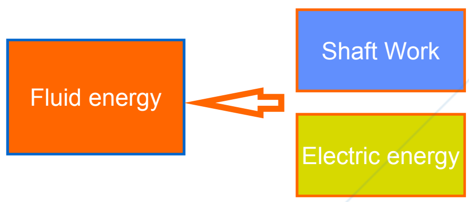
Machine classification

A system that converts energy by means of a working fluid is called a **fluid-flow machine,** or a fluid machine.

A **motor** (or power generating) **machine** transforms the energy stored in the fluid (which in turn comes from a **primary energy source**) into a mechanical energy output.



An **operating** (or power absorbing) **machine** absorbs mechanical power (usually coming from an electric motor) in order to increase the fluid energy (in terms of pressure and/or head and/or kinetic energy).



Introduction

Primary energy sources

- **Chemical energy of fuels**: it is converted into heat through an exothermic chemical reaction between the fuel and an oxidant (almost always air).
- **Energy from vegetation**: it can be converted into an intermediate energy product (biofuel), or used directly.
- **Geothermal energy**: thermal energy generated and stored in the underground. This energy can be exploited i.e. from hot springs, unfortunately to a limited extent.
- **Nuclear energy**: it is part of the energy of the atomic nucleus, which can be released by fission in nuclear power plants.
- **Hydraulic potential energy**: it is a hydraulic head, typically due to the difference in elevation of the fluid, with respect to a reference level.
- **Energy from tides**: it is a hydraulic head due to tides, i.e. the rise and fall of sea levels.
- **Energy from wind**
- **Solar energy**: it is connected to the exploitation of the energy which the Earth receives from the Sun. It can be used to produce thermal energy, electric energy, or both of them.

Introduction

Fluid machines classification

Hydraulic machine: the working fluid can be considered as incompressible and thermal phenomena (such as temperature variations or heat transfer) do not virtually affect the shaft work exchanged by the machine.

Thermal machine: the fluid compressibility must be taken into account during the energy conversion process; thermal phenomena might be relevant during the expansion and/or compression of the working fluid, thus affecting the shaft work exchanged by the machine.

Turbomachine: thermodynamic and dynamic actions between a continuosly flowing fluid and the machine rotating elements take place. The energy transfer is realized through changes in momentum within the flowing fluid.

Volumetric (or positive displacement) machine: a change of volume and/or displacement of the working fluid produces an interaction between the fluid and the moving parts of the machine. The action is nearly static (i.e., due to the fluid pressure) and the machine operates cyclically in time.

Introduction

Fluid machines classification

Fluid machines				
	Thermal machines		**Hydraulic machines**	
	Operating machines	**Motor machines**	**Operating machines**	**Motor machines**
Volumetric machines	Volumetric compressors	Internal combustion engines	Volumetric pumps	Positive displacement motors
Turbo-machines	Turbo-compressors	Gas and steam turbines	Turbopumps and fans	Hydraulic turbines

Introduction

Elements of thermodynamics: thermodynamic system

System: it is whatever one wants to study. It may be as simple as a free rigid body, or as complex as a steam turbine power plant. Whatever external to the system is said to be part of the system's **surroundings**.

Boundary: it is the surface separating the system from its surroundings. The boundary can be at rest or in motion. System and environment are in contact at the boundary and the value of a property that is measured at a point on the boundary must be shared by both the system and the environment. Across the boundary, **interactions** between the system and its surroundings take place.

It is essential for the boundary to be delineated carefully before proceeding with any thermodynamic analysis. However, the same problem can be studied with different choices of the system, boundary and surroundings. Usually, there is one choice which allows a higher convenience in the analysis.

Introduction

Elements of thermodynamics: thermodynamic system and properties

Closed system: it is a particular quantity of matter which is under study. It always contains the same matter, there can be no transfer of mass across the system's boundary. A special type of closed system that does not interact in any way with its surroundings is called an "isolated system".

Open system: it is a region of space (**control volume**) whose boundary (control surface) can be crossed by mass flow. This kind of system is usually defined in the analysis of devices such as turbines and compressors, through which the working fluid flows.

In order to describe a system, its properties must be defined.
A **property** is a macroscopic characteristic of a system, such as mass, volume, energy, pressure, temperature. It must be possible to assign a numerical value to a property, at a given time, without any knowledge of the previous behaviour (history) of the system.

Introduction

Elements of thermodynamics: thermodynamic processes

Extensive property: it is a property whose value for an overall system is the sum of its values for the parts into which the system is divided. Example: mass, volume, energy…

Intensive property: it is a property which is not additive in the sense above. The values of intensive properties are independent of the size or extent of a system. Example: pressure, temperature, specific volume…

The **state** of a system is a condition of it, as described by its properties. The system state is known when the composition and two independent properties are given (p and T, v and T,…).

When any of the properties of a system changes, the state changes and the system is said to have undergone a **process**. A system is said to be at **steady state** if none of its properties changes with time.

A **thermodynamic cycle** is a sequence of processes that begins and ends at the same state. Cycles that are repeated periodically play prominent roles in many applications (for example, steam circulating through an electrical power plant).

Outline

- Introduction and machines classification
- **The first law of thermodynamics and the mechanical energy balance for closed systems**
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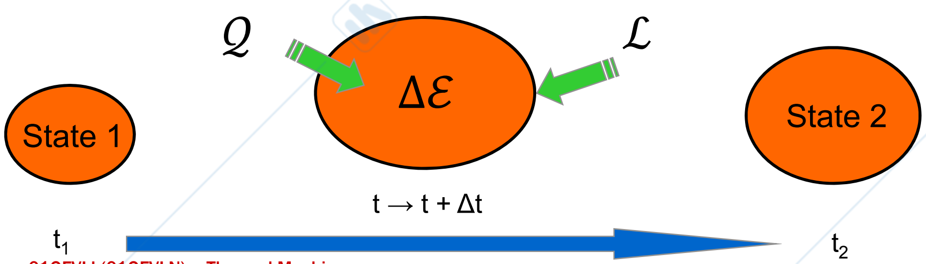
- Conservation laws for open systems

Closed systems: fundamental laws

First law of thermodynamics

The **first law of thermodynamics** is a fundamental concept and states that energy is conserved. The only ways the energy of a closed system can be changed are through transfer of energy by work and heat. Consider a closed system that experiences a change of state from an initial state (1) to a final state (2). The energy balance for this system can be written as:

$$\mathcal{Q} + \mathcal{L} = \Delta\mathcal{E}$$



Closed systems: fundamental laws

First law of thermodynamics

The equation in the previous slide stated that the net amount of energy transferred across the system boundary by heat and work between t_1 and t_2 equals the change in the amount of energy contained within the system during the same time interval.

$\mathcal{Q}$, $\mathcal{L}$ are **energy interactions at the boundary** (they are 'non-properties', that is, they depend on the process).

$$\mathcal{Q} = \int_{t_1}^{t_2} \dot{\mathcal{Q}} dt \qquad \mathcal{L} = \int_{t_1}^{t_2} \dot{\mathcal{L}} dt$$

$\Delta\mathcal{E} = \mathcal{E}_2 - \mathcal{E}_1$ is the **energy change** (property). The energy is

$$\mathcal{E} = \mathcal{U} + \mathcal{E}_c + \mathcal{E}_g + \mathcal{E}_\omega$$

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Closed systems: fundamental laws

First law of thermodynamics

$\boldsymbol{\mathcal{U} = internal\ energy}$

$$\mathcal{U} = \int_m U dm$$

$$U = specific\ internal\ energy.\ It\ depends\ on\ T\ and\ chemical\ composition.$$
$$It\ is\ not\ a\ macroscopic\ form\ of\ energy.$$

$\boldsymbol{\mathcal{E}_c = kinetic\ energy}$

$$\mathcal{E}_c = \int_m E_c dm$$

$$E_c = \frac{c^2}{2} = specific\ kinetic\ energy$$
$$c = absolute\ fluid\ velocity$$

Closed systems: fundamental laws

First law of thermodynamics

$\mathcal{E}_g = \boldsymbol{gravitational\ potential\ energy}$

$$\mathcal{E}_g = \int_m E_g dm$$

$$E_c = gz = specific\ potential\ energy$$
$$z = height\ (vertical\ position\ with\ respect\ to\ a\ reference\ level)$$
$$g = acceleration\ of\ gravity$$

With reference to other macroscopic forms of energy storage, energy storage due to centrifugal force field in a reference frame rotating about a fixed axis (non inertial frame of reference) will often be considered when dealing with turbomachines analysis. In this case, the axis about which the considered frame rotates is the machine axis.

$\mathcal{E}_\omega = \boldsymbol{potential\ energy\ from\ centrifugal\ force\ field}$

$$\mathcal{E}_\omega = \int_m E_\omega dm$$

$$E_\omega = -\frac{1}{2}\omega^2 r^2 = -\frac{u^2}{2}$$

Closed systems: fundamental laws

First law of thermodynamics

$$\Rightarrow \quad \boxed{Q + \mathcal{L} = \Delta\mathcal{U} + \Delta\mathcal{E}_c + \Delta\mathcal{E}_g + \Delta\mathcal{E}_\omega}$$

Other forms of the energy balance can be written. For example, in differential form:

$$dQ + d\mathcal{L} = d\mathcal{E} \qquad \text{or} \qquad \dot{Q}dt + \dot{\mathcal{L}}dt = d\mathcal{E}$$

For a homogeneous system, the balance in terms of quantities per unit mass can be written as follows.

$$Q + L = U_2 - U_1 + \frac{c_2^2 - c_1^2}{2} + g(z_2 - z_1) - \frac{1}{2}\omega^2(r_2^2 - r_1^2) \qquad \text{between } t_1 \text{ and } t_2$$

$$dQ + dL = dU + dE_c + dE_g + dE_\omega \qquad \text{between } t \text{ and } t + dt$$

Closed systems: fundamental laws

Mechanical energy balance

For a process between two states situated infinitely close to one another, the conservation of mechanical energy for a closed system can be derived from the integration of the Newton's law:

$$dL = -pdv + dE_c + dE_g + dE_\omega + dL_w$$

$dL_w = viscous\ (friction)\ dissipation\ per\ unit\ mass\ (non\ negative: dL_w \geq 0)$

$$\left.\begin{aligned} dQ + dL &= dU + dE_c + dE_g + dE_\omega \\ dL &= -pdv + dE_c + dE_g + dE_\omega + dL_w \end{aligned}\right\} \quad dQ + dL_w = dU + pdv$$

1st auxiliary equation

By introducing the property called 'enthalpy', defined as $h = U + pv$ the previous equation can be written as:

$$dQ + dL_w = dh - vdp$$

2nd auxiliary equation

Closed systems: fundamental laws

Sign convention

So far, in writing the first law of thermodynamics, both Q and L have been considered as positive when transferred into the system and done on the system, respectively. This is a sign convention that is convenient when dealing with power absorbing machines.

However, when studying devices whose purpose is to do work, such as ICEs or turbines, it might be more convenient to use another convention.
The work sign convention for motor machines considers that L>0 when the work is done by the system.
In this case:

$$dQ - dL = dU + dE_c + dE_g + dE_\omega$$

$$-dL = -pdv + dE_c + dE_g + dE_\omega + dL_w$$

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Closed systems: fundamental laws

Second law of thermodynamics

The basis of the second law of thermodynamics is the idea that an amount of heat Q cannot be entirely converted into work. Amongst the different formulations, the **Clausius inequality** states that for any thermodynamic cycle:

$$\oint \left(\frac{dQ}{T}\right)_b \leq 0$$

where dQ represents the heat transfer at the system boundary during each portion of the cycle and T is the corresponding boundary temperature. The equality applies when there are no internal irreversibilities as the system executes the cycle, and the inequality applies when irreversibilities are present.

The analysis of systems from a second law perspective is conveniently done by introducing a property called **entropy**. On a differential basis, i.e. by considering two states infinitely close to one another, one has:

$$dS = \frac{dQ}{T} + \frac{dL_w}{T}$$

$dS = entropy\ change\ (property)$

$\frac{dQ}{T}, \frac{dL_w}{T} = entropy\ interactions\ (non\ properties)$

Closed systems: fundamental laws

Second law of thermodynamics

In particular: $\frac{dQ}{T} = entropy\ transfer\ accompanying\ heat\ transfer$

$$\frac{dL_w}{T} = entropy\ production\ (\geq 0)\ by\ internal\ irreversibility$$
$$(no\ external\ irreversibilities\ are\ considered)$$

When $dL_w = 0$ the system does not have irreversibility in it, and the process is called a reversible process. This means that the system and all its surroundings can be exactly restored to their respective initial states after the process has taken place. An **internally reversible process** is a process in which there are no irreversibilities within the system. From a thermodynamic point of view, an internally reversible process consists of a series of equilibrium states.

Closed systems: fundamental laws

Second law of thermodynamics and TdS equations

Hence, for a **reversible process**:

$$dS = \frac{dQ}{T} \qquad S_2 - S_1 = \int_1^2 \left(\frac{dQ}{T}\right)_{rev}$$

That is, the entropy change between states 1 and 2 is the integral for any internally reversible process linking the two states.

By combining the following statements:

$$TdS = dQ + dL_w \quad \text{and} \quad \begin{array}{l} dQ + dL_w = dU + pdv \\ dQ + dL_w = dh - vdp \end{array}$$

the so called **TdS equations** or **Gibbs equations** are obtained:

$$\boxed{TdS = dU + pdv} \qquad \boxed{TdS = dh - vdp}$$

These equations are written in terms of properties and this is very convenient for evaluating the changes in entropy.

Closed systems: fundamental laws

Thermodynamic cycles

By applying the following equations to a cycle:

$$dQ - dL = dU + dE_c + dE_g + dE_\omega \qquad dS = \frac{dQ}{T} + \frac{dL_w}{T}$$

One obtains: $\oint (dQ - dL) = 0$ $\quad \boxed{\oint dQ = \oint dL}$

Hence, the overall heat transferred to the system within any thermodynamic cycle equals the overall work done by the system within the same cycle. It is a common practice to **highlight the overall heat contributions** directed to the system Q_1 and transferred to the surroundings Q_2. $Q_1$and Q_2 are considered as positive, so the equation turns into:

$$\boxed{\oint dL = Q_1 - Q_2 = L}$$

$$\boxed{\oint dS = 0 = \oint \left(\frac{dQ}{T} + \frac{dL_w}{T}\right)}$$

(quantification of the Clausius inequality which holds when viscous effects are the only irreversibilities)

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Open systems: fundamental laws

Conservation of mass or continuity equation

With reference to the control volume above, the **time rate of change** of the mass contained in it equals the difference between the mass flow rate at the inlet section and the mass flow rate at the outlet section.
The differential variation of mass in the control volume is:

$$dm_{cv} = \dot{m}_{in}dt - \dot{m}_{out}dt \quad \Rightarrow \quad \dot{m}_{in} - \dot{m}_{out} = \frac{\partial}{\partial t}\int_{V_c} \rho dV_c$$

If the hypothesis of **steady-state (or stationary) conditions** is introduced, the system properties do not change with time. Thus, the time derivatives are all equal to zero and, for a system with one inlet port and one outlet port only, one has:

$$\dot{m}_1 - \dot{m}_2 = 0 \qquad \dot{m}_1 = \dot{m}_2$$

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Open systems: fundamental laws

Conservation of mass or continuity equation

Let's consider the mass flow rate between A_1 and A_2.
Under the hypothesis that the **flow is one dimensional**, one can write, in section A_2:

$$dm_2 = \rho_2 dV_2 = \rho_2 A_2 dx_2 = \rho_2 A_2 c_2 dt \qquad \frac{dm_2}{dt} = \dot{m}_2 = \rho_2 A_2 c_2$$

and, similarly:

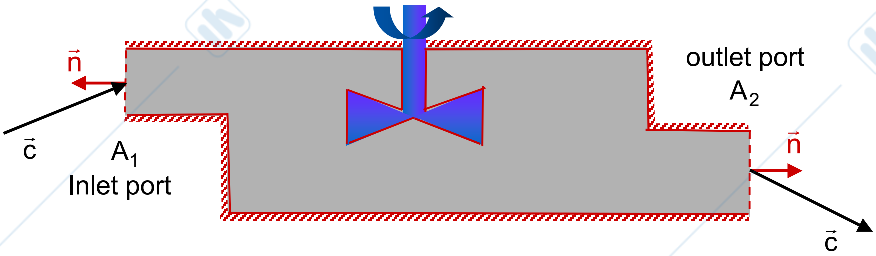
$$\dot{m}_1 = \rho_1 A_1 c_1$$

In fact, in a one-dimensional flow:

- The flow is normal to the boundary at locations where mass enters or exits the control volume.
- All the intensive properties, including velocity and density, are uniform with position over each area through which matter flows.

Open systems: fundamental laws

Conservation of mass or continuity equation



By introducing **the vector $\bar{c}$ and the unit vector $\bar{n}$**, which is normal to the boundary surface and directed outwards, the mass flow rate can be generalized:

$$\dot{m} = \int_A \rho(\bar{c} \cdot \bar{n}) dA$$

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Open systems: fundamental laws

Conservation of mass or continuity equation

Notice that $(\bar{c} \cdot \bar{n})$ is negative when the velocity enters the domain. Furthermore, $(\bar{c} \cdot \bar{n}) = 0$ at the walls. It follows that the integral over the whole control surface indicates the whole mass flow rate that exits from the system.
In other words, the previous equation can be generalized into:

$$\dot{m} = \int_{A_c} \rho(\bar{c} \cdot \bar{n}) dA_c = 0$$

in which the **only hypothesis is that the flow is steady-state**.

For a non-stationary flow:

$$\frac{\partial}{\partial t}\int_{V_c} \rho dV_c + \int_{A_c} \rho(\bar{c} \cdot \bar{n}) dA_c = 0$$

which is the **general formulation** of the conservation of mass for open systems.

Open systems: fundamental laws

Conservation of energy: FLT

Let's first consider the control volume of a generic open system.
The first law of thermodynamics for this open system can be **derived from the correspondent equation for the closed system** which is contained in the control volume at the time t.

In fact, for any open system, the total rate of change of any extensive property is made up of a **transient term** (involving a time derivative) and a flux term, due to the mass crossing the boundary.
This holds for any extensive property and is the principal difference between the closed system and the open system forms of a conservation law.

For the closed system which is contained in the control volume at the time t, by considering an **evolution between t and t+dt**:

$$dQ + d\mathcal{L} = d\mathcal{E} \qquad (\dot{Q} + \dot{\mathcal{L}})dt = d\mathcal{E} \qquad \dot{Q} + \dot{\mathcal{L}} = \frac{d\mathcal{E}}{dt}$$

Open systems: fundamental laws

Conservation of energy: FLT

Based on the previous discussion, **the variation of energy for the open system** is:

$$\dot{Q} + \dot{\mathcal{L}} = \frac{d\mathcal{E}_{cv}}{dt} + \dot{m}_2 E_2 - \dot{m}_1 E_1$$

$\dot{\mathcal{L}}$ is the net work per unit time, which is done on the system by interaction at its boundary.

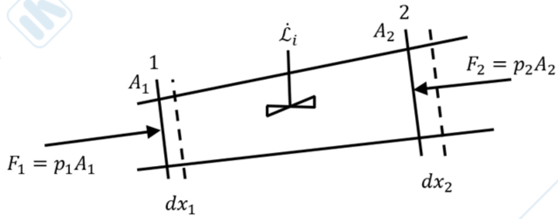
It is convenient to **separate the work term $\dot{\mathcal{L}}$ into two contributions**:

- one contribution is the work associated with the fluid pressure, as mass is introduced at inlets and removed at outlets;
- the other contribution ($\dot{\mathcal{L}}_i$), is associated with interaction between the fluid and moving elements, and is usually transferred to a rotating shaft in fluid machines applications.

Open systems: fundamental laws

Conservation of energy: FLT

$1-D\ flow$



$$\dot{\mathcal{L}}dt = \dot{\mathcal{L}}_i dt + F_1 dx_1 - F_2 dx_2 = \dot{\mathcal{L}}_i dt + p_1A_1dx_1 - p_2A_2dx_2 = \dot{\mathcal{L}}_i dt + p_1v_1dm_1 - p_2v_2dm_2$$

$$\boxed{\dot{\mathcal{L}} = \dot{\mathcal{L}}_i + \dot{m}_1p_1V_1 - \dot{m}_2p_2V_2}$$

By introducing this expression in the equation of the previous slide, one has:

$$\dot{Q} + \dot{\mathcal{L}}_i + \dot{m}_1p_1V_1 - \dot{m}_2p_2V_2 = \frac{d\mathcal{E}_{cv}}{dt} + \dot{m}_2E_2 - \dot{m}_1E_1$$

Open systems: fundamental laws

Conservation of energy: FLT

That is:

$$\dot{Q} + \dot{\mathcal{L}}_i = \frac{\partial}{\partial t}\int_{V_c} \rho E dV_c + \dot{m}_2\left(U + pv + E_{c,g,\omega}\right)_2 - \dot{m}_1\left(U + pv + E_{c,g,\omega}\right)_1$$

$$\boxed{\dot{Q} + \dot{\mathcal{L}}_i = \frac{\partial}{\partial t}\int_{V_c} \rho E dV_c + \dot{m}_2\left(h + E_{c,g,\omega}\right)_2 - \dot{m}_1\left(h + E_{c,g,\omega}\right)_1}$$

Which is the **FLT for a 1-D flow open system**, with one inlet and on outlet section.

By operating similarly for the continuity equation, the flux integral at the boundary can be introduced and this allows the general formulation of the FLT to be written:

$$\boxed{\dot{Q} + \dot{\mathcal{L}}_i = \frac{\partial}{\partial t}\int_{V_c} \rho\left(U + E_{c,g,\omega}\right) dV_c + \int_{A_c} \rho\left(h + E_{c,g,\omega}\right)(\bar{c}\cdot\bar{n}) dA_c}$$

Open systems: fundamental laws

Conservation of energy: FLT

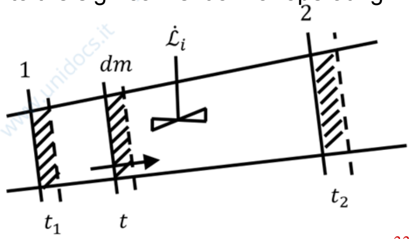
For a **1-D flow under steady-state conditions**, the previous equation can be simplified, because $\frac{\partial}{\partial t} = 0$ and $\dot{m}_1 = \dot{m}_2$

$$\dot{Q} + \dot{\mathcal{L}}_i = \dot{m}\left(h_2 - h_1 + E_{c,g,\omega_2} - E_{c,g,\omega_1}\right)$$

$$Q + L_i = \Delta h + \Delta E_{c,g,\omega}$$

Where the Δ operator indicates a difference between the values of the properties in two sections at the same time.
The equations have been written with reference to the sign convention for operating machines.

With reference to a 1-D stationary flow, a **mass dm, which enters the control volume at the instant t_1 and leaves it at the instant t_2**, can be considered.
This mass is a closed system.



Open systems: fundamental laws

Conservation of energy: FLT

By integrating the 2nd auxiliary equation between t_1 and t_2, with reference to the closed system **dm**, the following equation can be written:

$$\int_{t_1}^{t_2} (dQ + dL_w) = h_2 - h_1 - \int_{t_1}^{t_2} v dp$$

Due to the stationary conditions, the following **correspondences** amongst heat transfer/properties of the open system and those of the mass dm within the time interval $t_1 - t_2$ can be shown:

$$Q = \frac{\dot{Q}}{\dot{m}} = \int_1^2 dQ_{cv} = \int_{t_1}^{t_2} dQ_{cs} \qquad \int_1^2 v dp = \int_{t_1}^{t_2} v dp \qquad \int_1^2 dL_w = \int_{t_1}^{t_2} dL_w$$

Open systems: fundamental laws

Conservation of energy and generalized Bernoulli's equation

The **conservation of mechanical energy for open systems** (also known as **generalized Bernoulli's equation)** can thus be written. By using the sign convention for operating machines :

$$Q + L_i = \Delta h + \Delta E_{c,g,\omega}$$

$$\int_{t_1}^{t_2} (dQ + dL_w) = h_2 - h_1 - \int_{t_1}^{t_2} vdp \quad \longrightarrow \quad L_i = \int_1^2 vdp + \Delta E_{c,g,\omega} + L_{w1-2}$$

With reference to the **motor machines sign convention**:

$$Q - L_i = \Delta h + \Delta E_{c,g,\omega} \qquad -L_i = \int_1^2 vdp + \Delta E_{c,g,\omega} + L_{w1-2}$$

On a **differential basis**:

$$dQ \pm dL_i = dh + dE_{c,g,\omega} \qquad \pm dL_i = vdp + dE_{c,g,\omega} + dL_{w1-2}$$

Open systems: fundamental laws

Second law of Thermodynamics

As it was done before, by considering the **closed system dm**, the mass particle which flows through the control volume 1-2. For such a system, the 2nd law of Thermodynamics states:

$$\int_{t_1}^{t_2} dS = \int_{t_1}^{t_2} \frac{dQ + dL_w}{T}$$

As discussed before, it can be shown that:

$$\int_{t_1}^{t_2} dS = S_2 - S_1 \qquad \int_{t_1}^{t_2} \frac{dL_w}{T} = \int_1^2 \frac{dL_w}{T} \qquad \int_{t_1}^{t_2} \frac{dQ}{T} = \int_1^2 \frac{dQ}{T}$$

Consequently, the **SLT for the control volume 1-2** is:

$$S_2 - S_1 = \int_1^2 \frac{dQ + dL_w}{T}$$

The equation can also be used on a differential basis.

Open systems: fundamental laws

Conservation of momentum

For the **closed system contained in the control volume at a time t**, the conservation of momentum is:

$$\bar{R} = \frac{d\bar{Q}}{dt} = \frac{\partial}{\partial t}\int_m \bar{c}dm$$

$\bar{R}$ is the vector sum of all the surface and body forces acting on the closed system.
By considering the **open system 1-2**, and by recalling that the total variation of any extensive property can be split into a "transient-2 and a "flux" contribution, one has:

$$\bar{R} = \frac{\partial}{\partial t}\int_{V_c} \rho\bar{c}dV_c + \int_{A_c} \rho\bar{c}(\bar{c}\cdot\bar{n})dA_c$$

For a **steady-state, 1-D flow**, in which viscosity effects can be neglected, the equation becomes:

$$\dot{m}(\bar{c_2} - \bar{c_1}) = -\rho_1 A_1 \overline{n_1} - \rho_2 A_2 \overline{n_2} + \int_{V_c} \rho\bar{g}dV_c + \bar{F}$$

in which $\bar{F}$ is the sum of the forces acting on the fluid within the control volume at the walls (either moving or fixed).

Open systems: fundamental laws

Conservation of angular momentum

For the **closed system** within the control volume at the time t, the conservation of angular momentum is:

$$\overline{M_o} = \frac{d}{dt}\int_m \bar{r}\wedge\bar{c}dm$$

In which $\overline{M_o}$ is the vector sum of the moments of all external forces acting on the system about the point O, $\bar{r}$ is the vector distance from O of each element dm and $\bar{c}$ is the absolute velocity.

For an **open system under steady-state conditions**:

$$\overline{M_o} = \int_{A_c} \bar{r}\wedge\rho\bar{c}(\bar{c}\cdot\bar{n})dA_c$$

Open systems: fundamental laws

Conservation of angular momentum

In the context of fluid machines the most useful formulation is obtained by considering the fluid moment of momentum and the force **moments about an axis which is fixed in space** (usually the rotation axis of a turbomachine).

The corresponding equation is obtained by projecting the conservation of angular momentum (vector equation) along the axis. By indicating with c_u the tangential component of $\bar{c}$, with r the distance from the axis, for an open system with one entry and one exit, stationary and 1-D flow one has:

$$M_a = \dot{m}\left(r_2 c_{u_2} - r_1 c_{u_1}\right)$$

In which M_a is the moment of all the external forces about the axis.

Introduction to turbomachines

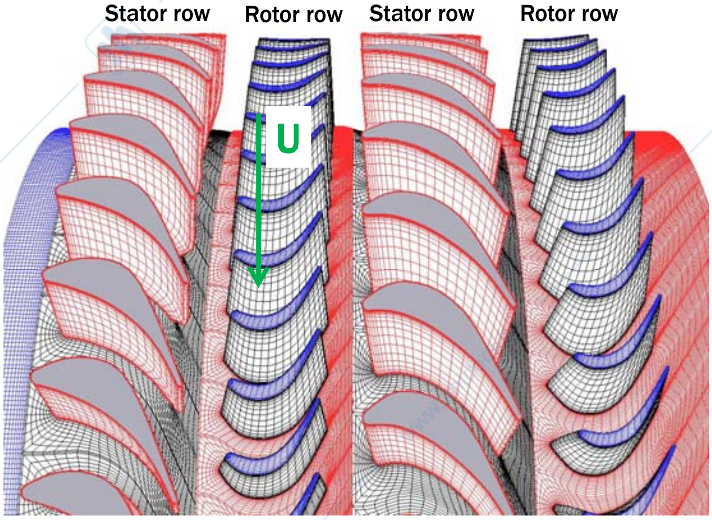
Prof. Mirko Baratta

Dipartimento Energia
Politecnico di Torino

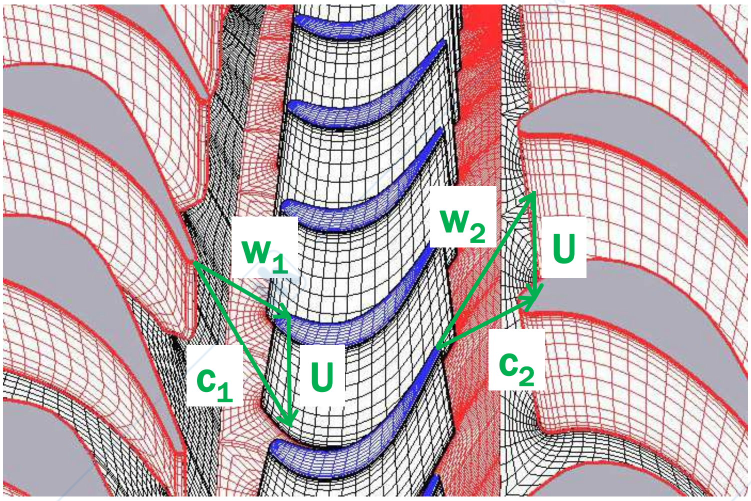
Outline

- **Examples of turbomachines configurations**
- **Turbomachines classification**
- **Velocity triangles and blading terminology**
- **Turbomachines: mass flow rate, torque and power**
- **Examples of velocity triangles**

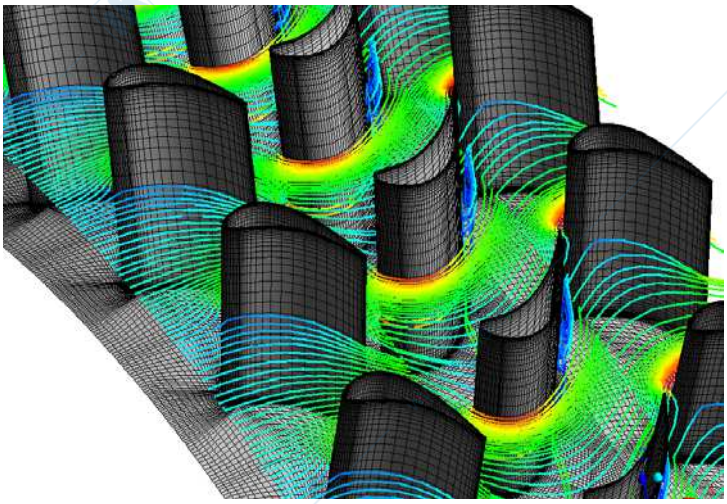
Axial turbine stages



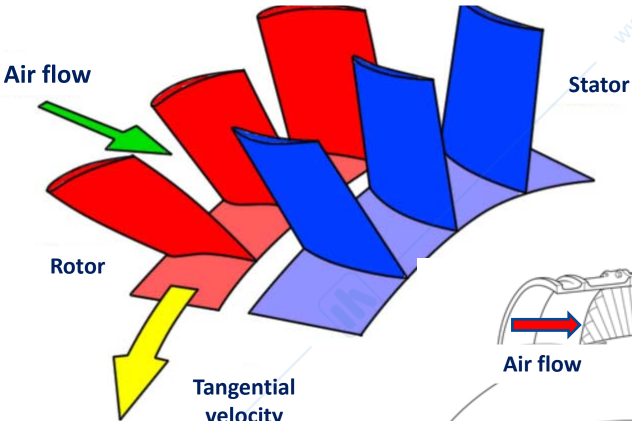
Axial turbine stages

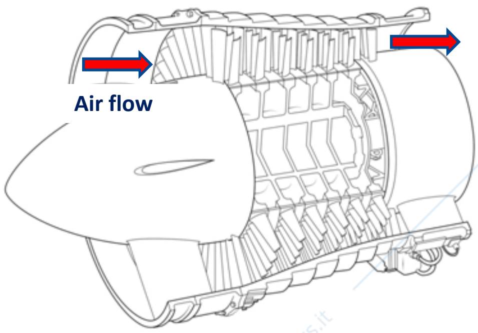


Axial turbine blades with streamlines from 3D simulation

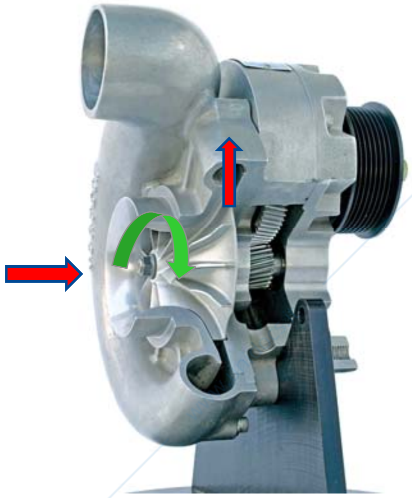


Axial turbocompressor



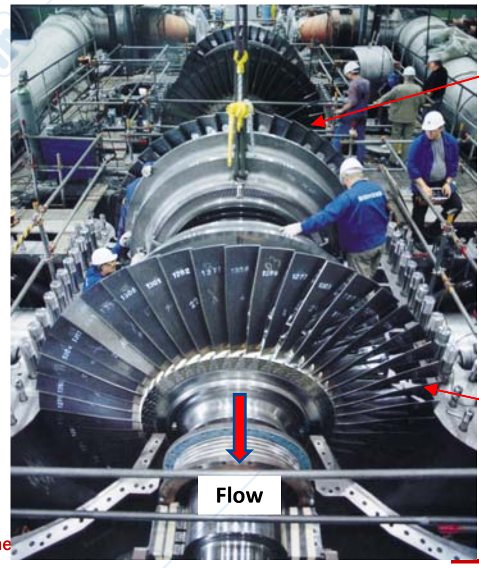


Centrifugal turbocompressor



Rotor of a multistage mixed-flow steam turbine

Rotor rows of an axial steam turbine



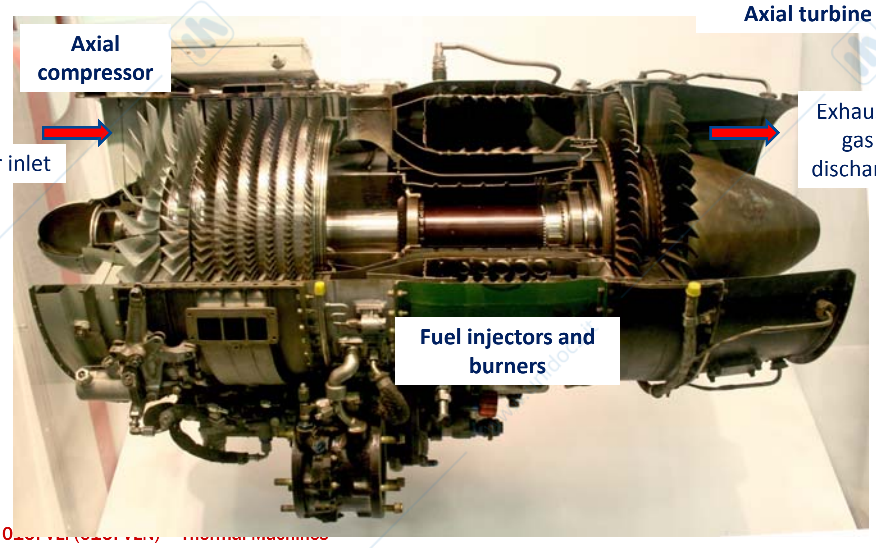
"Untwisted" blades (HP stages)

"Twisted" blades (LP stages)

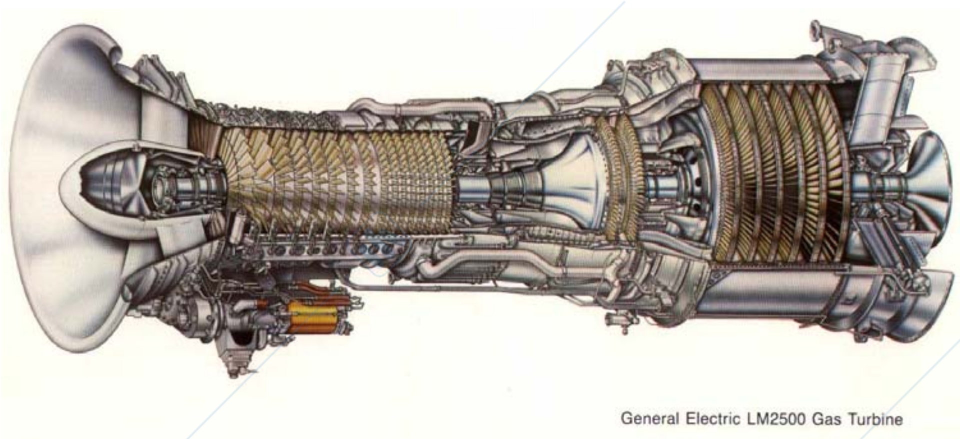
Steam turbine blades

"Twisted" blades

Gas turbine



Gas turbine



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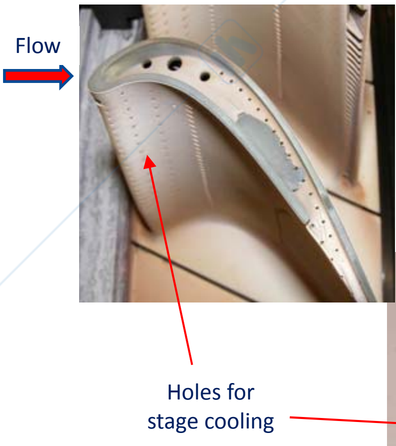
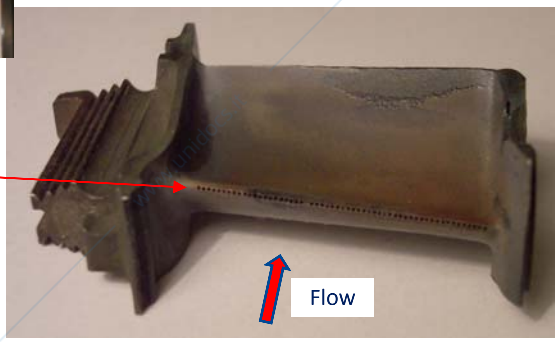
Gas turbine blades





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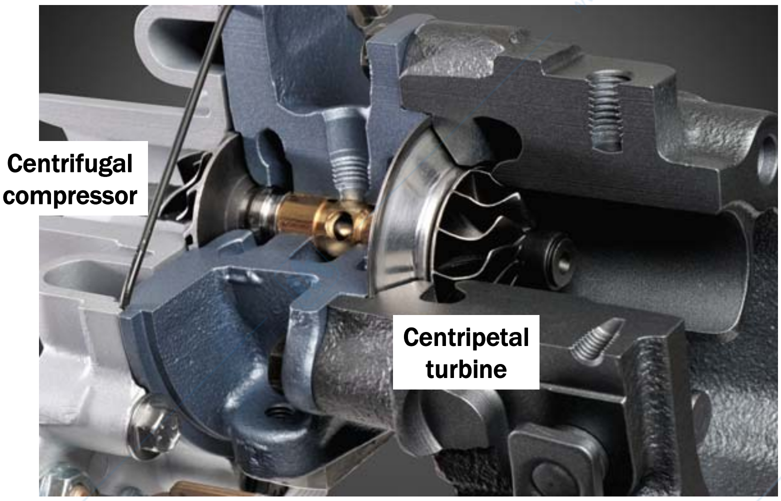
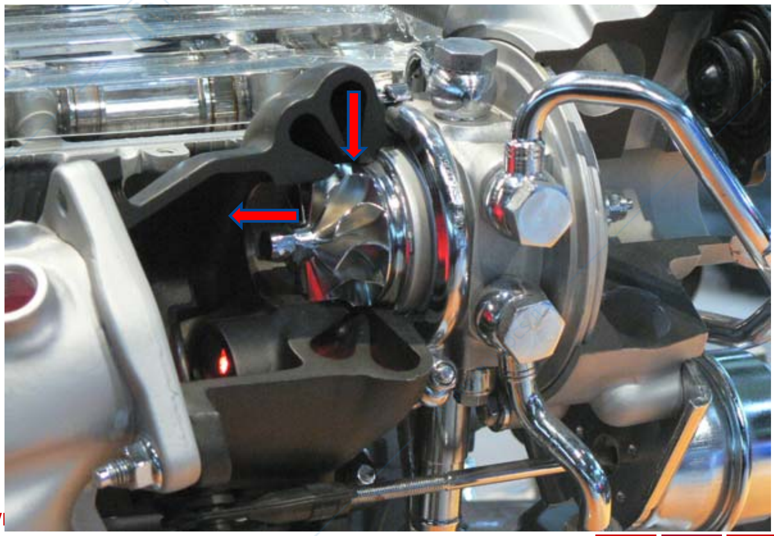
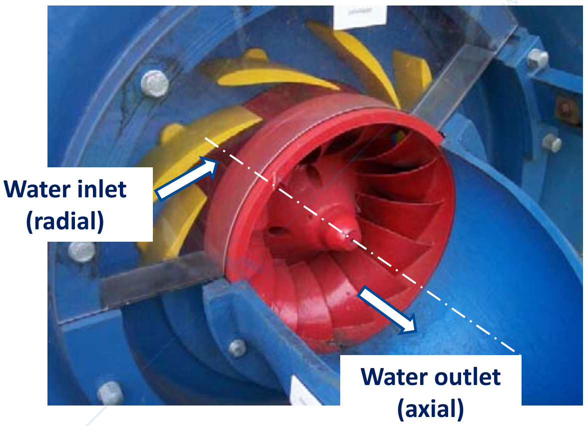
IC engine turbocharger group



Twin-entry turbine for turbocharger-group applications

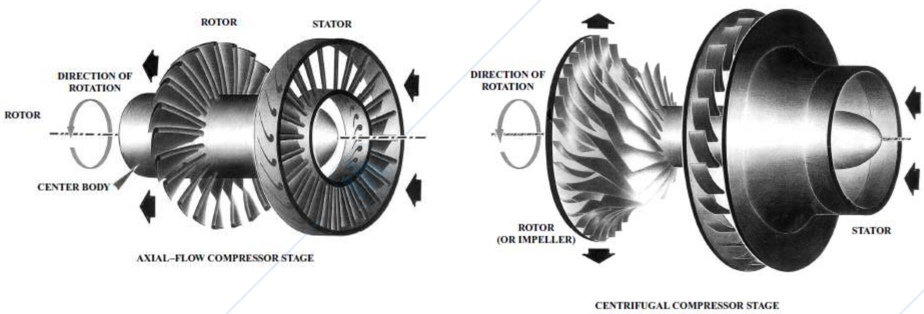


Francis turbine (hydraulic turbine)

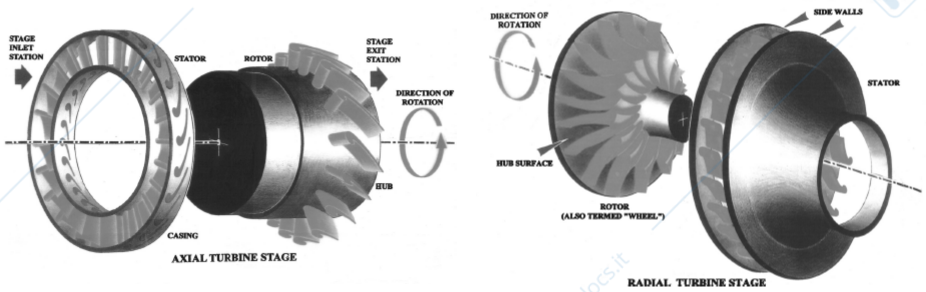


Rotor of a 110 MW Kaplan turbine (axial hydraulic turbine)

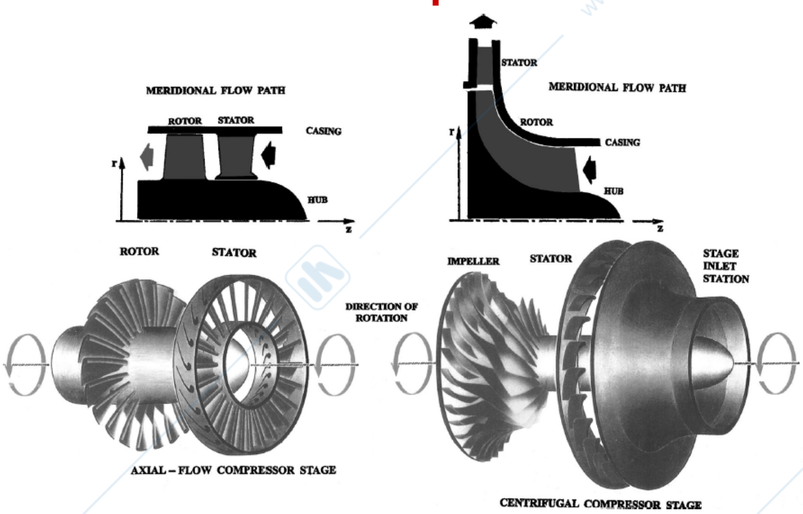
Axial and centrifugal flow turbocompressor stages



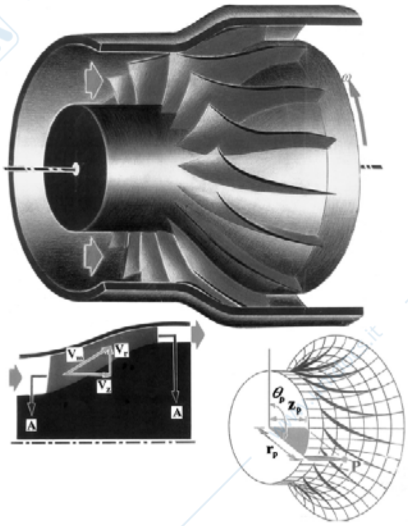
Axial and centripetal turbine stages



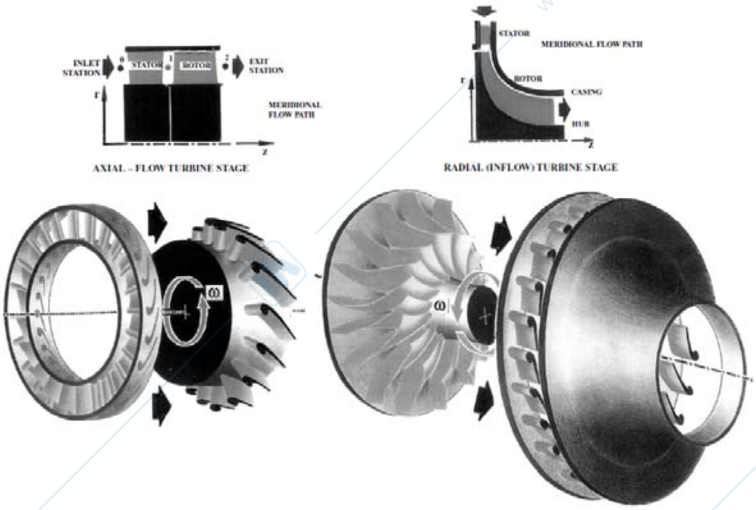
Turbocompressor stages: meridional flow path



Mixed-flow turbocompressor



Turbine stages: meridional flow path



Outline

- Examples of turbomachines configurations
- Turbomachines classification
- Velocity triangles and blading terminology
- Turbomachines: mass flow rate, torque and power
- Examples of velocity triangles

Turbomachines

Classification

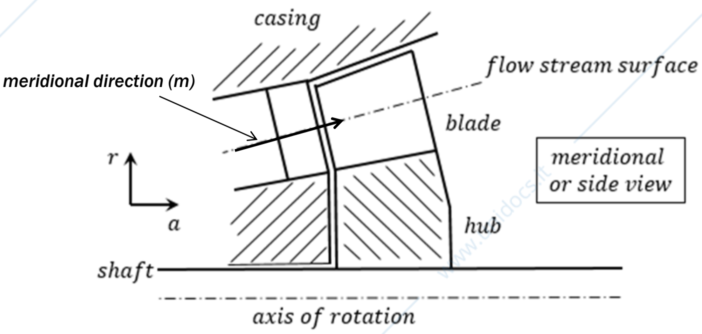
In this course, the study of different types of turbomachines will be accomplished. The reason for which so **many different types** of either operating or motor machines are in use, is because of the almost infinite range of service requirements. Generally speaking, for a given application there is one type of compressor/turbine that is best suited to provide optimum conditions of operation.

From the point of view of the **geometrical configuration**, turbomachines are categorised according to the nature of the flow path through the passages of the rotor:

- when the path of the through-flow is wholly or mainly parallel to the axis of rotation, the machine is called "axial flow turbomachine".
- when the path of the through-flow is wholly or mainly in a plane perpendicular to the rotation axis the device is termed a "radial flow turbomachine".
- "mixed flow turbomachines" are those in which both radial and axial velocity components are present in significant amount at the rotor outlet.

Turbomachine views

Turbomachines consist of rotating and stationary blades arranged around a common axis, which means that they tend to have some form of cylindrical shape. It is therefore natural to use a **cylindrical polar coordinate system**, aligned with the axis of rotation, for their description and analysis. This coordinate system is represented in the following picture, with reference to a generic mixed-flow turbine.



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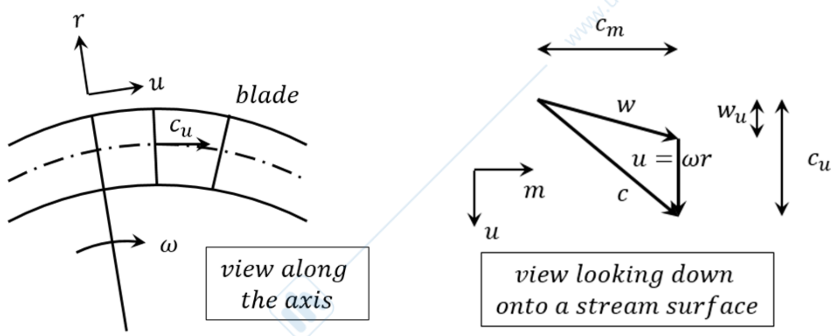
Turbomachine views



The **three axes** are referred to as axial (a), radial (r) and tangential or circumferential (u). In general, the flow in a turbomachine has velocity components along all the three axes, which vary in all directions.

Velocity components

However, to simplify the analysis, it is usually assumed that the **flow does not vary in the tangential direction**. In this case, the flow moves through the machine on axial-symmetric stream surfaces. The component of velocity along a stream surface is called "meridional component".
In purely axial-flow machines, the radius of the flow path is constant, so $c_m = c_a$.
Similarly, in purely radial-flow machines, $c_m = c_r$.

For simplicity, the assumption of **one-dimensional flow** will be made in the course.
This means:
- Flow properties are constant along the tangential direction.
- The flow conditions at the intermediate flow surface are representative of all the flow-stream surfaces. In other words, the flow properties are independent of the radius.

It is very convenient to analyse the flow within the rotating blades in a **frame of reference that is stationary relative to the blades**. This means that the frame rotates about the machine rotation axis with the same angular velocity. The relative flow velocity will hence be denoted with $\overline{w}$.

Outline

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Velocity components

Conventional **notation**:

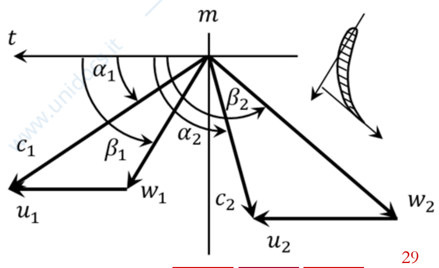
- 0: fixed row inlet
- 1: fixed row outlet-rotating row inlet
- 2: rotating row outlet

It is usual practice to draw the velocity diagrams (or triangles) at the rotor inlet and outlet sections. These are drawn by taking the **relation between the absolute and the relative fluid velocity** into account:

$$\bar{c} = \bar{w} + \bar{u}$$

Where:

- $\bar{c}$ is the absolute fluid velocity
- $\bar{w}$ is the relative fluid velocity
- $\bar{u}$ is the local velocity of the blade ($u = \omega r$)



Flow angles

Velocity triangles are drawn with reference to a **plane tangential to the average stream surface**. Tangential and meridional directions are considered.

The meridional direction might be different between sections 1 and 2. Still, also in this case the triangles will be sketched onto the same plane.

The positive direction of $\bar{u}$ indicates the positive direction to choose the sign of the fluid tangential velocities. Furthermore, **angles will be measured starting from $\bar{u}$** and rotating towards the positive direction of the meridional components.

Particular care should be taken to distinguish between the **blade or metal angles** α_{b0} at inlet and α_{b1} at exit, and the corresponding **flow or gas angles** α_0 and α_1 (for a rotor blade, β_1 and β_2 replace α_0 and α_1, respectively).

Flow angles

The difference between blade and flow angles at the leading edge of the blade is the **incidence angle i**, defined as:

$$i = \alpha_0 - \alpha_{b0} \quad \textit{for stator blades}$$

$$i = \beta_1 - \beta_{b1} \quad \textit{for rotor blades}$$

At the trailing edge, the difference between the flow and blade angles is the **deviation angle** δ:

$$\delta = \alpha_1 - \alpha_{b1} \quad \textit{for stator blades}$$

$$\delta = \beta_2 - \beta_{b2} \quad \textit{for rotor blades}$$

Flow angles

In designing a blade, the **flow angles** are set by the velocity triangles.

In order to **determine the metal angles**, the designer must be able to decide upon or calculate the amount of incidence and deviation, in order that the blade will produce the correct flow angles.

Within 1-D approach, the usual approach is to set:

$$\delta = 0 \quad \textit{for both stator and rotor blades (all operating conditions)}$$

$$i = 0 \quad \textit{for both stator and rotor blades (design conditions only)}$$

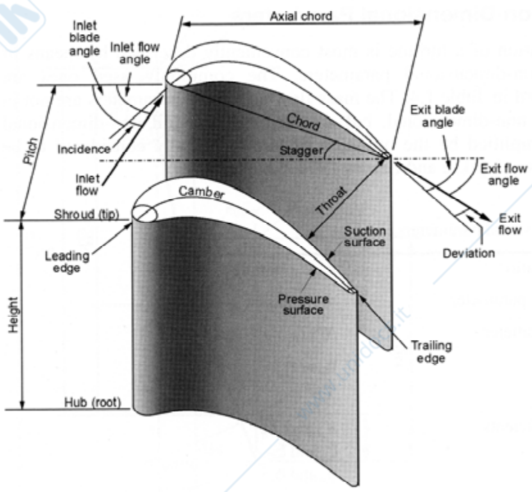
The **camber angle** is a measure of the turning done by the blade itself:

$$\theta = \alpha_{b0} - \alpha_{b1} \quad \textit{for stator blades} \qquad \theta = \beta_{b1} - \beta_{b2} \quad \textit{for rotor blades}$$

and the **deflection angle** is the corresponding turning done by the flow:

$$\varepsilon = \alpha_0 - \alpha_1 \quad \textit{for stator blades} \qquad \varepsilon = \beta_1 - \beta_2 \quad \textit{for rotor blades}$$

Blading terminology



Blading terminology

The geometric terms and parameters used to define a blade passage are shown in previous slide. The concave side of the blade is the **pressure surface**, and the convex side is the **suction surface**. The gas pressure is higher on the pressure than on the suction surface, due to the curvature of the blade passage, and the resultant force in the tangential direction causes the work interaction between the fluid and the blades.

The **overall size** of the blade is defined by the chord (or the axial chord), the blade height or span, the pitch or spacing, and the stagger angle. The throat is the point of minimum area in the passage.

Since a turbine blade passage is arranged to accelerate the flow, the **throat** is normally located close to the trailing edge. In a supersonic turbine choking occurs at the throat where the Mach number is equal to one, and the flow then accelerates beyond Ma = 1 downstream of the throat.

The region of the suction surface downstream of the throat is called **the uncovered surface** because there is no blade opposite to it in a direction normal to the surface.

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Continuity equation

The meridional components in 1 and 2 can be linked through the **continuity equation**, applied to the open system between 1 ad 2:

$$\dot{m}_1 = \dot{m}_2$$

For example, for an **axial turbine**:

$$\dot{m}_1 = \rho_1 A_1 c_{a_1} = \rho_2 A_2 c_{a_2} = \dot{m}_2$$

where:

$$A_1 = \xi_1 \pi d l_1 \qquad A_2 = \xi_2 \pi d l_2$$

$$\begin{cases} d = mean\ rotor\ diameter \\ l = blade\ height \\ \xi = restrictive\ coefficient \end{cases}$$

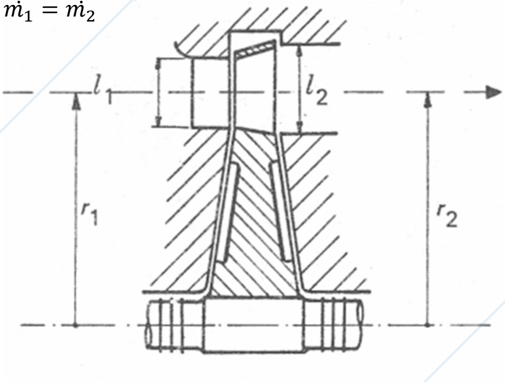

Fig. 4

Continuity equation

Other turbine arrangements:

Radial (centrifugal)

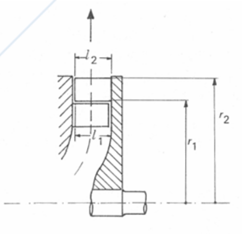

Fig. 5

$$\dot{m}_1 = \rho_1 A_1 c_{r_1} = \rho_2 A_2 c_{r_2} = \dot{m}_2$$

$$A_1 = \xi_1 \pi d_1 l_1 \qquad A_2 = \xi_2 \pi d_2 l_2$$

Radial (centripetal)

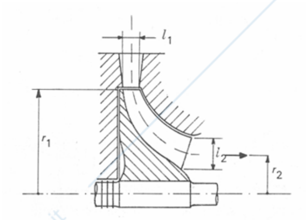

Fig. 6

$$\dot{m}_1 = \rho_1 A_1 c_{r_1} = \rho_2 A_2 c_{a_2} = \dot{m}_2$$

$$A_1 = \xi_1 \pi d_1 l_1 \qquad A_2 = \xi_2 \pi d_2 l_2$$

Torque

The **torque exchanged** between the fluid and he moving blades is related to the tangential velocities. In fact, recalling the conservation of the angular momentum:

$$M_a = \dot{m}\left(r_2 c_{u_2} - r_1 c_{u_1}\right)$$

M_a is positive if it is acting on the fluid.
Thus, for an operating machine, the equation directly gives the torque which the rotating-row blades exert on the fluid (internal torque).

For a **motor machine**, the fluid exerts a positive torque on the blades. Thus it is necessary to write:

$$-M_a = C = \dot{m}\left(r_1 c_{u_1} - r_2 c_{u_2}\right)$$

Power and Euler's Equations

The **internal power** of a motor machine is:

$$\dot{\mathcal{L}}_i = P_i = C \cdot \omega = \dot{m}\left(r_1 c_{u_1} - r_2 c_{u_2}\right) \cdot \omega$$

$$P_i = \dot{m}\left(u_1 c_{u_1} - u_2 c_{u_2}\right)$$

Where the internal work per unit mass is:

$$L_i = \frac{P_i}{\dot{m}} = u_1 c_{u_1} - u_2 c_{u_2}$$

which holds for motor machines, and is referred to as **Euler's Turbine Equation**.

For an operating machine the **Euler's Pump Equation** can be defined:

$$L_i = \frac{P_i}{\dot{m}} = u_2 c_{u_2} - u_1 c_{u_1}$$

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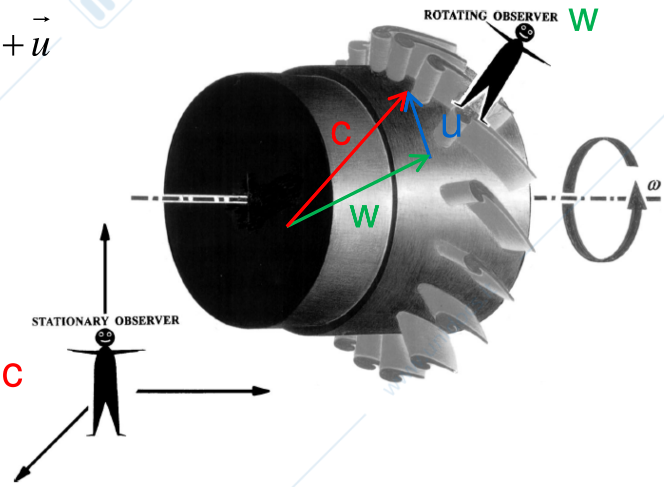
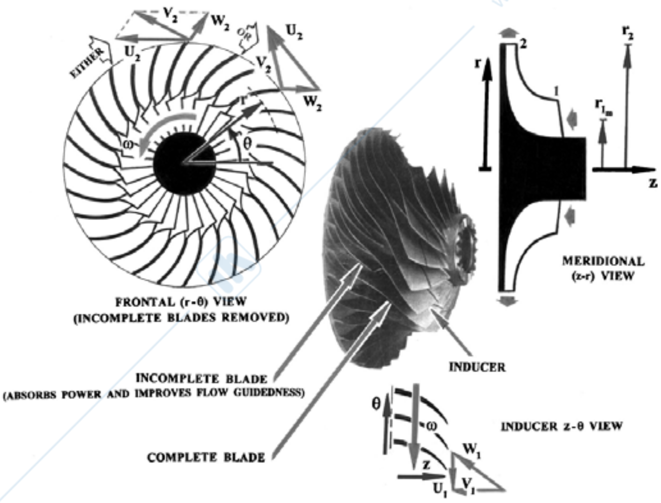
Fixed (inertial) and rotating reference frames

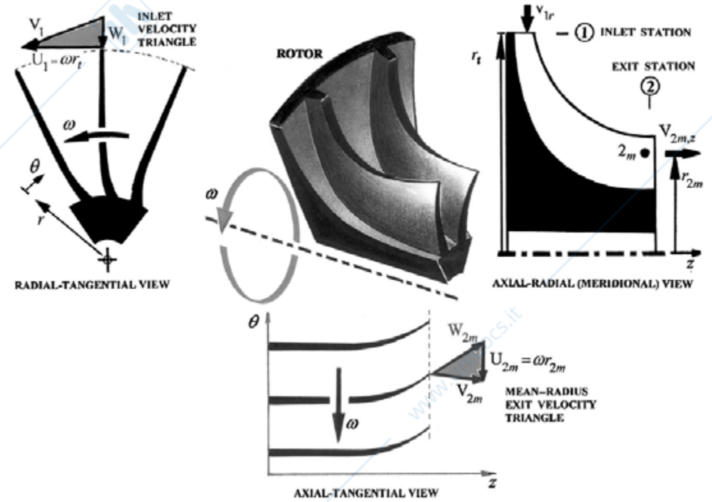
$$\vec{c} = \vec{w} + \vec{u}$$



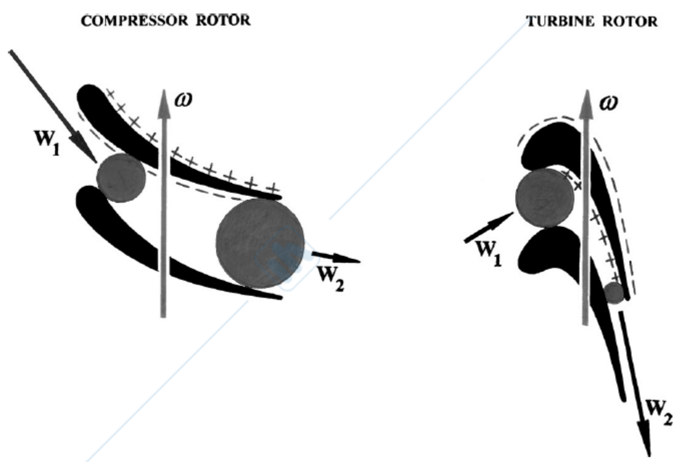
Centrifugal turbocompressor



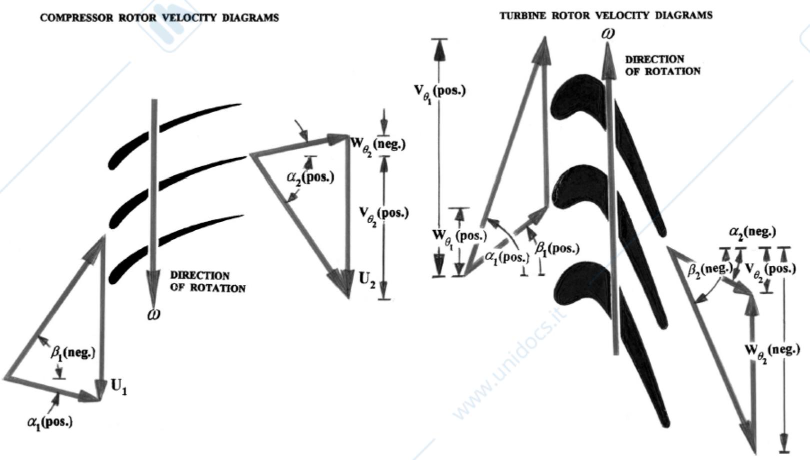
Centripetal turbine



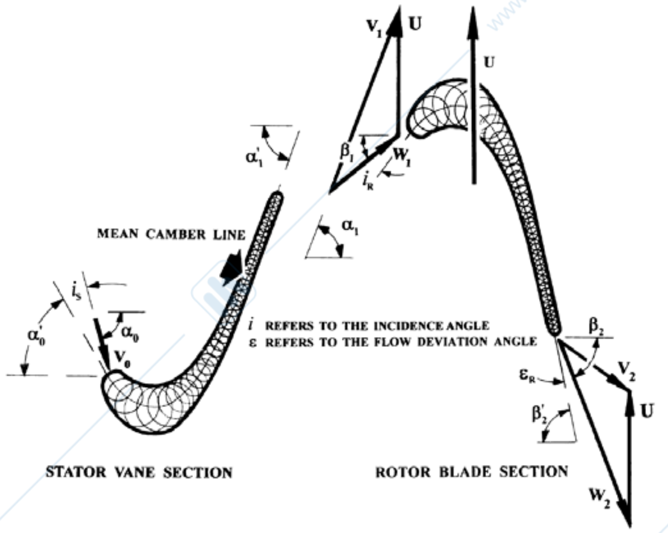
Blade velocity in compressor and turbine rotating rows



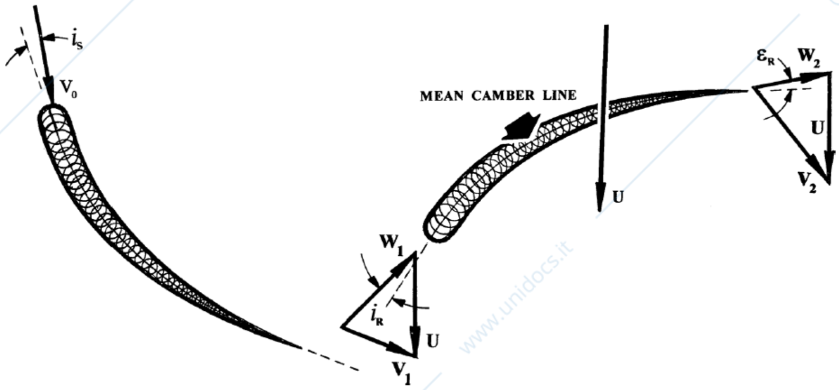
Axial compressors and turbines



Incidence angles for an axial turbine stage



Incidence angles for an axial compressor stage



Nozzles and diffusers

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Outline

- **Introduction**
- **Speed of sound and stagnation properties**
- **Isentropic compressible flow in nozzles and diffusers (steady state flow)**
- **Design of nozzles**
- **Off-design performance of convergent nozzles**
- **Off-design performance of convergent-divergent nozzles**
- **Effects of viscous dissipation**

Introduction

Turbomachines and flow evolution

Turbomachines are made up of a series of fixed and rotating rows through which the fluid flows. The **channels between adjacent blades** can be considered as pipes with a variable cross-section. The processes the fluid undergoes while flowing within these channels depend on the cross-section variation along the mean streamline.
It follows that, in order to get desired flow properties evolution, the channel cross section needs to be carefully designed.

In particular, the **flow evolution should be**:

- gradual;
- as close as possible to a reversible process (this means to reduce flow losses to a minimum);
- the flow stream at the channel exit should be organized into parallel streamlines; in this way, the flow kinetic energy can be exploited efficiently.

Introduction

Turbomachines and flow evolution

In turbomachinery field, the flow rate is usually high and this means high flow velocities. Furthermore, high blade tangential speed are required to guarantee a high internal work. Under these conditions, the fluid exhibits appreciable changes in density; the correspondent flow is named **"compressible flow"**. We will deal with compressible flow through nozzles and diffusers.

Nozzles: purposely designed pipes, in which the flow velocity is increased, whereas the pressure is decreased.
Diffusers: purposely designed pipes, in which the flow kinetic energy is decreased, because it is converted into pressure energy.

In our applications, we will consider the following **hypotheses**, in order to simplify the study:

- 1-D steady state flow
- adiabatic reversible process: $Q = 0, L_w = 0 \rightarrow S = const. (isentropic\ process)$

Outline

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- **Speed of sound and stagnation properties**
- Isentropic compressible flow in nozzles and diffusers (steady state flow)
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- Off-design performance of convergent-divergent nozzles
- Effects of viscous dissipation

Speed of sound

Definition

The speed of sound is the **propagation velocity of small waves** relative to the fluid flow. In particular, it can be shown that:

$$c_s = \sqrt{\left(\frac{\partial p}{\partial \rho}\right)_{S=const}}$$

The speed of sound is an <u>intensive property</u>.
For an ideal gas (**isentropic process**):

$$\frac{p}{\rho^k} = const \longrightarrow \frac{dp}{\rho^k} - kp\frac{d\rho}{\rho^{k+1}} = 0 \longrightarrow \frac{dp}{\rho} - k\frac{p}{\rho} = 0$$

$$c_s = \sqrt{k\frac{p}{\rho}} \qquad (1)$$

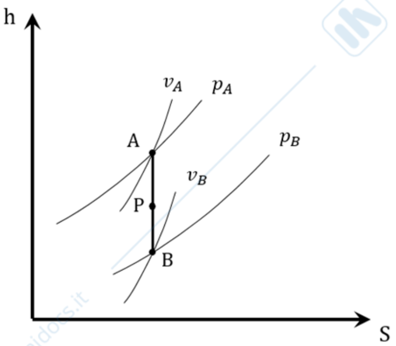
And, by using the **perfect gas equation of state**:

$$\frac{p}{\rho} = RT \longrightarrow c_s = \sqrt{kRT} \qquad (2)$$

Speed of sound

Isentropic coefficient for steam

For a real gas (such as, steam), eq. (2) cannot be used. Still, eq. (1) can be used, provided that an average value of the isentropic exponent is used. Such a value can be obtained by considering two points A ad B on the Mollier chart, close to the point of interest P.



$$p_A v_A^k = p_B v_B^k \qquad ln\left(\frac{p_A}{p_B}\right) = k \cdot ln\left(\frac{v_B}{v_A}\right)$$

$$k = \frac{ln\left({}^{p_A}/_{p_B}\right)}{ln\left({}^{v_B}/_{v_A}\right)}$$

The so-calculated value holds within the interval used for its calculation. The closer are A and B, the more precise is the value of k, but its validity will be more limited.
For saturated steam, one has, on average: $k \sim 1.15$
For highly super-heated steam: $k \sim 1.3$

Speed of sound

Mach number

The **Mach number**, Ma, is defined as the ratio between the local fluid velocity, to the local speed of sound:

$$Ma = \frac{c}{c_s}$$

When $Ma > 1$ the flow is said to be "supersonic", when $Ma < 1$, the flow is "subsonic" and when $Ma = 1$, the flow is "sonic".

The Ma plays a important role to establish weather the flow can be considered **compressible or incompressible**.
As a matter of fact, compressibility effects can usually be neglected for Ma values below 0.3.

Stagnation properties

Definition

When dealing with compressible flows, it is often convenient to work with properties evaluated at a **reference state** which is known as stagnation state.
The stagnation state is the state a flowing fluid would attain if it were decelerated to zero-velocity isentropically, without work transfer, in an horizontal duct, under steady-state conditions.
By applying the FLT for open systems between the actual state and the stagnation state, one has:

$$\cancel{Q} + \cancel{L_i} = \Delta h + \Delta E_c + \cancel{\Delta E_g} + \cancel{\Delta E_\omega}$$

$$0 = h° - h + \frac{c^{\circ 2} - c^2}{2} \qquad \text{where } c^\circ = 0 \text{ by definition}$$

$$h^\circ = h + \frac{c^2}{2} \quad \textbf{stagnation enthalpy}$$

$$for\ a\ perfect\ gas \qquad \Delta h = c_p \Delta T \qquad T^\circ = T + \frac{c^2}{2c_p} \quad \textbf{stagnation temperature}$$

Stagnation properties

Definition and application

By using the equation $pv^k = const.$ for the isentropic process:

$$p^\circ = p\left(\frac{T^\circ}{T}\right)^{\frac{k}{k-1}} \quad \textbf{stagnation pressure} \qquad \rho^\circ = \rho\left(\frac{T^\circ}{T}\right)^{\frac{1}{k-1}} \quad \textbf{stagnation density}$$

With respect to **entropy**, by considering two states A and B, one has $S_A^\circ = S_A$, $S_B^\circ = S_B$:

$$S_B^\circ - S_A^\circ = S_B - S_A = c_p \cdot ln\left(\frac{T_B^\circ}{T_A^\circ}\right) - R \cdot ln\left(\frac{p_B^\circ}{p_A^\circ}\right)$$

The FLT for open systems can be written as follows:

$$Q + L_i = \Delta h^\circ + \Delta E_g + \Delta E_\omega$$

For gases $\Delta E_g = 0$. For a fixed frame of reference, $\Delta E_\omega = 0$.

$$Q + L_i = \Delta h^\circ$$

Stagnation properties

Application: throttling valve

In a fixed, adiabatic duct, we have $Q = 0$ and $L_i = 0$:

$$h^{\circ} = const$$

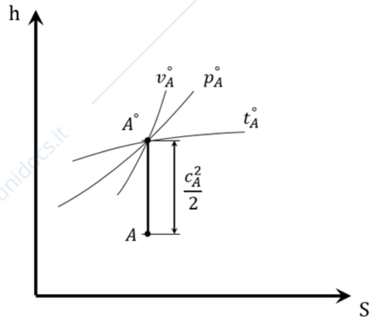
For example, the total enthalpy is conserved when crossing a throttling valve or a fixed blade row.

For a perfect gas, $h^{\circ} = const$ means $T^{\circ} = const$:

$$S_B - S_A = -R \cdot ln\left(\frac{p_B^{\circ}}{p_A^{\circ}}\right)$$

The above equation indicates that, in a fixed, adiabatic duct, any irreversibility effects leads to an increase in entropy, and, thus, a decrease in the stagnation pressure.

For **steam**, stagnation properties must be read on the Mollier diagram by increasing the enthalpy by an amount $^{c^2}/_2$.



Outline

Isentropic compressible flow

Fundamental equations

For an arbitrary control volume enclosed in the pipe, the **continuity equation** states:

$$\rho A c = const.$$

In differential form, and on dividing each term by $\rho A c$, one gets: $$\frac{d\rho}{\rho}+\frac{dA}{A}+\frac{dc}{c}=0 \quad (1)$$

From the conservation of mechanical energy (in differential form): $$0=\frac{dp}{\rho}+cdc \quad (2)$$

$$\left(\frac{dp}{d\rho}\right)_S\frac{d\rho}{\rho}+cdc=0 \longrightarrow \frac{d\rho}{\rho}+\frac{c^2}{c_S^2}\frac{dc}{c}=0 \longrightarrow \frac{d\rho}{\rho}=-Ma^2\frac{dc}{c}=0 \quad (3)$$

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Isentropic compressible flow

Fundamental equations

$$(1)+(3) \longrightarrow \boxed{\frac{dA}{A}=(Ma^2-1)\frac{dc}{c}} \quad (4)$$

$$(2) \longrightarrow \frac{dp}{p}\frac{p}{\rho}+cdc=0 \qquad \frac{dp}{p}\frac{c_S^2}{k}+cdc=0 \qquad \frac{dp}{p}+kMa^2\frac{dc}{c}=0$$

$$\boxed{\frac{dc}{c}=-\frac{1}{kMa^2}\frac{dp}{p}} \quad (5)$$

By combining eq. (4) and eq. (5): $$\boxed{\frac{dA}{A}=\frac{1-Ma^2}{kMa^2}\frac{dp}{p}} \quad (6)$$

Isentropic compressible flow

Nozzles vs diffusers

Based on equations (4) and (6), the following table can be built:

	Ma<1	Ma>1
dA<0	dc>0; dp<0 (nozzle)	dc<0; dp>0 (diffuser)
dA>0	dc<0; dp>0 (diffuser)	dc>0; dp<0 (nozzle)

In particular, we have determined **whether a duct should have a converging, diverging, or converging-diverging shape**:

- to accelerate a fluid flowing subsonically, a converging nozzle must be used, but once Ma=1 is achieved, further acceleration can occur only in a diverging nozzle.

- on the other hand, a converging diffuser is required to decelerate a fluid flowing supersonically, but once Ma=1 is achieved, further deceleration can occur only in a diverging diffuser.

Isentropic compressible flow

Throat

In both cases, a continuous evolution is obtained, and equations (4) and (6) show that a Mach number of unity **can only occur at the location where the cross-sectional area is minimum**. In fact, one has, for a nozzle:

$$\frac{dA}{A} = (Ma^2 - 1)\frac{dc}{c} = 0 \qquad (because\ dA = 0)$$

If $dA = 0$:
- $Ma = 1, dc > 0$ (expansion in the divergent portion of the duct).
- $dc = 0$, $Ma < 1$ (since $Ma < 1$, a compression will occur in the divergent part).

For a continuous expansion (or compression), $dA = 0$ thus means $Ma = 1$.
The location of minimum area is called the throat.

So far, no equation of state has been specified. Thus, the conclusions hold for all gases.
Moreover, although the conclusions have been drawn under the restriction of isentropic flow through nozzles and diffusers, they are **qualitatively valid for actual flows**, because the flow through well-designed nozzles and diffusers is nearly isentropic.

Isentropic compressible flow

Fundamental equations

In what follows, the analytical expressions that **link the area variation to the flow properties** will be derived for a given mass-flow rate, which is isentropically expanding from $Ma < 1$ to $Ma > 1$.
Given the upstream conditions, p_1, ρ_1, c_1, we consider an isentropic expansion from 1° to the downstream pressure p_2. By using the conservation of mechanical energy:

$$\frac{c^2}{2} = -\int_{p_1^0}^{p} \frac{dp}{\rho}$$

For an isentropic expansion of an ideal gas from the stagnation conditions 2° to the generic pressure p:

$$\frac{p}{\rho^k} = \frac{p_1^0}{(\rho_1^0)^k} \quad \Rightarrow \quad \frac{1}{\rho} = \frac{1}{\rho_1^0}\left(\frac{p_1^0}{p}\right)^{\frac{1}{k}}$$

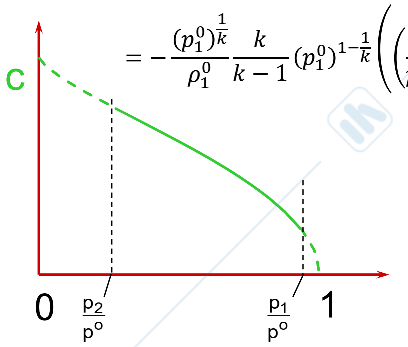
Isentropic compressible flow

Velocity variation along nozzle axis

Solving the integral and making c explicit:

$$\frac{c^2}{2} = -\frac{(p_1^0)^{\frac{1}{k}}}{\rho_1^0}\int_{p_1^0}^{p} p^{-\frac{1}{k}}dp = -\frac{(p_1^0)^{\frac{1}{k}}}{\rho_1^0}\frac{1}{1-\frac{1}{k}}\left[p^{1-\frac{1}{k}}\right]_{p_1^0}^{p} = -\frac{(p_1^0)^{\frac{1}{k}}}{\rho_1^0}\frac{k}{k-1}\left(p^{1-\frac{1}{k}}-(p_1^0)^{1-\frac{1}{k}}\right) =$$

$$= -\frac{(p_1^0)^{\frac{1}{k}}}{\rho_1^0}\frac{k}{k-1}(p_1^0)^{1-\frac{1}{k}}\left(\left(\frac{p}{p_1^0}\right)^{1-\frac{1}{k}}-1\right) = \frac{k}{k-1}\frac{p_1^0}{\rho_1^0}\left(1-\left(\frac{p}{p_1^0}\right)^{\frac{k-1}{k}}\right)$$



$$c = \sqrt{2\frac{k}{k-1}\frac{p_1^0}{\rho_1^0}\left(1-\left(\frac{p}{p_1^0}\right)^{\frac{k-1}{k}}\right)} \quad (7)$$

Isentropic compressible flow

Area variation along nozzle axis

Now, multiplying by ρ and using the equation of state:

$$\rho c = \rho\sqrt{2\frac{k}{k-1}\frac{p_1^0}{\rho_1^0}\left(1-\left(\frac{p}{p_1^0}\right)^{\frac{k-1}{k}}\right)} = \rho_1^0\left(\frac{p}{p_1^0}\right)^{\frac{1}{k}}\sqrt{2\frac{k}{k-1}\frac{p_1^0}{\rho_1^0}\left(1-\left(\frac{p}{p_1^0}\right)^{\frac{k-1}{k}}\right)}$$

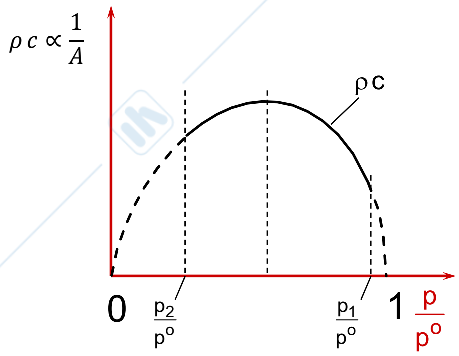
$$\rho c = \sqrt{2\frac{k}{k-1}p_1^0\rho_1^0\left(\left(\frac{p}{p_1^0}\right)^{\frac{2}{k}}-\left(\frac{p}{p_1^0}\right)^{\frac{k+1}{k}}\right)} = \frac{\dot{m}}{A} \quad (8)$$

$$A = \frac{\dot{m}}{\rho c} = \frac{\dot{m}}{\sqrt{2\frac{k}{k-1}p_1^0\rho_1^0\left(\left(\frac{p}{p_1^0}\right)^{\frac{2}{k}}-\left(\frac{p}{p_1^0}\right)^{\frac{k+1}{k}}\right)}} \propto \frac{1}{\rho c} \quad (9)$$

Isentropic compressible flow

Area variation along nozzle axis

Equation (8) gives the value of ρc **as a function of the pressure**. In the nozzle, the flow continuously expands from p_1 to p_2. In each section along the nozzle, a different pressure level is reached, p. The area is then connected quantitatively to the pressure. Note that the nozzle length is not important. The nozzle can be shorter or longer; correspondingly pressure and velocity gradients along it will be higher or lower.



Isentropic compressible flow

Critical pressure ratio

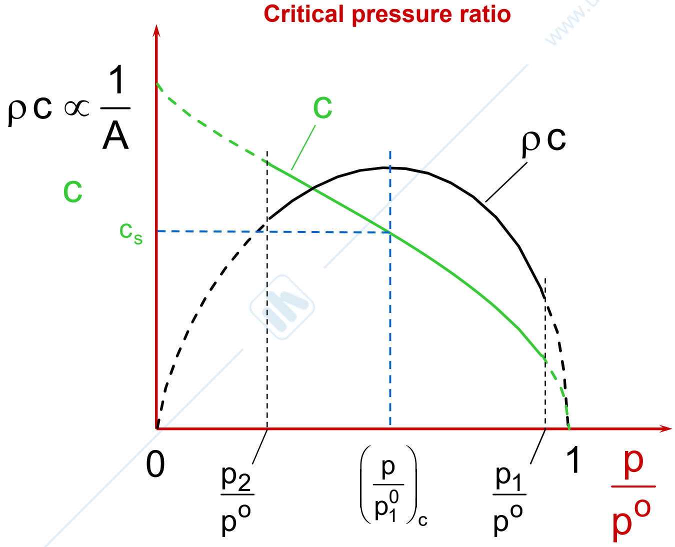
The **throat area can be found** by minimizing eq. (9), i.e. for a fixed flow rate by finding out the maximum of eq. (8):

$$\frac{d(\rho c)}{d(p/p_1^0)} = 0 \quad \Rightarrow \quad \left(\frac{p}{p_1^0}\right)_c = \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}} \qquad (10)$$

For a nozzle expanding from Ma<1 to Ma>1 the flow velocity c equals the local velocity of sound c_s in the throat area, therefore eq. (10) gives the pressure ratio that produces choking (called "**critical pressure ratio**").

Isentropic compressible flow

Critical pressure ratio



Isentropic compressible flow

Properties in the critical section

By replacing eq. (10) in eq. (7) one obtains:

$$c_c = \sqrt{2\frac{k}{k-1}\frac{p_1^0}{\rho_1^0}\left\{1-\left[\left(\frac{p}{p_1^0}\right)_c\right]^{\frac{k-1}{k}}\right\}} = \sqrt{2\frac{k}{k+1}\frac{p_1^0}{\rho_1^0}} = \sqrt{k\frac{p_c}{\rho_c}} = c_{s,c} \qquad (11)$$

$$(\rho c)_c = \rho_c c_c = \rho_1^0\left[\left(\frac{p}{p^o}\right)_c\right]^{\frac{1}{k}}\sqrt{2\frac{k}{k+1}\frac{p_1^0}{\rho_1^0}} = \rho_1^0\left[\left(\frac{2}{k+1}\right)^{\frac{k}{k-1}}\right]^{\frac{1}{k}}\sqrt{2\frac{k}{k+1}\frac{p_1^0}{\rho_1^0}} =$$

$$= \rho_1^0\left(\frac{2}{k+1}\right)^{\frac{1}{k-1}}\sqrt{2\frac{k}{k+1}\frac{p_1^0}{\rho_1^0}}$$

$$(\rho c)_c = \sqrt{p_1^0\rho_1^0\, k\left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}} \qquad (12)$$

Isentropic compressible flow

Summary of critical ratios

$$\left(\frac{p}{p_1^0}\right)_c = \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}} \qquad (10)$$

$$\rho_c = \rho^o\left(\frac{p_c}{p^o}\right)^{\frac{1}{k}} = \rho^o\left(\frac{2}{k+1}\right)^{\frac{1}{k-1}} \qquad (13)$$

$$T_c = T^o\left(\frac{p_c}{p^o}\right)^{\frac{k-1}{k}} = \frac{2}{k+1}\,T^o \qquad (14)$$

For **steam**, eqs. (7)-(13) can be still applied using a suitable average value of k. However, T_c cannot be evaluated from eq. (14) and must be read from steam Mollier diagram, based on p_c and ρ_c.

Outline

- Introduction
- Speed of sound and stagnation properties
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- Design of nozzles
- Off-design performance of convergent nozzles
- Off-design performance of convergent-divergent nozzles
- Effects of viscous dissipation

Design of nozzles

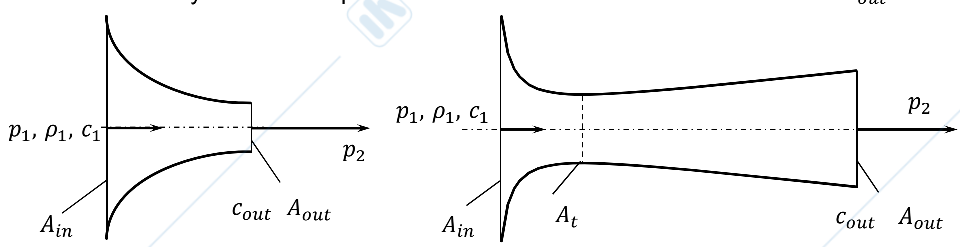
Data and unknowns

Design data:

- mass flow rate: $\dot{m}$
- fluid properties upstream of the nozzle: $p_1, \rho_1, c_1 \Rightarrow p_1^0, \rho_1^0$
- back pressure (in the environment downstream of the nozzle): p_2

Quantities to be evaluated:

- type of nozzle and characteristic nozzle cross sections: A_{in}, A_{out}, A_t
- fluid velocity at the outlet port: c_{out}



Design of nozzles

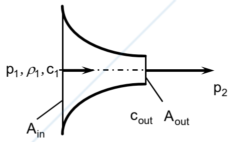
Convergent nozzle

Cross section of the inlet port (independent on the kind of nozzle):

$$A_{in} = \frac{\dot{m}}{\rho_1 c_1} \qquad c_1 \to 0 \implies \begin{cases} \rho_1 \to \rho_1^0 \\ A_{in} \to \infty \end{cases}$$

$$\frac{p_2}{p_1^0} > \left(\frac{p}{p_1^0}\right)_c$$

Subcritical efflux: convergent nozzle



- 1st method:

$$A_{out} = \frac{\dot{m}}{\rho_{out} c_{out}} \quad (9') \qquad c_{out} = \sqrt{2(h_1^0 - h_{out})} \quad (7')$$

Perfect gas: $T_{out} = T_1^0 \left(\frac{p_2}{p_1^0}\right)^{\frac{k-1}{k}}$ $\quad \rho_{out} = \frac{p_2}{RT_{out}}$ $\quad c_{out} = \sqrt{2c_p(T_1^0 - T_{out})}$

Steam: h_{out} and v_{out} from the Mollier chart $\quad c_{out} = \sqrt{2(h_1^0 - h_{out})}$

Design of nozzles

Convergent nozzle

- 2nd method:
  Equation (9) can be applied to the outlet section.
  Let's consider that $p_{out} = p_2$, in fact, when the flow is properly guided, the whole process from p_1^0 to p_2 occurs within the nozzle.

$$A_{out} = \frac{\dot{m}}{\rho_{out} c_{out}} = \frac{\dot{m}}{\sqrt{2\frac{k}{k-1} p_1^0 \rho_1^0 \left[\left(\frac{p_2}{p_1^0}\right)^{\frac{2}{k}} - \left(\frac{p_2}{p_1^0}\right)^{\frac{k+1}{k}}\right]}} \quad (9'')$$

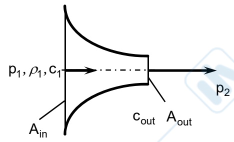
$$c_{out} = \sqrt{2\frac{k}{k-1}\frac{p_1^0}{\rho_1^0}\left[1 - \left(\frac{p_2}{p_1^0}\right)^{\frac{k-1}{k}}\right]} \quad (7'')$$

Design of nozzles

Convergent nozzle

$$\frac{p_2}{p_1^0} = \left(\frac{p}{p_1^0}\right)_c$$

Critical efflux: convergent nozzle



- 1st method: as before
- 2nd method:

$$A_{out} = A_c = \frac{\dot{m}}{\rho_c c_c} = \frac{\dot{m}}{\sqrt{p_1^0 \rho_1^0 \, k \left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}}} \quad (12')$$

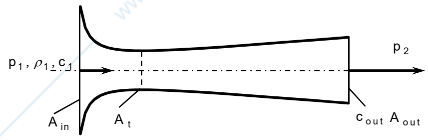
$$c_{out} = c_c = \sqrt{2 \frac{k}{k+1} \frac{p_1^0}{\rho_1^0}} \quad (11')$$

Design of nozzles

Convergent – divergent nozzle

$$\frac{p_2}{p_1^0} < \left(\frac{p}{p_1^0}\right)_c$$

Critical efflux: convergent-divergent nozzle



- 1st method:

$$A_{out} = \frac{\dot{m}}{\rho_{out} c_{out}} \qquad c_{out} = \sqrt{2(h_1^0 - h_{out})}$$

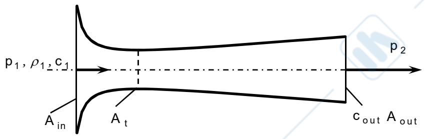
$$p_c = p_1^0 \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}} \quad => \quad A_t = \frac{\dot{m}}{\rho_c c_c} \qquad c_c = \sqrt{2(h_1^0 - h_c)}$$

Design of nozzles

Convergent – divergent nozzle

$$\frac{p_2}{p_1^0} < \left(\frac{p}{p_1^0}\right)_c$$

Critical efflux: convergent-divergent nozzle



- 2nd method:

$$A_{out} = \frac{\dot{m}}{\sqrt{2\frac{k}{k-1}p_1^0\rho_1^0\left[\left(\frac{p_2}{p_1^0}\right)^{\frac{2}{k}} - \left(\frac{p_2}{p_1^0}\right)^{\frac{k+1}{k}}\right]}}$$

$$c_{out} = \sqrt{2\frac{k}{k-1}\frac{p_1^0}{\rho_1^0}\left(1-\left(\frac{p_2}{p_1^0}\right)^{\frac{k-1}{k}}\right)}$$

$$A_t = A_c = \frac{\dot{m}}{\rho_c c_c} = \frac{\dot{m}}{\sqrt{p_1^0\rho_1^0\, k\left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}}}$$

$$c_t = c_c = \sqrt{2\frac{k}{k+1}\frac{p_1^0}{\rho_1^0}}$$

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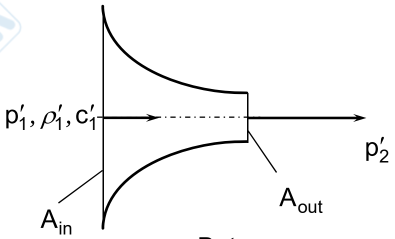
Off-design: convergent nozzles

Data and unknowns



Data:

- characteristic nozzle cross sections: A_{in}, A_{out}
- fluid properties upstream of nozzle: $p'_1, \rho'_1, c'_1 \Rightarrow p'^0_1, \rho'^0_1$
- back pressure (in the environment downstream from nozzle): p'_2

Quantities to be evaluated:

- mass flow rate: $\dot{m}'$
- fluid velocity at outlet port: c'_{out}

Off-design: convergent nozzles

Downstream pressure change

Consider a convergent nozzle with a given area A_{out}, and fixed upstream conditions p_1^0, ρ_1^0. Let us evaluate the mass flow rate for **different values of the downstream pressure, p_2**:

- When $p_2 = p_1$, there is no flow and $\dot{m} = 0$. The pressure is constant along the nozzle. If p_2 is decreased, a flow appears through the nozzle.
- As long as the flow is subsonic at the exit ($p_2 > p_c$), information about changing conditions in the exhaust region can be transmitted upstream. Decreases in the backpressure thus result in greater mass-flow rates and new pressure variations within the nozzle. In each instance, the velocity is subsonic throughout the nozzle and the exit pressure equals the backpressure.
- For $p_2 = p_c$, the flow will expand up to Ma=1 in the outlet section.
- For $p_2 < p_c$, since $c = c_s$ in the outlet section, information about the decrease in p_2 can no longer be transmitted upstream past the exit plane. Accordingly, further reduction in p_2 have no effect on flow conditions in the nozzle. Neither the pressure variation within the nozzle nor the mass flow rate is affected. Under these conditions, the nozzle is said to be chocked.

Off-design: convergent nozzles

Mass flow rate

When a **nozzle is chocked**, $\dot{m}$ is the maximum possible mass flow rate for the given stagnation conditions.
For $p_2 < p_c$, the flow expands outside the nozzle to match the lower backpressure. The pressure variation outside the nozzle cannot be predicted using the one-dimensional flow model.

Concerning the **mass flow rate**, when $p_2 \geq p_c$ (then $p_{out} = p_2$):

$$\dot{m} = A_{out}(\rho c)_{out} = A_{out}\sqrt{2\frac{k}{k-1}p_1^0\rho_1^0\left[\left(\frac{p_2}{p_1^0}\right)^{\frac{2}{k}} - \left(\frac{p_2}{p_1^0}\right)^{\frac{k+1}{k}}\right]}$$

That is:

$$\dot{m} = A_{out}\frac{p_1^0}{\sqrt{p_1^0 v_1^0}}\sqrt{2\frac{k}{k-1}\left[\left(\frac{p_2}{p_1^0}\right)^{\frac{2}{k}} - \left(\frac{p_2}{p_1^0}\right)^{\frac{k+1}{k}}\right]} \qquad (15)$$

Off-design: convergent nozzles

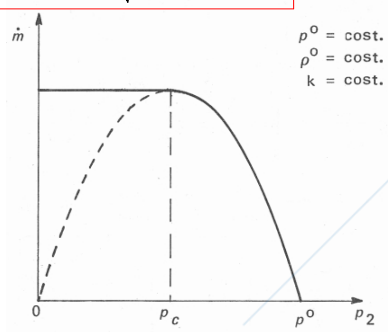
Mass flow rate

When $p_2 = p_c$, eq. (15) can still be used, by considering the p_c value.
Alternatively, one can use eq. (12) to express $(\rho c)_c$ and thus:

$$\dot{m}_c = A_{out}(\rho c)_c = A_{out}\frac{p_1^0}{\sqrt{p_1^0 v_1^0}}\sqrt{k\left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}} = A_{out}\frac{p_1^0}{\sqrt{p_1^0 v_1^0}}\cdot\Gamma(k) \qquad (16)$$

Eq. (16) holds when $p_2 < p_c$, because nothing can change inside the nozzle. The mass flow rate is thus the same as for the $p_2 = p_c$ case.

After having collected this information, one can **plot $\dot{m}$ as a function of the backpressure p_2**, for a given fluid and upstream stagnation conditions.

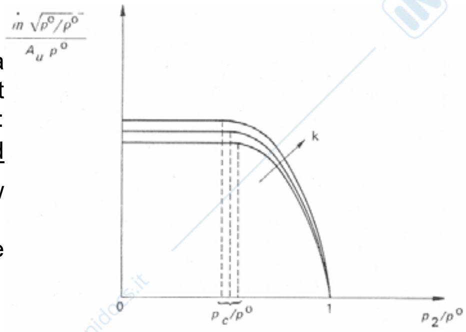


Off-design: convergent nozzles

Characteristic diagram

In both equations (15) and (16) the same term can be collected as a pre-multiplying factor.

This suggests that we can plot a "**characteristic diagram**" for the convergent nozzle in terms of normalized quantities: nozzle expansion ratio $\frac{p_2}{p_1^0}$ and corrected mass flow rate (or "flow capacity" or "flow parameter"), $\frac{\dot{m}\sqrt{p_1^0 v_1^0}}{p_1^0 A_{out}}$. This diagram is more general than the previous one.



It is worth noting that, whenever upstream conditions are **changed in a way such that** $\boldsymbol{h_1^0 = const}$, the chocked nozzle mass flow rate is proportional to p_1^0. This can be easily shown for a perfect gas, because $p_1^0 v_1^0 = RT_1^0$, whereas in the case of steam one can use the Mollier chart to verify that, when $h = const$, the product pv keeps almost unchanged.

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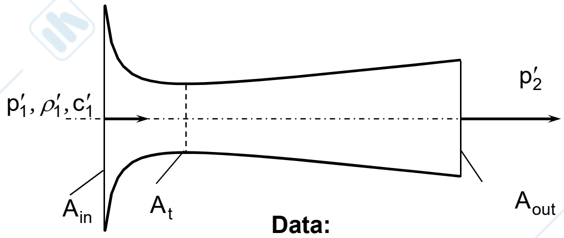
Off-design: De Laval nozzles

Data and unknowns



Data:

- characteristic nozzle cross sections: A_{in}, A_t, A_{out}
- fluid properties upstream of nozzle: $p'_1, \rho'_1, c'_1 \Rightarrow p_1'^0, \rho_1'^0$
- back pressure (in the environment downstream from nozzle): p'_2

Quantities to be evaluated:

- mass flow rate: $\dot{m}'$
- fluid velocity at throat and outlet port: c'_t, c'_{out}

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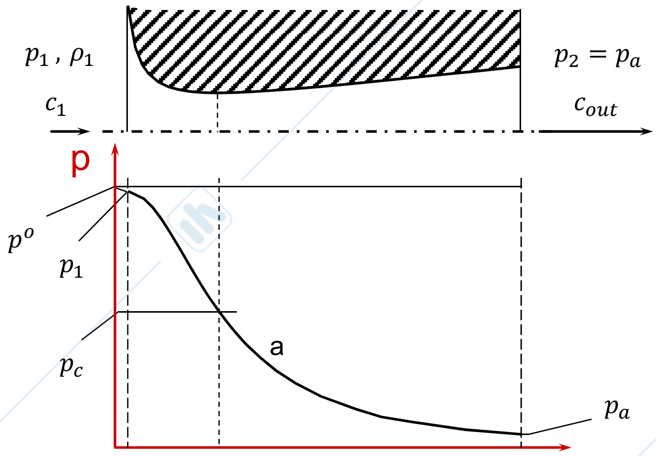
Off-design: De Laval nozzles

Design conditions

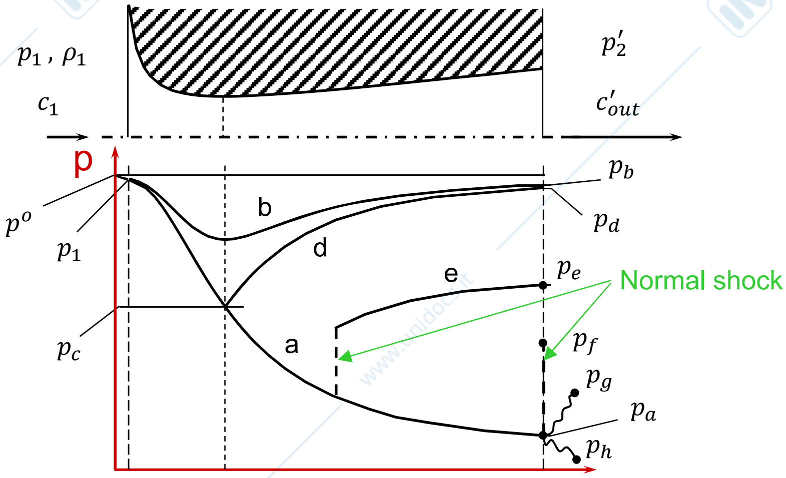
Under design conditions:



Off-design: De Laval nozzles

Off-design conditions

When p_2' is changed with respect to design conditions:



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Off-design: De Laval nozzles

Discharge pressures

- $p_2' = p°_1$: there is no flow across the nozzle.
- p_2' > p_d (see for instance p_b): <u>subsonic flow</u>, the nozzle behaves like a Venturi duct (maximum velocity and lowest pressure in the throat) and an isentropic subsonic diffusion occurs in the divergent part. If the back pressure is reduced, then the mass flow rate and velocity get bigger.
- $p_2' = p_d$ is the higher pressure for which the flow becomes sonic in the throat section. The flow is still isentropic everywhere.
- p_d > p_2' > p_a (see for instance p_e): the flow is <u>sonic in the throat</u>, therefore the nozzle is chocked. However, a <u>shock</u> occurs in the divergent duct. In fact, the duct over-expands the gas so that before the gas can discharge into the surroundings some recompression and deceleration of it must occur. Across the shock the flow is no more isentropic and becomes subsonic downstream of it. By reducing the back pressure, the <u>location of the shock wave moves downstream</u>.

Off-design: De Laval nozzles

Discharge pressures

- $p_2' = p_f$: the shock occurs at the exit section: from now on, the nozzle will be always internally isentropic (even though the whole expansion cannot be considered isentropic).
- $p_f > p_2' > p_a$ (see for instance p_g): the flow is still over-expanded, and the adjustment of the pressure to the downstream value occurs through recompression oblique shocks outside the nozzle.
- $p_2' = p_a$: the back-pressure is exactly the discharge pressure achieved by the nozzle: no shockwaves occur, neither within nor outside the duct; the whole expansion can be considered isentropic.
- $p_2' < p_a$ (see for instance p_h): the fluid expands isentropically through the nozzle and then, since it comes out to be under-expanded, further expansion occurs outside the nozzle to the back pressure, through oblique expansion waves (i.e., not isentropically).

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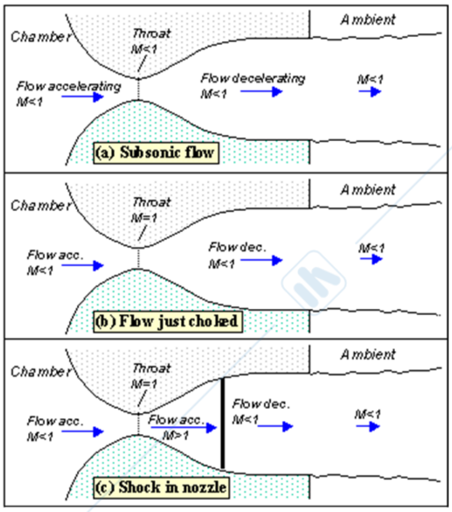
Off-design: De Laval nozzles

Flow patterns in a De Laval nozzle



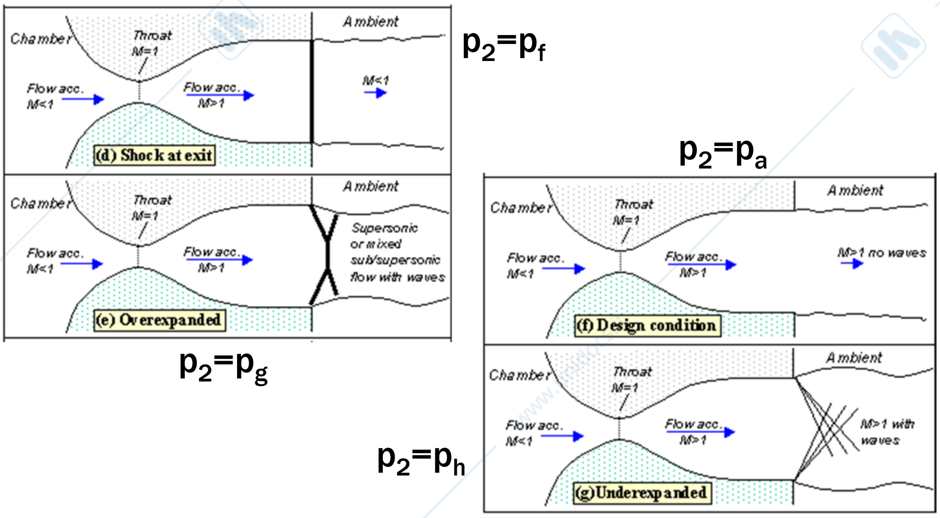
$p_2 = p_b$

$p_2 = p_d$

$p_2 = p_e$

Off-design: De Laval nozzles

Flow patterns in a De Laval nozzle



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Off-design: De Laval nozzles

Review

- a De Laval nozzle is **chocked** when $\frac{p_2'}{p_1'^0} \le \left(\frac{p}{p_1^0}\right)_d$:
  in this case, the maximum mass flow rate has been attained for the given stagnation conditions; further reductions in the back pressure cannot result in an increase in the mass flow rate. They can only affect the flow evolution in the diverging part, i.e. introducing shock waves within or outside the nozzle.

- when $\left(\frac{p}{p_1^0}\right)_f \le \frac{p_2'}{p_1'^0} \le \left(\frac{p}{p_1^0}\right)_d$
  shock waves occur in the divergent part of a De Laval nozzle; the flow is still isentropic in the convergent part (as it always is), but it is not in the divergent one.

- when $\frac{p_2'}{p_1'^0} \le \left(\frac{p}{p_1^0}\right)_f$
  shock waves occur outside the nozzle (except for $p_2' = p_a$: design backpressure) and it can be considered internally isentropic; the pressure variations induced by these oblique waves cannot be predicted using a one-dimensional flow model.

Off-design: De Laval nozzles

Characteristic diagram

It is useful to express the nozzle performance in terms of **non-dimensional parameters**: the expansion ratio and the reduced mass flow rate:

$$\frac{p_2'}{p_1'^0} \geq \left(\frac{p}{p_1^0}\right)_d \quad \Rightarrow \quad \frac{\dot{m}'\sqrt{p_1'^0/\rho_1'^0}}{p_1'^0 A_t} = \frac{A_{out}}{A_t}\sqrt{2\frac{k}{k-1}\left[\left(\frac{p_2'}{p_1'^0}\right)^{\frac{2}{k}} - \left(\frac{p_2'}{p_1'^0}\right)^{\frac{k+1}{k}}\right]}$$

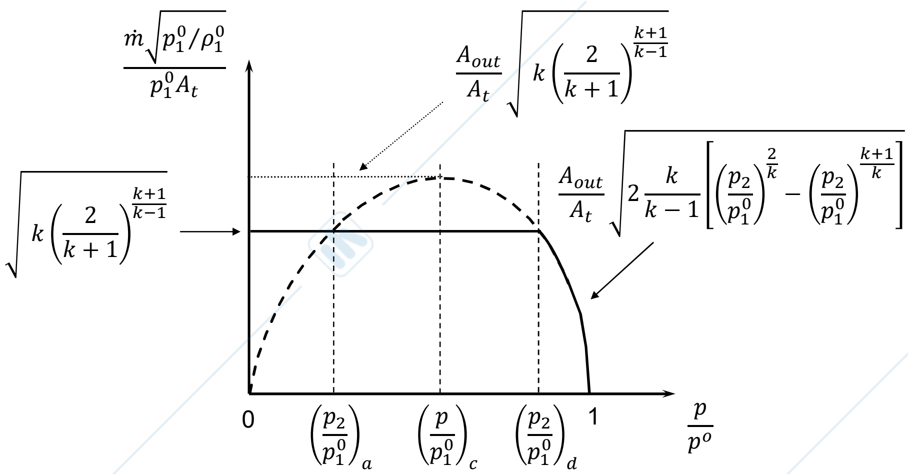
Subsonic nozzle

$$\frac{p_2'}{p_1'^0} \geq \left(\frac{p}{p_1^0}\right)_d \quad \Rightarrow \quad \frac{\dot{m}'\sqrt{p_1'^0/\rho_1'^0}}{p_1'^0 A_t} = \sqrt{k\left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}}$$

Chocked nozzle

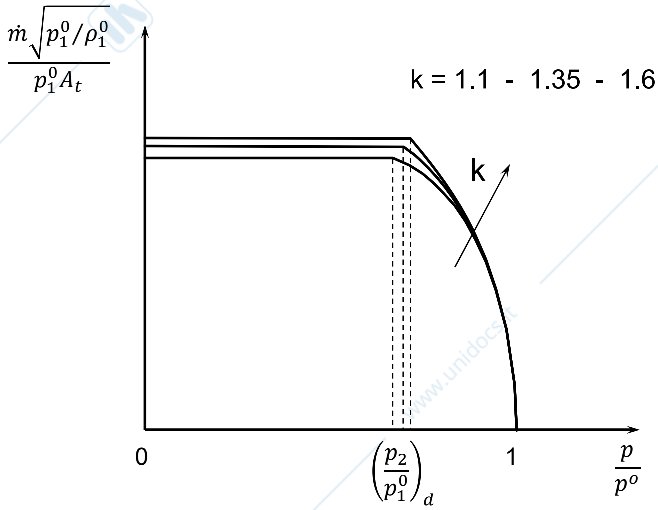
Off-design: De Laval nozzles

Characteristic diagram



Off-design: De Laval nozzles

Characteristic diagram



Outline

- Introduction
- Speed of sound and stagnation properties
- Isentropic compressible flow in nozzles and diffusers (steady state flow)
- Design of nozzles
- Off-design performance of convergent nozzles
- Off-design performance of convergent-divergent nozzles
- **Effects of viscous dissipation**

Effects of viscous dissipation

Description of the phenomenon

- Formation of a **boundary layer**, so that the values of the cross section to use in the previously developed equations are not equal to the geometrical ones. In a turbomachine, such an effect can be included in a coefficient that also accounts for blade thickness:

$$\xi = \frac{A}{A_{geom.}}$$

In a turbomachine, ξ is the ratio of the effective and geometric flow areas, and is therefore a measure of the aerodynamic blockage due to blade surface and endwall boundary layers.

- The process is **no more isentropic**: this implies a reduction of the velocity in the outlet port of a nozzle and a lower pressure increase in a diffuser with respect to the isentropic case. This effect can be accounted by introducing:
  - Loss coefficients to account for kinetic energy reduction at the nozzle outlet, with respect to the isentropic case;
  - Isentropic efficiency of nozzle and diffuser.

Effects of viscous dissipation

Loss coefficient for a nozzle

$$\varphi = \frac{c_{out}}{c_{out,is}}$$

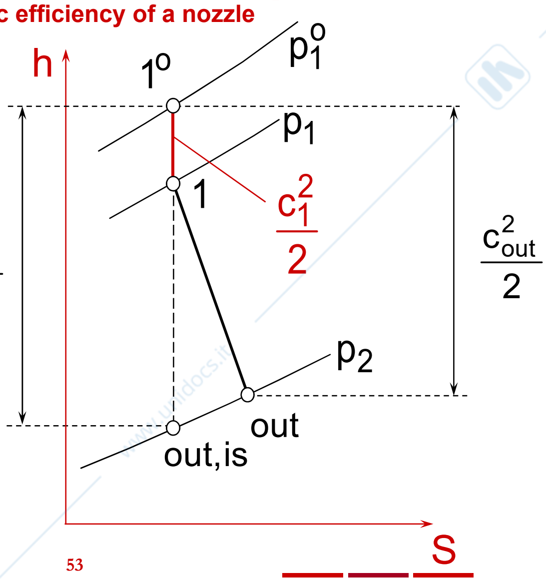
Under design conditions, **the value of** φ might range from 0.97-0.99 for a convergent nozzle and from 0.94-0.97 in a convergent-divergent nozzle.

$$\varphi = \frac{c_{out}}{\sqrt{2\frac{k}{k-1}\frac{p_1^0}{\rho_1^0}\left[1-\left(\frac{p_2}{p_1^0}\right)^{\frac{k-1}{k}}\right]}}$$

Effects of viscous dissipation

Isentropic efficiency of a nozzle

$$\eta_{is,n} = \frac{c_{out}^2 - c_1^2}{c_{out,is}^2 - c_1^2}$$



For the 1st law of thermodynamics:

$$h_1^0 = h_{out}^0 = h_{out,is}^0$$

$$\eta_{is,n} = \frac{h_1 - h_{out}}{h_1 - h_{out,is}}$$

Effects of viscous dissipation

Isentropic efficiency of a diffuser

Is it possible to adopt for the diffuser the same definition as for the nozzle?

First of all, it does make sense that the **pressure levels p_1 and p_2 are the same** in the actual and in the ideal process. In fact they are boundary conditions for the diffuser.

For an **irreversible process** with $p_2 > p_1$, the isentropic enthalpy drop is lower than the ideal one:

$$\eta_{is,d} = \frac{h_{2,is} - h_1}{h_2 - h_1}$$

$h_{2,is} < h_2$: this means that, for a given kinetic energy at the outlet, a lower inlet total enthalpy is required in the isentropic case, in order to reach the same pressure level:

$$h_{2,is} + \frac{c_2^2}{2} < h_2 + \frac{c_2^2}{2}$$

Effects of viscous dissipation

Isentropic efficiency of a diffuser

$$\eta_d = \frac{h_{2,is} - h_1}{h_2 - h_1} = \frac{\Delta h_{is}}{\Delta h}$$

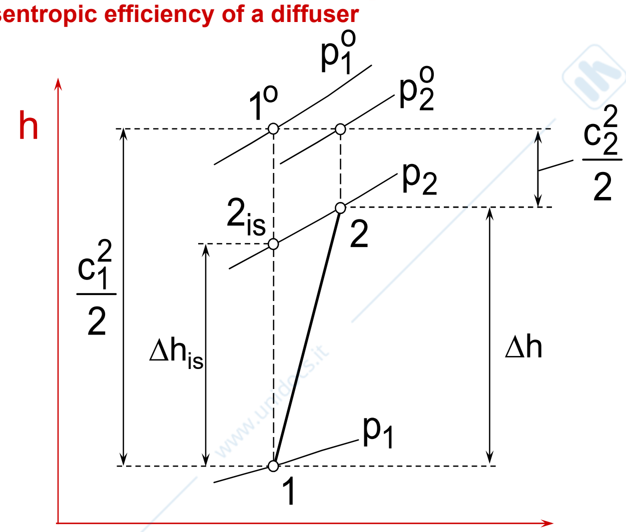
$$h_1 + \frac{c_1^2}{2} = h_2 + \frac{c_2^2}{2}$$

$$h_2 - h_1 = \frac{c_1^2 - c_2^2}{2}$$

$$h_{2,is} - h_1 = \eta_d \frac{c_1^2 - c_2^2}{2}$$

If $\frac{c_2^2}{2} \sim 0$

$$\Delta h_{is} = h_{2,is} - h_1 = \eta_d \frac{c_1^2}{2}$$



Gas and steam turbines

Prof. Mirko Baratta

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Outline

- **Components**
- **Flow losses**
- **1-D analysis of the flow in a turbine stage**
- **Stage and Turbine efficiency**
- **Degree of reaction**
- **Stage loading and flow coefficient**

Components

A **turbine** is a **motor machine** made up of fixed and rotating blade rows. The working fluid (gas, steam) expands across the blade passages and transfers work to the walls of rotating blade passages.

Single and multiple-stage turbines are present in engineering applications. A **turbine stage** features two essential parts: a fixed, swirl generating component (stator) in which the working fluid is expanded and turned to give it a circumferential velocity along the axis of the machine (tangential velocity component), and a rotor, through which the flow passes and in doing so does work. Turbine stages can be either axial or radial.

An **action** or **impulse stage** is characterized by rotor blades moving in an environment featuring the same pressure value both upstream from and downstream of the rotating row. Therefore, the working fluid expansion occurs in the stator blades only. When the pressure downstream of rotor blades is lower than the pressure upstream from them, the stage is called **reaction stage**.

Outline

- **Components**
- **Flow losses**
- **1-D analysis of the flow in a turbine stage**
- **Stage and Turbine efficiency**
- **Degree of reaction**
- **Stage loading and flow coefficient**

Flow losses

Meanline analysis

Any turbine design or optimization study must inevitably include some means of accounting for the **losses in the flow**. While it is correct to say that losses can be calculated by means of viscous, three-dimensional CFD analysis today, that approach has a number of drawbacks.

The main problem is the **place of CFD in the design cycle**. The principal geometric parameters, such as radii and blade heights, are determined early in the cycle from a 1-D, meanline analysis. At this point the precise blade shapes are unknown, and until they are determined at a later stage in the cycle a CFD analysis cannot be made.

There is, therefore, a need for the **meanline analysis** that incorporates its own system of aerodynamic losses expressed as functions of the blade row inlet and exit velocity triangles and only the overall geometric features.

Flow losses

Meanline analysis

The approach to develop such a meanline loss system is, therefore, to **identify the various loss-generating processes** which take place in the flow field. Even this implies a considerable simplification, because there is no way in which this can be done completely unambiguously.

All of the sources of loss are **interrelated and interact**, often in quite fundamental ways, and any division is inevitably artificial. Nevertheless, with some understanding of the physical processes which control the blade passage flow, a reasonable and useful division is possible.

Flow losses

Aerodynamic losses

1) The **profile loss** is the loss due to the skin friction on the blade surface, and thus it depends on the area of the blade in contact with the fluid, the surface finish, and the Reynolds number and the Mach number of the flow through the passage. All of these effects are governed by the geometry of the airfoil.

2) The **annulus losses** are similarly caused by friction on the endwall surfaces.

A division between profile and annulus losses is maintained because the **boundary layer** growth and development tends to be quite different. A fresh blade boundary layer always grows from the leading edge, whereas the annulus boundary layer may have its origin some way upstream of the leading edge, depending on the details of the annulus itself.

Flow losses

Velocity coefficients

The main way to account for the profile losses is to introduce the **velocity coefficients**, that consider the decrease in output velocity with respect to the ideal case.

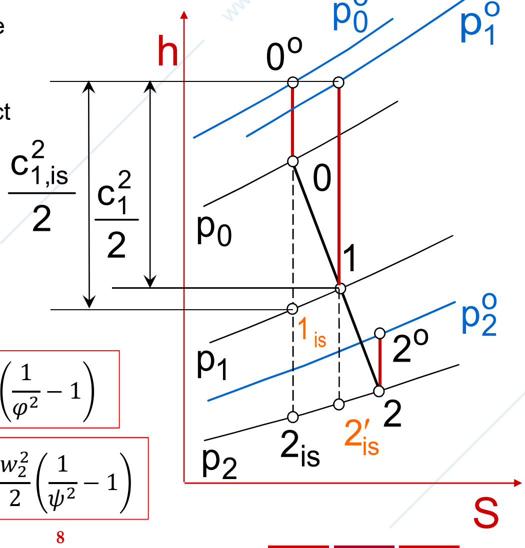
$$stator:\ \varphi = \frac{c_1}{c_{1,is}}$$

$$rotor:\ \psi = \frac{w_2}{w_{2',is}}$$

Moreover, **from the FLT**:

$$h_1 = h_{1,is} + \left(\frac{c_{1,is}^2}{2} - \frac{c_1^2}{2}\right) = h_{1,is} + \frac{c_1^2}{2}\left(\frac{1}{\varphi^2} - 1\right)$$

$$h_2 = h_{2',is} + \left(\frac{w_{2',is}^2}{2} - \frac{w_2^2}{2}\right) = h_{2',is} + \frac{w_2^2}{2}\left(\frac{1}{\psi^2} - 1\right)$$

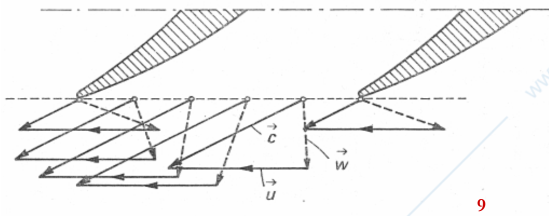


Flow losses

Flow impingement

The velocity coefficients also take into account **flow impingement**. This is an additional source of loss, due to the flow striking on the rotating blades, that typically arises when the incidence is not zero: the flow, not well guided by the blade, starts some kind of recirculation that produces dissipation. This phenomenon usually affects just the ψ coefficient and makes it lower than φ.

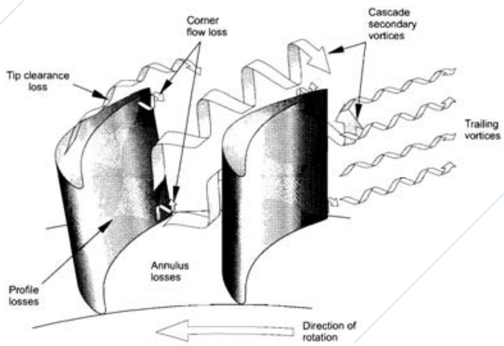
The flow impingement is both due to the **thickness of the blades** and to the **presence of a boundary layer** inside the channel between two blades. This last effect, in particular, leads to a not uniform distribution of absolute velocity, that in turn implies a significant spatial variation of the direction of relative velocity, that is the basis for impingement in rotating rows, as well as in fixed rows that are placed right downstream of a rotor (e.g. in the case of a multistage turbine). It follows that the blade row flow coefficients has different values according to the row position: it ranges between 0.94 and 0.98 for the stator of a single stage turbine or the first stage of a multistage one, and between 0.90 and 0.94 for rotor and for any stator of intermediate stages.



Flow losses

Aerodynamic losses

3) The third source of aerodynamic loss is **secondary flow**. Secondary flows are vortices that occur as a result of the boundary layers and the curvature of the passage, and cause some parts of the fluid to move in directions other than the principal direction of flow. At the trailing edge of the blade the flow separates, forming a sheet of trailing vortices which make up the wake. The momentum deficit of the wake and the kinetic energy bound up in the trailing vortices are sources of loss.



Flow losses

Flow leakages and discharge kinetic energy

4) Tip clearance losses occur in rotors. Some fluid leaks in the gap between the blade tip and the shroud, and therefore contributes little or not at all to expansion work. The **leakage flow** under the differential shear of the moving blade and the stationary casing forms a tip leakage vortex which is driven into the passage by the pressure difference between the pressure and suction surfaces of the blades, where it interacts with the corner and passage vortices to form very complex flow patterns.

5) The last source of loss considered here is the **kinetic energy discharge**. If the flow that leaves the row keeps inside a consistent amount of kinetic energy, this must be considered a loss, since clearly it could not be transformed into mechanical work. As a matter of fact, this actually happens if the stage is the only one or the last one of the machine; otherwise, this energy would be recovered, at least partially.

Outline

- **Components**
- **Flow losses**
- **1-D analysis of the flow in a turbine stage**
- **Stage and Turbine efficiency**
- **Degree of reaction**
- **Stage loading and flow coefficient**

1-D Flow analysis in a turbine stage

Data input

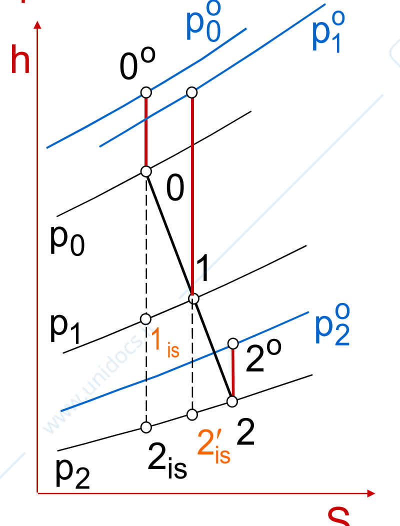
Input data:

$p_0,\ t_0,\ c_0 \quad \Rightarrow \quad p_0^O,\ t_0^O$

$p_1,\ \varphi,\ \alpha_1$

$p_2,\ \psi,\ \beta_2$

$d_1,\ d_2,\ l_1,\ \xi_1,\ \xi_2,\ n$



1-D Flow analysis in a turbine stage

Stator outlet - Rotor inlet

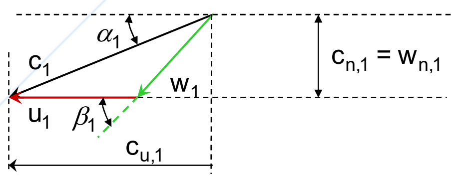
$$c_{1,is} = \sqrt{2(h_0^O - \underline{h_{1,is}})} \quad \xrightarrow{perfect\ gas} \quad c_{1,is} = \sqrt{2c_p(T_0^O - T_{1,is})} \qquad T_{1,is} = T_0^O\left(\frac{p_1}{p_0^O}\right)^{\frac{k-1}{k}}$$

$$c_1 = \varphi c_{1,is}$$

$$u_1 = \pi\, d_1\, n$$

$$w_1 = \sqrt{u_1^2 + c_1^2 - 2c_1 u_1 cos\alpha_1}$$

$$\beta_1 = arcsin\left(\frac{c_1 sin\alpha_1}{w_1}\right)$$



$$c_{u,1} = c_1 cos\alpha_1 \qquad c_{n,1} = c_1 sin\alpha_1 = w_{n,1} = w_1 sin\beta_1$$

$$\dot{m} = \frac{\xi_1\, \pi\, d_1\, l_1 c_{n,1}}{\underline{v_1}}$$

For steam, quantities underlined with —— should be read from steam Mollier diagram.

$$h_1 = h_0^O - \frac{c_1^2}{2} \quad \xrightarrow{perfect\ gas} \quad T_1 = T_0^O - \frac{c_1^2}{2c_p} \qquad v_1 = \frac{RT_1}{p_1}$$

1-D Flow analysis in a turbine stage

Rotor outlet

$u_2 = \pi\, d_2\, n$

$$w_{2',is} = \sqrt{2\left(h_1 - \underline{h_{2',is}}\right) + w_1^2 + u_2^2 - u_1^2} \xrightarrow{perfect\ gas} w_{2',is} = \sqrt{2c_p\left(T_1 - T_{2',is}\right) + w_1^2 + u_2^2 - u_1^2}$$

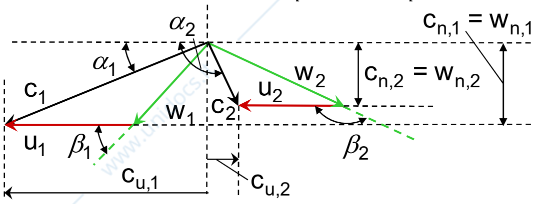
$w_2 = \psi w_{2',is}$

$$T_{2',is} = T_1 \left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}}$$

$c_{n,2} = w_{n,2} = w_2 sin\beta_2$

$$h_2 = h_1 - \frac{w_2^2 - w_1^2}{2} + \frac{u_2^2 - u_1^2}{2} \xrightarrow{perfect\ gas} T_2 = T_1 - \frac{w_2^2 - w_1^2}{2c_p} + \frac{u_2^2 - u_1^2}{2c_p}$$

$$l_2 = \frac{\dot m \underline{v_2}}{\xi_2\, \pi\, d_2\, c_{n,2}} \qquad v_2 = \frac{RT_2}{p_2}$$



$$c_2 = \sqrt{u_2^2 + w_2^2 - 2w_2u_2cos(\pi - \beta_2)}$$

$$\alpha_2 = \pi - asin\left(\frac{c_{n,2}}{c_2}\right)$$

$$c_{u,2} = c_2 cos\alpha_2$$

For steam, quantities underlined with —— should be read from steam Mollier diagram.

1-D Flow analysis in a turbine stage

Specific work and power

Specific work (L_i) and power output ($\dot{\mathfrak{L}}_i$ or P_i):

$L_i = u_1 c_{u,1} - u_2 c_{u,2}$ from Euler equation

$-L_i = h_2 - h_1 + \frac{c_2^2 - c_1^2}{2}$ from energy conservation

The latter equation can be re-arranged by replacing the change in enthalpy with the expression derived from the application of energy conservation to the rotor, in a reference system relative to the rotor:

$$0 = h_2 - h_1 + \frac{w_2^2 - w_1^2}{2} - \frac{u_2^2 - u_1^2}{2} \Rightarrow h_1 - h_2 = \frac{w_2^2 - w_1^2}{2} - \frac{u_2^2 - u_1^2}{2}$$

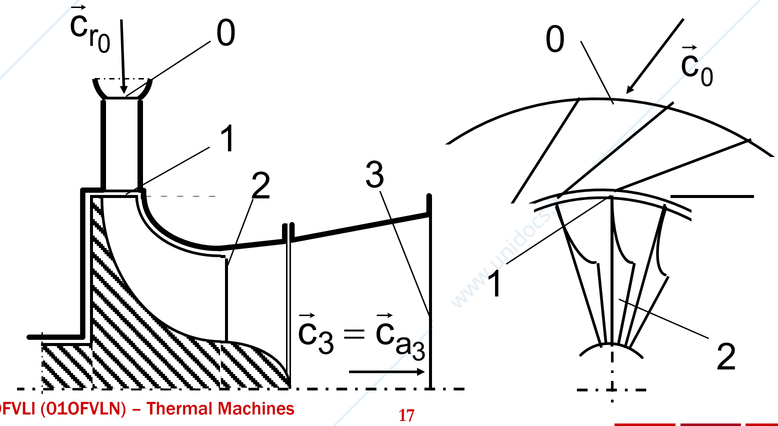
$$L_i = \frac{w_2^2 - w_1^2}{2} - \frac{c_2^2 - c_1^2}{2} - \frac{u_2^2 - u_1^2}{2} \qquad (1)$$

$$\dot L_i = P_i = \dot m L_i$$

1-D Flow analysis in a turbine stage

Centripetal effect

Equation (1) shows clearly the contribution made to the work output by the **change in blade speed u**, and hence radius, in a radial centripetal turbine, where the fluid enters the rotor in the radial inward direction, is turned in the axial-radial plane, and leaves in the axial direction.



1-D Flow analysis in a turbine stage

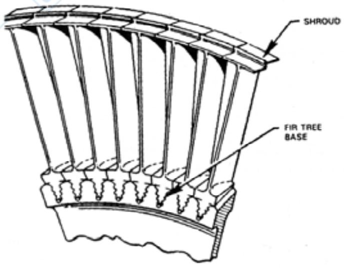
Axial vs radial turbines

In an **axial stage**, u is approximately constant, and there is no significant contribution. However, in an axial stage the radius is not always constant across the stage. It often increases from the leading to the trailing edge in order to increase the area ratio across the blade (thereby increasing the expansion of the flow), and this must be taken into account. Nevertheless, in an axial stage the enthalpy change due to the increase in blade speed is normally very much smaller than that due to the increase in relative velocity w thorough the blade row.

From this consideration, it can be seen that a **radial turbine stage** can deliver a greater specific power than an equivalent axial stage. The reason for selecting a radial turbine in preference to an axial turbine may be the particular installation requirements, that a compact power plant is required or that a radial turbine rotor can be manufactured as a one-piece casting whereas an axial rotor often demands separated blades and disk.

1-D Flow analysis in a turbine stage

Maximizing the work output



Rotor of an axial stage

Equation (1) also shows several other design that are necessary in order to achieve a high specific work output L_i. The **relative velocity term** is additive, and so it must be arranged so that $w_2 > w_1$ so that this term makes a net positive contribution to the work output. The **absolute velocity term** is subtracted. The stator should therefore be designed to accelerate the flow at inlet in order to maximize the stator exit velocity and hence the rotor inlet velocity c_1, thus resulting in $c_1 > c_2$.

1-D Flow analysis in a turbine stage

Maximizing the work output

There is also a clear **connection between the velocity triangles and the Euler turbomachine equation**. It follows from this that a high level of specific work output can be achieved if $c_{u,1}$ is large and $c_{u,2}$ is small. The rotor inlet velocity triangle shows that this is achieved by turning the flow in the stator, so that the exit flow angle α_1 is small. In axial stages, values of α_1 of 15-25 deg are typical, and α_1 may be lower than 10 deg in some impulse turbines. Conversely, the rotor is arranged so that the resultant exit $c_{u,2}$ is fairly small, therefore β_2 should be set in order to have α_2 virtually equal to 90 deg.

Theoretically, equation (1) shows that specific work can be increased if **$c_{u,2}$ is negative**, and this is often a choice in design conditions. However, large values of exit swirl, either positive or negative, determine an increase in the discharge kinetic energy, which might be difficult to be recovered due to its high tangential component.

Outline

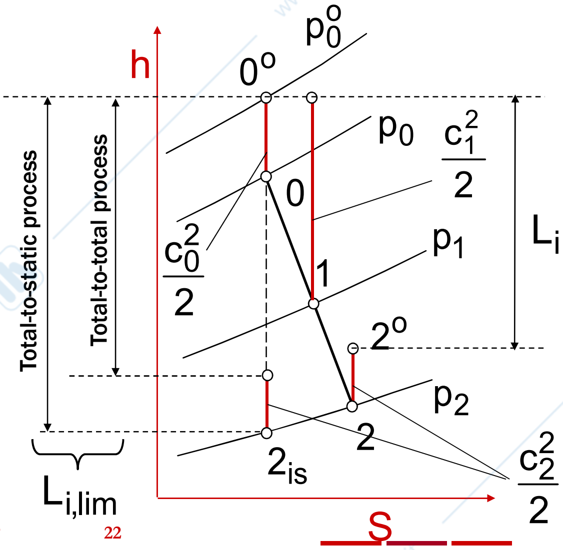
- Components
- Aerodynamic losses
- 1-D analysis of the flow in a turbine stage
- **Stage and Turbine efficiency**
- Degree of reaction
- Stage loading and flow coefficient

Stage and turbine efficiency

Processes comparison

The **isentropic efficiency** of the turbine stage is defined as the ratio of the actual work output to that of an equivalent isentropic process.

Depending on the application, this latter can either be considered as **a total-to-total** process or **a total-to-static** one.



Stage and turbine efficiency

Total2Static and Total2Total efficiencies

Total-to-static efficiency:
the ideal machine has no kinetic energy discharge

$$\eta_\theta = \frac{L_i}{L^\theta_{i.lim}} = \frac{h^O_0 - h^O_2}{h^O_0 - h_{2,is}}$$

It is a measure of the stage capability to convert flow energy into mechanical work.
Single stage turbine or last stage of a multistage turbine: wasted kinetic energy output.

Total-to-total efficiency:
the ideal machine has the same kinetic energy discharge as the real machine

$$\eta_\Theta = \frac{L_i}{L^\Theta_{i,lim}} = \frac{h^O_0 - h^O_2}{h^O_0 - h_{2,is} - \frac{c_2^2}{2}}$$

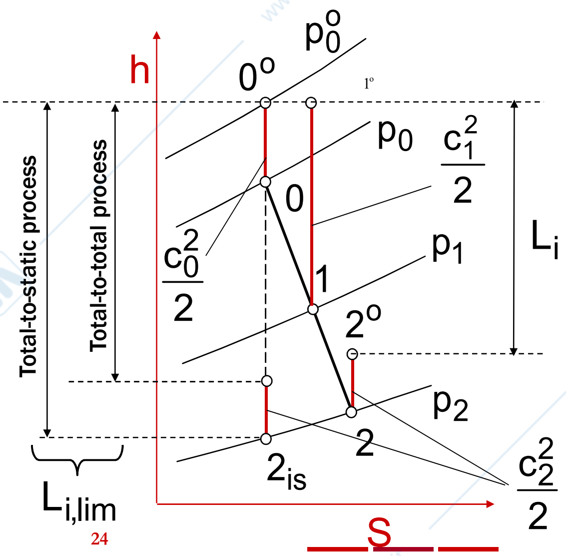
It is a measure of the fluid dynamic quality of the stage.

Stage and turbine efficiency

Total2Static and Total2Total efficiencies

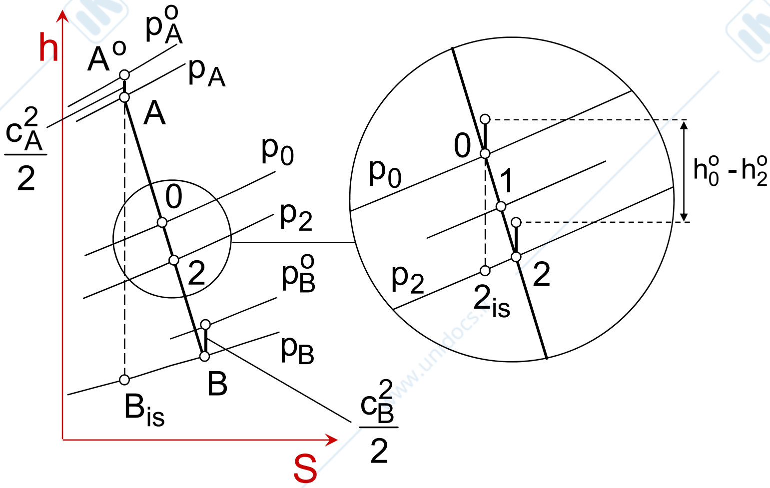
The total-to-total efficiency is conveniently used whenever the discharge kinetic energy of the stage is recovered. In fact, in this case it does not constitute a loss and must thus be excluded from the limit work.

Viceversa, the total-to-static efficiency is more significant if the kinetic discharge energy is lost. In this case, the most physically consistent reference process is a process in which the discharge kinetic energy is nil.



Stage and turbine efficiency

Stage efficiency vs overall efficiency



Stage and turbine efficiency

Stage efficiency vs overall efficiency

A problem with isentropic efficiency arises in **multistage turbines** because of the difference between the isentropic efficiency of a stage and that of the whole machine.

Because of the divergence of the isobars, the isentropic work of the whole machine is clearly less than the sum of the isentropic works of each stage. As a matter of fact, the isentropic work of stage j is **apparently "reheated"** by the previous stage j-1, thus producing an isentropic efficiency of the whole turbine which is higher than that of each stage.

However, keep in mind that, in multistage turbines, the overall enthalpy drop is much **higher than the kinetic energy term**; thus the corresponding contribution can be usually neglected in the formulas.

Stage and turbine efficiency

Re-heating factor

Let's consider (as an example) the **total-to-total efficiency** of the multistage turbine:

Constant for all stages

$$\eta_\Theta = \frac{h_A - h_B + \frac{c_A^2 - c_B^2}{2}}{h_A - h_{B,is} + \frac{c_A^2 - c_B^2}{2}} = \frac{\sum_j \left(h_0 - h_2 + \frac{c_0^2 - c_2^2}{2}\right)_j}{h_A - h_{B,is} + \frac{c_A^2 - c_B^2}{2}} = \frac{\eta_{\Theta,j} \sum_j \left(h_0 - h_{2,is} + \frac{c_0^2 - c_2^2}{2}\right)_j}{h_A - h_{B,is} + \frac{c_A^2 - c_B^2}{2}}$$

$$\eta_\Theta = \eta_{\Theta,j} \frac{h_A - h_{B,is}}{h_A - h_{B,is} + \frac{c_A^2 - c_B^2}{2}} \sum_j \left[\frac{\left(h_0 - h_{2,is}\right)_j}{h_A - h_{B,is}} + \frac{(c_0^2 - c_2^2)_j}{2(h_A - h_{B,is})}\right] \approx \eta_{\Theta,j} \sum_j \left[\frac{\left(h_0 - h_{2,is}\right)_j}{h_A - h_{B,is}}\right]$$

$$\eta_\Theta = \eta_{\Theta,j} y$$

Re-heating factor or recovery index
y>1 (usually not higher than 1.1)

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Stage and turbine efficiency

Polytropic efficiency

A different definition of stage and turbine efficiency can be given, by considering a reference process other than the isentropic one.

The **polytropic** efficiency is obtained if the actual work is related to the **polytropic** one, the latter being the work that would be obtained from a reversible process from the same starting and ending points as the actual process.

$$L_{i,pol} = -\int_0^2 v\,dp + \frac{c_0^2 - c_2^2}{2}$$

$$L_i = -\int_0^2 v\,dp + \frac{c_0^2 - c_2^2}{2} - L_w$$

$$L_{i,pol} = L_i + L_w$$

$$\eta_{pol} = \frac{L_i}{L_{i,pol}} = \frac{L_i}{L_i + L_w}$$

Stage and turbine efficiency

Polytropic efficiency

$$-\int_0^2 v\,dp = -\frac{m}{m-1}p_0 v_0\left[\left(\frac{p_2}{p_0}\right)^{\frac{m-1}{m}}-1\right] = \frac{m}{m-1}p_0 v_0\left[1-\left(\frac{p_2}{p_0}\right)^{\frac{m-1}{m}}\right]$$

$$L_{i,pol} = \frac{m}{m-1}p_0 v_0\left[1-\left(\frac{p_2}{p_0}\right)^{\frac{m-1}{m}}\right] + \frac{c_0^2-c_2^2}{2}$$

For a perfect gas:

$$L_{i,pol} = \frac{m}{m-1}RT_0\left[1-\left(\frac{p_2}{p_0}\right)^{\frac{m-1}{m}}\right] + \frac{c_0^2-c_2^2}{2}$$

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Stage and turbine efficiency

Polytropic efficiency

For a perfect gas:

$$L_i = h_0 - h_2 + \frac{c_0^2-c_2^2}{2} = c_p(T_0-T_2) + \frac{c_0^2-c_2^2}{2}$$

$$L_i = \frac{k}{k-1}RT_0\left(1-\frac{T_2}{T_0}\right) + \frac{c_0^2-c_2^2}{2} = \frac{k}{k-1}RT_0\left[1-\left(\frac{p_2}{p_0}\right)^{\frac{m-1}{m}}\right] + \frac{c_0^2-c_2^2}{2}$$

$$\eta_{pol} = \frac{L_i}{L_{i,pol}} = \frac{\frac{k}{k-1}RT_0\left[1-\left(\frac{p_2}{p_0}\right)^{\frac{m-1}{m}}\right] + \frac{c_0^2-c_2^2}{2}}{\frac{m}{m-1}RT_0\left[1-\left(\frac{p_2}{p_0}\right)^{\frac{m-1}{m}}\right] + \frac{c_0^2-c_2^2}{2}}$$

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Stage and turbine efficiency

Polytropic efficiency

In the case of a multistage turbine:

$$\Delta E_c \approx 0$$

$$\Downarrow$$

$$\eta_{pol} = \frac{L_i}{L_{i,pol}} = \frac{\frac{k}{k-1} R T_A \left[1-\left(\frac{p_B}{p_A}\right)^{\frac{m-1}{m}}\right] + \frac{c_A^2 - c_B^2}{2}}{\frac{m}{m-1} R T_A \left[1-\left(\frac{p_B}{p_A}\right)^{\frac{m-1}{m}}\right] + \frac{c_A^2 - c_B^2}{2}} \approx \frac{k}{k-1}\,\frac{m-1}{m}$$

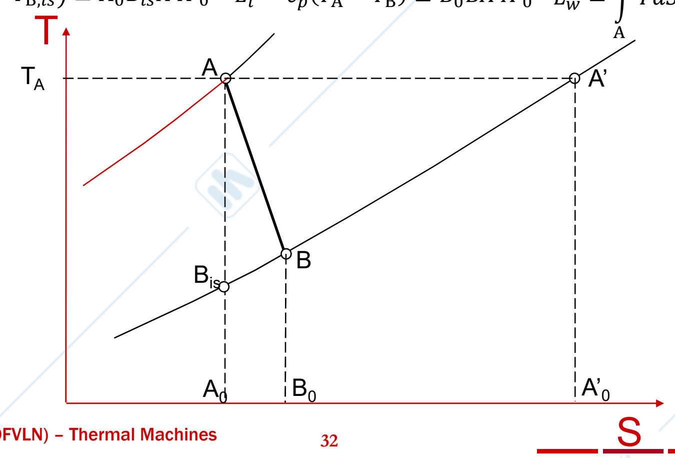
No reference is made to the pressure ratio: the polytropic efficiency is affected just by the **slope of the evolution "m"**, which in turn depends on the irreversibility of the process.

Stage and turbine efficiency

Re-heating effect

Let's consider a **perfect gas** and let's **neglect the kinetic energy term** (Multistage turbine).

$$L_{i,is} = c_p(T_A - T_{B,is}) \triangleq A_0B_{is}A'A'_0 \quad L_i = c_p(T_A - T_B) \triangleq B_0BA'A'_0 \quad L_w = \int_A^B T\,dS \triangleq A_0ABB_0$$

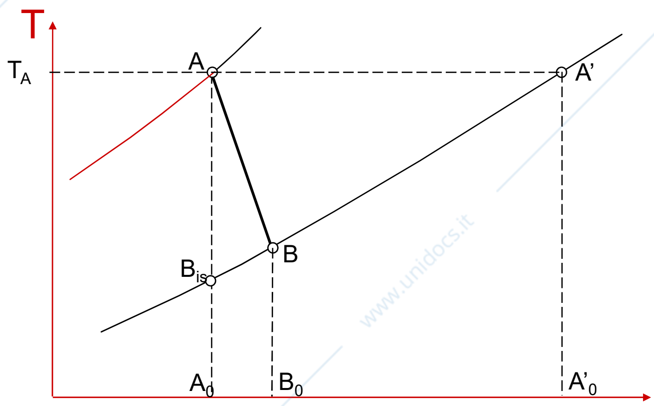


Stage and turbine efficiency

Re-heating

$$L_{i,pol} = L_i + L_w \triangleq A_0ABA'A'_0 = A_0B_{is}A'A'_0 + B_{is}AB = L_{i,is} + RHE$$

$$L_i = L_{i,is} + B_{is}AB - L_w = L_{i,is} + RHE - L_w$$



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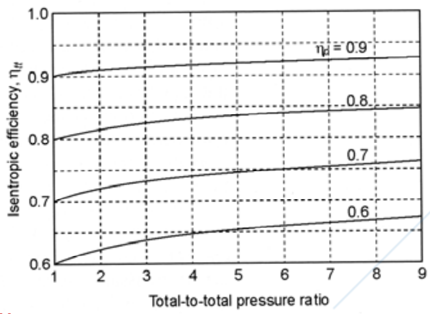
Stage and turbine efficiency

Polytropic efficiency

The **polytropic work of the whole machine** (or of the stage) is clearly larger than the isentropic work at fixed L_i (i.e., polytropic efficiency is usually lower than isentropic one). More specifically, during the expansion the working fluid is reheated by viscous dissipation, thus allowing a work transfer which is higher than L_i-L_w of a quantity equal to the area $2_{is}02$. When p_2/p_0 tends to 1, the contribution of this area tends to zero and η_{is} tends to η_{pol}. For p_2/p_0 values lower than unity, the area $2_{is}02$ tends to increase, thus increasing η_{is} with respect to η_{pol}.

Relationship between **polytropic and isentropic efficiencies** for a turbine with a working fluid of specific heat ratio k = 1.4



Stage and turbine efficiency

Polytropic efficiency

Therefore, the polytropic efficiency eliminates the apparent reheat effect and has the advantage that a **multistage turbine** comprised of identical stages has the same polytropic efficiency as that of each individual stage.

$$\eta_{pol_{(i)}} = \frac{-\int_0^2 v\,dp + \frac{c_0^2 - c_2^2}{2} - L_{w_{0-2}}}{-\int_0^2 v\,dp + \frac{c_0^2 - c_2^2}{2}} \qquad \eta_{pol} = \frac{-\int_A^B v\,dp + \frac{c_A^2 - c_B^2}{2} - L_{w_{A-B}}}{-\int_A^B v\,dp + \frac{c_A^2 - c_B^2}{2}}$$

$$-\int_A^B v\,dp + \frac{c_A^2 - c_B^2}{2} - L_{w_{A-B}} = \sum_i \left(-\int_0^2 v\,dp - L_{w_{0-2}} + \frac{c_0^2 - c_2^2}{2}\right)$$

$$-\int_A^B v\,dp + \frac{c_A^2 - c_B^2}{2} = \sum_i \left(-\int_0^2 v\,dp + \frac{c_0^2 - c_2^2}{2}\right)$$

$$\eta_{pol} = \frac{\sum_i \left(-\int_0^2 v\,dp - L_{w_{0-2}} + \frac{c_0^2 - c_2^2}{2}\right)}{\sum_i \left(-\int_0^2 v\,dp + \frac{c_0^2 - c_2^2}{2}\right)} = \frac{\sum_i \eta_{pol_{(i)}} \left(-\int_0^2 v\,dp + \frac{c_0^2 - c_2^2}{2}\right)}{\sum_i \left(-\int_0^2 v\,dp + \frac{c_0^2 - c_2^2}{2}\right)} = \eta_{pol_{(i)}}$$

Outline

- **Components**
- **Aerodynamic losses**
- **1-D analysis of the flow in a turbine stage**
- **Stage and Turbine efficiency**
- **Degree of reaction**
- **Stage loading and flow coefficient**

Degree of reaction

Definition

The degree of reaction defines how the complete turbine stage **expansion process is split** between the rotor and the stator. A high reaction implies a large acceleration of the fluid in the rotor, and a rather smaller acceleration in the stator.

Various definitions of reaction exist and are used in different industries. The degree of reaction can be defined **with reference to the isentropic enthalpy drops** across the rotor and the stator:

$$\chi = \frac{\Delta h_{is,rotor}}{\Delta h_{is,rotor} + \Delta h_{is,stator}}$$

$\chi = 0$ *impulse stage*

$\chi = 1$ *full reaction stage*

It is also common to define the kinematic degree of reaction, **according to "real" enthalpy drop** in the rotor and the specific work of the stage:

$$R = \frac{\Delta h_{rotor}}{\Delta h^{o}_{stage}} = \frac{\Delta h_{rotor}}{L_i}$$

Degree of reaction

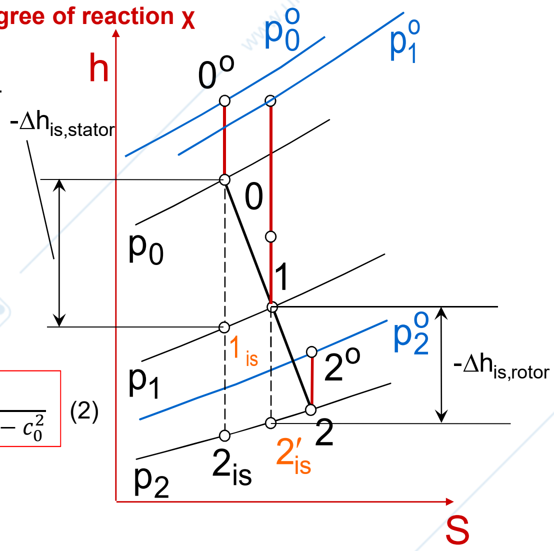
Degree of reaction χ

$$\Delta h_{is,stator} = h_{1,is} - h_0 = -\frac{c_{1,is}^2 - c_0^2}{2}$$

$$\Delta h_{is,rotor} = h_{2',is} - h_1 =$$

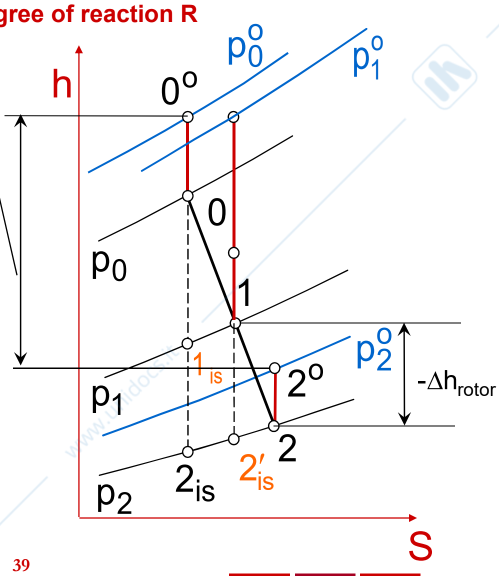
$$= -\frac{w_{2,is}^2 - w_1^2}{2} + \frac{u_2^2 - u_1^2}{2}$$

$$\chi = \frac{w_2^2/\psi^2 - w_1^2 - u_2^2 + u_1^2}{w_2^2/\psi^2 - w_1^2 - u_2^2 + u_1^2 + c_1^2/\varphi^2 - c_0^2} \quad (2)$$



Degree of reaction

Kinematic degree of reaction R



$$\Delta h_{rotor} = h_2 - h_1 = -\frac{w_2^2 - w_1^2}{2} + \frac{u_2^2 - u_1^2}{2}$$

$$\Delta h_{stage}^{O} = L_i$$

By expressing L_i according to eq.(1):

$$R = \frac{w_2^2 - w_1^2 - u_2^2 + u_1^2}{w_2^2 - w_1^2 - u_2^2 + u_1^2 + c_1^2 - c_2^2}$$

Degree of reaction

Axial turbine stage

For a **stage of an axial turbine**, it can be assumed that $u_1 \approx u_2$ (=u) at the mean diameter, therefore the expression of the degree of reaction can be written as:

$$\chi = \frac{w_2^2/\psi^2 - w_1^2}{w_2^2/\psi^2 - w_1^2 + c_1^2/\varphi^2 - c_0^2}$$

$$R = \frac{w_2^2 - w_1^2}{w_2^2 - w_1^2 + c_1^2 - c_2^2}$$

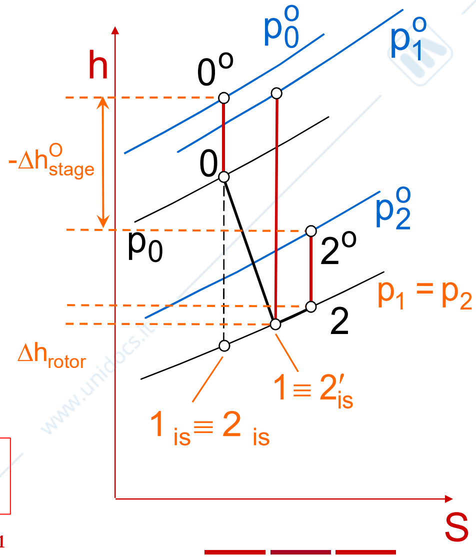
Impulse stage (χ=0)

For an impulse stage, $p_1 = p_2$, thus $1 \equiv 2'_{is}$ (see diagram on this slide). From Eq. (2), this implies χ=0, i.e. an impulse stage has the **degree of reaction χ equal to zero**.
In an impulse stage the **degree of reaction R is lower than zero** (R < 0) because $h_1 < h_2$.
Based on Eq. (2), it can also be deduced that for $u_2=u_1$ (axial stage), $w_1 = w_{2,is}$ therefore:

$$w_2 = \psi w_{2',is} = \psi w_1$$

Then we get:

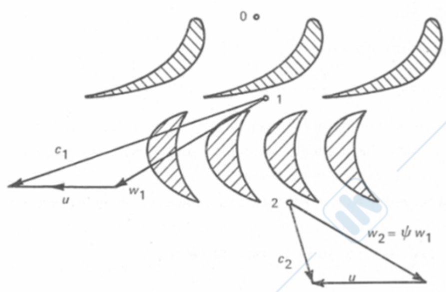
$$h_2 = h_1 + \frac{w_2^2}{2}\left(\frac{1}{\psi^2} - 1\right) = h_1 + \frac{w_1^2}{2}(1 - \psi^2)$$

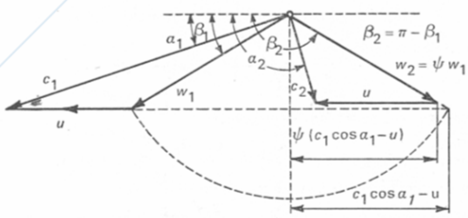


Impulse stage

Velocity triangles: axial turbine

If a **symmetrical rotor airfoil** is considered (i.e., for design conditions $\beta_2=\pi-\beta_1$):





It follows that:

$$c_{a,2} = w_{a,2} = w_2 sin\beta_2 = \psi w_1 sin(\pi - \beta_1) = \psi w_1 sin\beta_1 = \psi w_{a,1} = \psi c_{a,1}$$

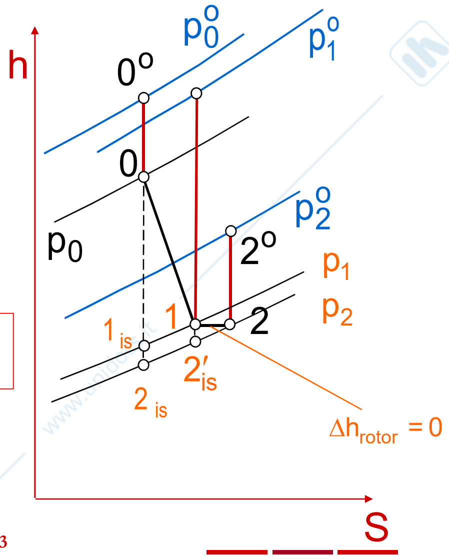
$$l_2 = l_1 \frac{\xi_1}{\xi_2}\frac{c_{a,1}}{c_{a,2}}\frac{v_2}{v_1} = l_1 \frac{\xi_1}{\xi_2}\frac{1}{\psi}\frac{v_2}{v_1}$$

Axial stage with R = 0

In the **stage with R = 0** the degree of reaction χ is slightly higher than zero.

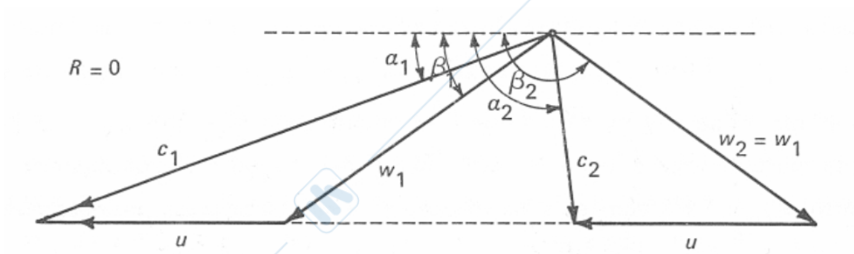
As a matter of fact, since $h_1 = h_2$, p_2 must be slightly lower than p_1.

$$R = \frac{w_2^2 - w_1^2}{w_2^2 - w_1^2 + c_1^2 - c_2^2} = 0 \quad \Rightarrow \quad w_2 = w_1$$



Axial stage with R = 0

If the **axial component of velocity c_a is constant** across the rotor, symmetrical directions of w_1 and w_2 are obtained with respect to the turbine axis:



design conditions; 1-D flow

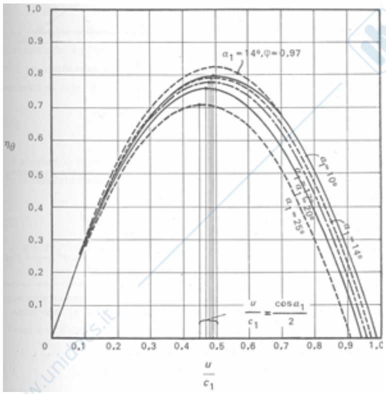
$$\beta_2 = \pi - \beta_1 \quad \Rightarrow \quad \beta_{b2} = \pi - \beta_{b1}$$

Therefore, a **symmetrical rotor airfoil** also results.

Maximum efficiency for the impulse stage

From the velocity triangles:

$$L_i = u(c_{1u} - c_{2u}) = u\big(c_1 cos\alpha_1 - (w_{2u} + u)\big) =$$
$$= u(c_1 cos\alpha_1 + \psi w_{1u}) =$$
$$= u(c_1 cos\alpha_1 + \psi(c_1 cos\alpha_1 + u) - u) =$$
$$= u(1+\psi)(c_1 cos\alpha_1 - u)$$



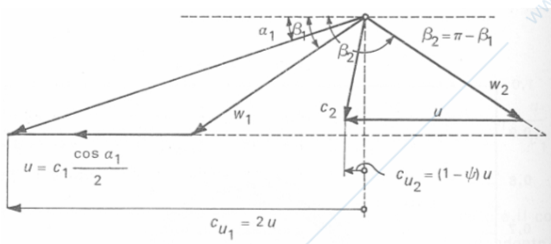
$$\eta_\theta = \frac{L_i}{h_0^o - h_{2,is}} = \frac{L_i}{h_0^o - h_{1,is}}$$

$$h_0^o - h_{1,is} = \frac{c_{1,is}^2}{2} = \frac{c_1^2}{2\varphi^2}$$

$$\eta_\theta = \frac{u(1+\psi)(c_1 cos\alpha_1 - u)}{\frac{c_1^2}{2\varphi^2}} = 2\varphi^2(1+\psi)\frac{u}{c_1}\left(cos\alpha_1 - \frac{u}{c_1}\right)$$

$$\eta_\theta = 0 \begin{cases} \dfrac{u}{c_1} = 0 \\ \dfrac{u}{c_1} = cos\alpha_1 \end{cases}$$

Maximum efficiency for the impulse stage



$$\chi = 0$$

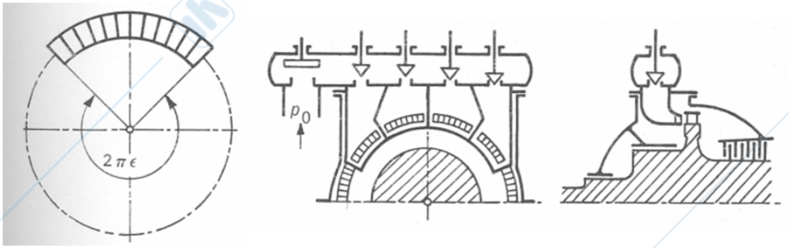
$$\left(\frac{u}{c_1}\right)_{opt} = \frac{cos\alpha_1}{2}$$

For **optimal** blade speed ratio, the **velocity triangles** of an impulse stage are reported in the above picture. It follows that:

$$L_{i,opt} = u(1+\psi)\left(\frac{2u}{cos\alpha_1}cos\alpha_1 - u\right) = u^2(1+\psi)$$

$$\eta_{\theta,opt} = 2\varphi^2\frac{cos\alpha_1}{2}(1+\psi)\left(cos\alpha_1 - \frac{cos\alpha_1}{2}\right) = \varphi^2\frac{1+\psi}{2}cos^2\alpha_1$$

Impulse stage with partial admission



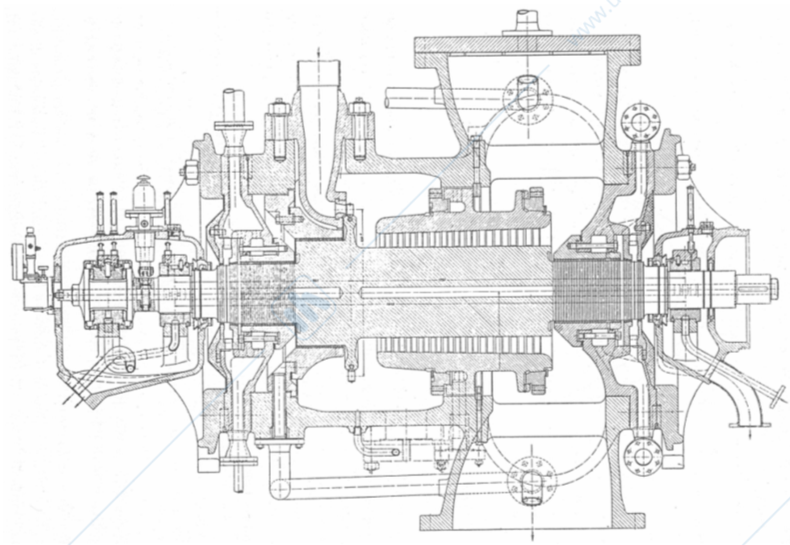
$\chi = 0$

Impulse stages can also work at **partial admission**: "ε" is the fraction of the stator blade rows in which no admission takes place, therefore the fraction of admission results to be "$1-\varepsilon$". With reference to the mass flow rate:

$$\dot{m} = \frac{\xi_1 \pi d l_1 (1-\varepsilon) c_{a,1}}{v_1} = \frac{\xi_2 \pi d l_2 (1-\varepsilon) c_{a,2}}{v_2}$$

it can be easily controlled acting on ε.

Impulse stage with partial admission

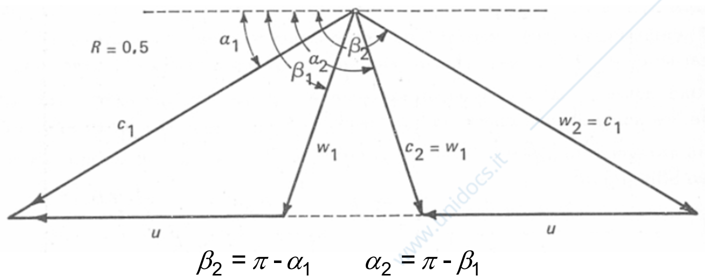


Degree of reaction

Axial turbine, R = 0.5

$$R = \frac{w_2^2 - w_1^2}{w_2^2 - w_1^2 + c_1^2 - c_2^2} = 0.5 \quad \Rightarrow \quad w_2 = c_1 \,\&\, c_2 = w_1$$

Although it is not a general rule. the **axial component of velocity c_a** is often **constant** across the rotor.



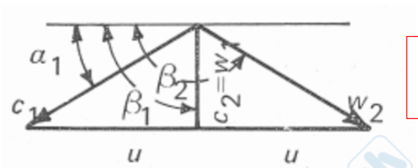
$$\beta_2 = \pi - \alpha_1 \qquad \alpha_2 = \pi - \beta_1$$

Therefore, **symmetrical triangles** result (i.e., stator and rotor blades can be mirror images of each other).

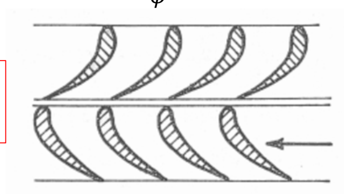
Maximum efficiency for R = 0.5 stages

$$L_i = u(c_{1u} - c_{2u}) = u(c_{1u} + w_{1u})$$
$$= u(2c_1 cos\alpha_1 - u)$$

$$\eta_\Theta = \frac{L_i}{h_0^o - h_{2,is}^o} = \frac{u(2c_1 cos\alpha_1 - u)}{\frac{c_1^2}{\varphi^2} - w_1^2}$$



$$\left(\frac{u}{c_1}\right)_{opt} = cos\alpha_1$$



$$L_{i,opt} = u(2u - u) = u^2$$

$$\eta_{\Theta,opt} = \frac{cos\alpha_1(2cos\alpha_1 - cos\alpha_1)}{\left(\frac{1}{\varphi^2} - 1\right) + cos\alpha_1(2cos\alpha_1 - cos\alpha_1)} = \frac{cos^2\alpha_1}{\frac{1}{\varphi^2} - 1 + cos^2\alpha_1} = \frac{cos^2\alpha_1}{\frac{1}{\varphi^2} - sin^2\alpha_1}$$

Degree of reaction

Impulse vs reaction stages

From the **efficiency** point of view, reaction stages are better than impulse ones.

From the **enthalpy drop** point of view, impulse stages allow a higher enthalpy drop leading both to a sudden reduction of p and T of the fluid and to a reduction in the number of stages, while reaction stages are less effective.

It is possible to introduce **partial admission** in an impulse stage but not in a reaction stage, since on the second one there is some pressure drop across the rotor. Partial admission can give a further increase in the enthalpy drop across the impulse stage. Moreover, it is a very effective way to control a multistage turbine without penalizing too much the overall efficiency.

Usually **turbine layouts** show an impulse stage upstream and some reaction stages downstream of the impulse one.

Outline

- **Components**
- **Aerodynamic losses**
- **1-D analysis of the flow in a turbine stage**
- **Stage and Turbine efficiency**
- **Degree of reaction**
- **Stage loading and flow coefficient**

Stage loading and flow coefficient

Definitions

The **stage loading** is a measure of the work output of a turbine stage.
With reference to an **axial stage** (u_1=u_2), it is defined as:

$$\Psi = \frac{L_i}{u^2}$$

It is often used in conjunction with the **flow coefficient**:

$$\Phi = \frac{c_n}{u}$$

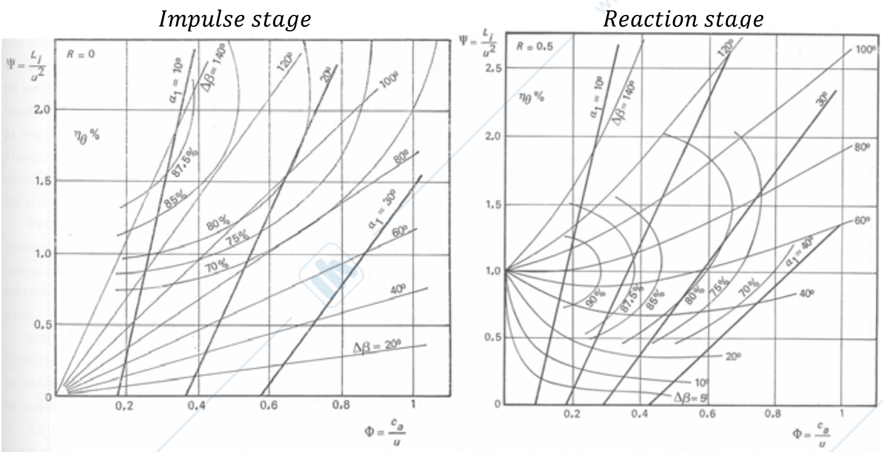
For an **axial stage with $c_{n,1}$ = $c_{n,2}$** at all operating conditions, it can be easily shown that:

$$\Psi = \Phi(cotg\alpha_1 - cotg\beta_2) - 1$$

Stage performance are usually presented in Ψ-Φ charts (Smith charts).

Stage loading and flow coefficient

Charts



$$\Psi_{opt} \cong 2 \overset{i.e.}{\Rightarrow} L_i \cong 2u^2$$

$$\Psi_{opt} \cong 1 \overset{i.e.}{\Rightarrow} L_i \cong u^2$$

Stage loading and flow coefficient

Conclusions

Thus as Ψ increases (from reaction to impulse stages), the **deflection angle** (β_2-β_1) must increase and the stage can work with higher enthalpy drops.

High-turning blades invariably have a larger surface area in contact with the fluid, and therefore larger friction losses. Furthermore, as the amount of turning done by the fluid in the blade passage increases, the scope for secondary flow generation and the associated losses also increases.

As the flow coefficient increases at **constant loading coefficient**, the deflection angle decreases but the mass flow rate and therefore the axial component of velocity increase, leading to a general increase in velocity levels through the stage.

So for any given loading coefficient there is an **optimum flow coefficient value** where the efficiency is a maximum in the Smith chart. At lower than optimum flow coefficients the deflection angle becomes large, and at higher than optimum values the velocity level and the associated friction losses become large, eventually leading to supersonic flow and shock losses.

Stage loading and flow coefficient

Conclusions

Clearly the **best efficiency occurs at small values of loading and flow coefficients**. Since the flow rate and power output of a stage are commonly fixed by the application, a reduction in the stage loading coefficient can only be achieved by increasing the blade speed. This also has the effect of reducing the flow coefficient. Set against this is the increase in blade stress levels. Almost invariably the largest component of the blade stress is centrifugal, which increases as the square of the blade speed.

Alternatively, if a **reduction in the flow coefficient** is achieved by keeping the blade speed constant and reducing the axial component of velocity, this can only be done by increasing the annulus area and therefore the height of the blades. Once again, this increases the stress in the root of the blade.

To generalize, **minimizing stress and maximizing blade life** imply a choice of large Ψ and Φ, but maximizing efficiency demands small Ψ and Φ. Effective designs must provide the best compromise between these conflicting requirements.

Turbocompressors

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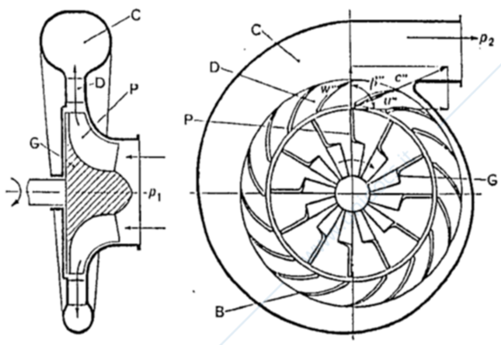
Outline

- **Introduction**
- **Basic operation**
- **Centrifugal compressors**
- **Characteristic diagram**
- **Surge and stall line**
- **Axial compressor**
- **Control of turbocompressors**

Introduction

Centrifugal compressor

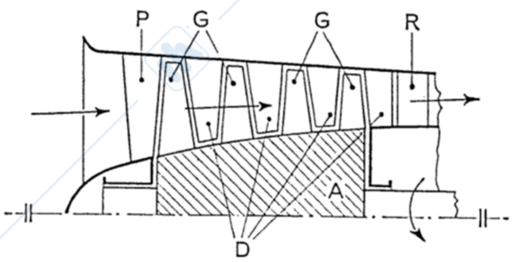
The **centrifugal compressor** consists essentially of a stationary casing containing a rotating impeller which imparts a high velocity to the air, and a number of fixed (usually diverging) passages in which the air is decelerated with a consequent rise in static pressure. The part of the machine containing the diverging passages is known as the diffuser.



Introduction

Axial compressor

The **axial compressor** consists of a series of stages, each stage comprising a row of rotor blades followed by a row of stator blades. The working fluid is initially accelerated by the rotor blades (however, the fluid velocity relative to the rotor is decreased) and then decelerated in the stator blade passages wherein the kinetic energy transferred in the rotor is converted to static pressure. The process is repeated in as many stages as are necessary to yield the required overall pressure ratio



Outline

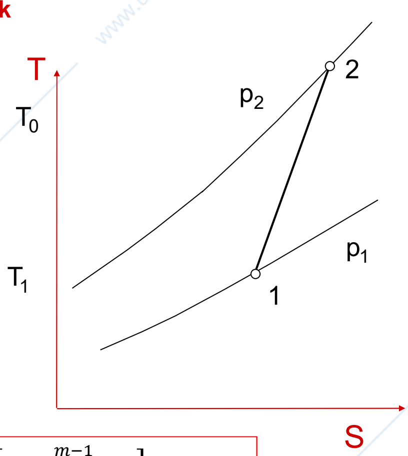
Basic operation

Specific work

Let's write the **mechanical energy conservation law**, with reference to the compressor operating between (p_1,T_1) and p_2:

$$L_i = \int_{p_1}^{p_2} V\,dp + \frac{c_2^2 - c_1^2}{2} + g(z_2 - z_1) + L_w \qquad (1)$$

However, the change in kinetic energy across the whole machine can be usually neglected, the working fluid (air) can be considered as ideal and for compressible flows the change in gravimetric energy can be neglected. Therefore, by introducing the exponent m of the polytropic 1-2:



$$L_i = \int_{p_1}^{p_2} V\,dp + L_w = \frac{m}{m-1} R T_1 \left[\left(\frac{p_2}{p_1} \right)^{\frac{m-1}{m}} - 1 \right] + L_w \qquad (2)$$

Basic operation

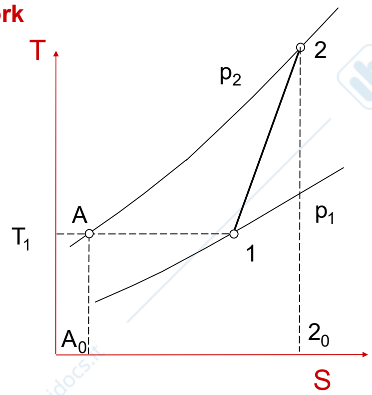
Specific work

Under the same hypothesis (change in kinetic energy between 1 and 2 is virtually zero; air is an ideal gas; change in gravitational energy is negligible) it is possible to **consider the FLT**:

$$L_i = h_2 - h_1 - Q = c_p(T_2 - T_1) - Q \qquad (3)$$

For adiabatic processes:

$$L_i = c_p(T_2 - T_1) \qquad (4)$$



Therefore, the area A_0A22_0 in the diagram corresponds to the **work interaction per unit mass**. By rearranging eq. (4):

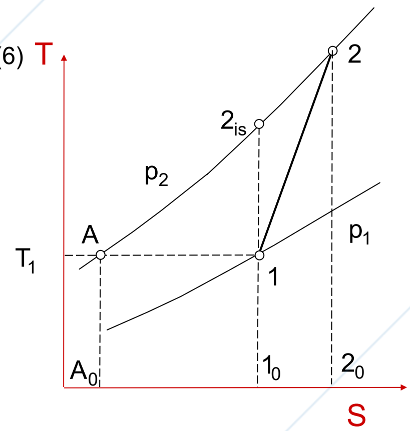
$$L_i = \frac{k}{k-1} R T_1 \left(\frac{T_2}{T_1} - 1 \right) = \frac{k}{k-1} R T_1 \left[\beta_c^{\frac{m-1}{m}} - 1 \right] \qquad (5)$$

Basic operation

Isentropic efficiency

For **isentropic processes** (L_w = 0; m = k), from either eq. (2) or eq. (5) the following expression can be derived for work interaction per unit mass:

$$L_{i,is} = c_p(T_{2,is} - T_1) = \frac{k}{k-1} R T_1 \left[\beta_c^{\frac{k-1}{k}} - 1 \right] \qquad (6)$$



Such isentropic work corresponds to area $A_0A2_{is}1_0$ on the T-s diagram to the right-hand side.

The **isentropic efficiency** of the compressor can be defined as:

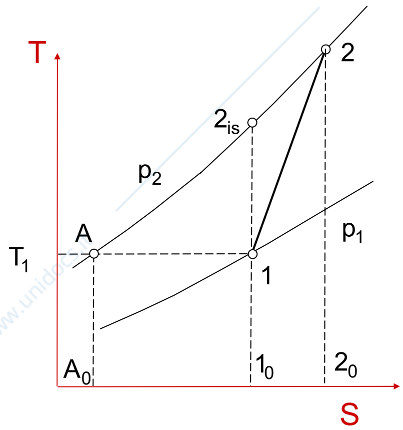
$$\eta_{is} = \frac{L_{i,is}}{L_i} = \frac{c_p(T_{2,is} - T_1)}{c_p(T_2 - T_1)}$$

Basic operation

Polytropic process

If a reversible path (L_w = 0) is chosen for the **polytropic process** 1-2 of exponent m, eq. (2) shows that:

$$L_{i,pol} = \frac{m}{m-1} R T_1 \left[\left(\frac{p_2}{p_1} \right)^{\frac{m-1}{m}} - 1 \right] \qquad (7)$$



By comparing eq. (7) to eq. (2):

$$L_{i,pol} = L_i - L_w \qquad (8)$$

However, from eq. (3):

$$L_{i,pol} = c_p (T_2 - T_1) - Q_{rev}$$

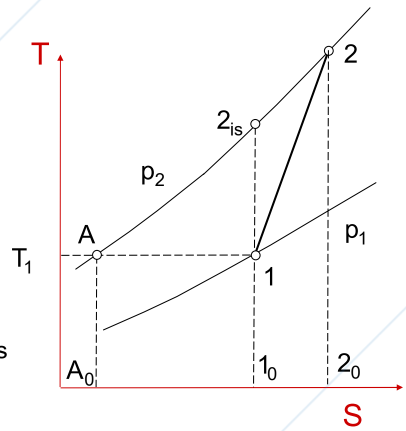
This expression, taking account of (8) and (4) recalls that Q_{rev}=L_w.

Basic operation

Polytropic process

From the second law of thermodynamics applied to the "real" adiabatic path 1-2:

$$\int_1^2 T dS = Q + L_w = L_w$$



thus showing that L_w corresponds to the area $1_0 1 2 2_0$ on the T-s diagram. Therefore:

$$L_{i,pol} = L_i - L_w \triangleq A_0 A 2 2_0 - 1_0 1 2 2_0 \triangleq A_0 A 2 1 1_0$$

It follows that $L_{i,pol} > L_{i,is}$ and

$$L_{i,pol} - L_{i,is} \triangleq 1 2_{is} 2$$

as a result of the **reheating effect** due to viscous dissipation along the real adiabatic path 1-2.

Basic operation

Polytropic efficiency

The **polytropic efficiency** can be defined as

$$\eta_{pol} = \frac{L_{i,pol}}{L_i} = \frac{\int_1^2 v dp}{c_p(T_2 - T_1)} \qquad (9)$$

Taking account of eqs. (8), (7) and (5), eq. (9) can be rewritten as:

$$\eta_{pol} = \frac{L_i - L_w}{L_i} = \frac{\frac{m}{m-1} R T_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{m-1}{m}} - 1 \right]}{\frac{k}{k-1} R T_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{m-1}{m}} - 1 \right]} = \frac{m}{m-1} \frac{k-1}{k} \qquad (10)$$

Basic operation

Isentropic vs polytropic efficiency

The **isentropic efficiency** can be rewritten as:

$$\eta_{is} = \frac{\beta_c^{\frac{k-1}{k}} - 1}{\beta_c^{\frac{m-1}{m}} - 1}$$

It depends on the compression ratio β_c and on the polytropic index m. The isentropic efficiency accounts both for L_w and RHE i.e. it is a measure of the quality of the whole compression.

From eq. 10 the **polytropic efficiency** can be rewritten as:

$$\eta_{pol} = \frac{\frac{k-1}{k}}{\frac{m-1}{m}} \qquad (11)$$

It depends just on the polytropic index m. The polytropic efficiency only accounts for the flow losses L_w i.e. it is a measure of the fluid dynamic quality of the compression.

Basic operation

Commonly used expressions for L_i

Coupling eq.(11) & eq.(5) and eq.(6) & eq. (7), the following expressions of **work transfer interaction** can be obtained:

$$L_i = \frac{k}{k-1} R T_1 \left[\beta_c^{\frac{1}{\eta_{pol}} \frac{k-1}{k}} - 1 \right] \qquad (12)$$

with reference to the polytropic efficiency.

$$L_i = \frac{1}{\eta_{is}} \frac{k}{k-1} R T_1 \left[\beta_c^{\frac{k-1}{k}} - 1 \right] \qquad (13)$$

with reference to the isentropic efficiency.

Basic operation

Example

Let's assume that: $T_1 = 293\ K$ $\quad k = 1.4\ (air)$ $\quad \eta_{pol} = 0{,}90$

$$\frac{m-1}{m} = \frac{1}{\eta_{pol}} \cdot \frac{k-1}{k} = 0.3175 \qquad m = 1.465 > k$$

$\boldsymbol{\beta_c = 2}$

$$T_2 = T_1 \cdot \beta_c^{\frac{m-1}{m}} = 365.12\ K \qquad T_{2,is} = T_1 \cdot \beta_c^{\frac{k-1}{k}} = 357.17\ K$$

$$\eta_{is,c} = \frac{c_p (T_{2,is} - T_1)}{c_p (T_2 - T_1)} = 0.890$$

$\boldsymbol{\beta_c = 4}$

$$T_2 = T_1 \cdot \beta_c^{\frac{m-1}{m}} = 455.01\ K \qquad T_{2,is} = T_1 \cdot \beta_c^{\frac{k-1}{k}} = 435.40\ K$$

$$\eta_{is,c} = \frac{c_p (T_{2,is} - T_1)}{c_p (T_2 - T_1)} = 0.879$$

Hence, the isentropic efficiency of the machine gets progressively smaller while the compression ratio increases.

Basic operation

Isothermal reversible process

Let's consider the **isothermal reversible compression** from 1 to 1_T. Under the same hypotheses previously done (change in kinetic energy between 1 and 2 is virtually nihil; air is an ideal gas; change in gravitational energy is negligible), eq. (1) reduces to:

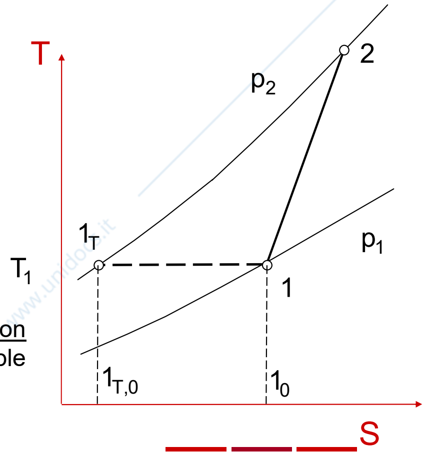
$$L_{i,T} = \int_{p_1}^{p_2} v\,dp = \int_{p_1}^{p_2} \frac{RT}{p}\,dp = RT_1 \int_{p_1}^{p_2} \frac{dp}{p}$$

$$L_{i,T} = RT_1 ln\left(\frac{p_2}{p_1}\right) = RT_1 ln\beta_c \quad (14)$$

From 1[st] and 2[nd] laws of thermodynamics:

$$L_{i,T} = -Q_{1-1,T} \triangleq 1_{T,0}1_T 1 1_0$$

thus showing that $L_{i,T}$ is the minimum compression work with respect to all considered reversible paths from p_1 to p_2.



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Basic operation

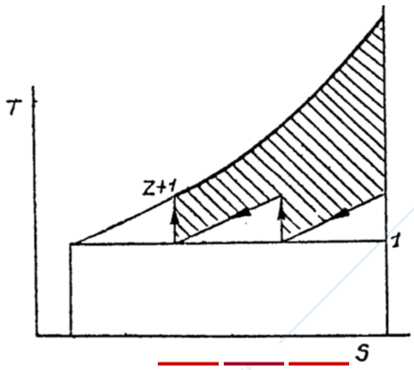
Intercooled compression

Isothermal compression cannot be practically achieved, since it is not possible to cool down the fluid during the compression process. Nevertheless, it is possible to realize a multiple stage **intercooled compression**. After each stage of compression, the air is intercooled to T_1, by extracting as much heat as the compression work of the stage.

The diagram below suggests that if the number of stages z tends to be infinite, an isothermal compression is virtually achieved.

It can also be shown that the specific work input is minimum when the compression ratios of all the stages are equal (**uniform intercooled compression**):

$$\beta_j = \sqrt[z]{\frac{p_{z+1}}{p_1}} = \sqrt[z]{\beta_c} \quad (15)$$



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Outline

- Introduction
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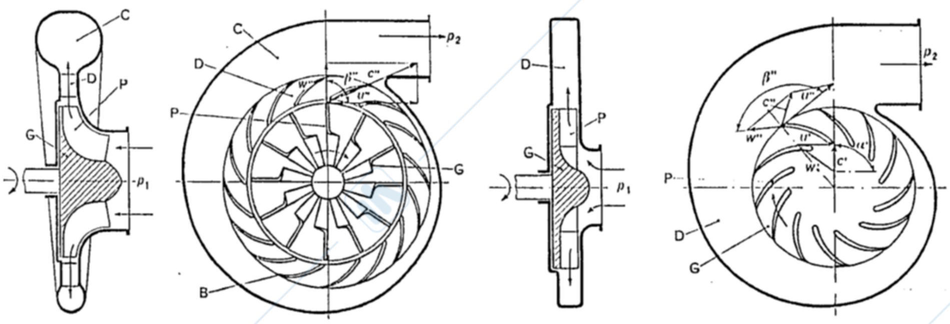
- Centrifugal compressors
- Characteristic diagram
- Surge and stall line
- Axial compressor
- Control of turbocompressors

Centrifugal compressor

Examples



Centrifugal compressor

Velocity triangles

The energy conservation applied to the whole machine (between 1 and 2) and the Euler equation applied to the impeller leads to the following expression of the **specific work**:

$$L_i = h_2^0 - h_1^0 = c''_u u'' - c'_u u'' = c''_u u'' \quad (16)$$

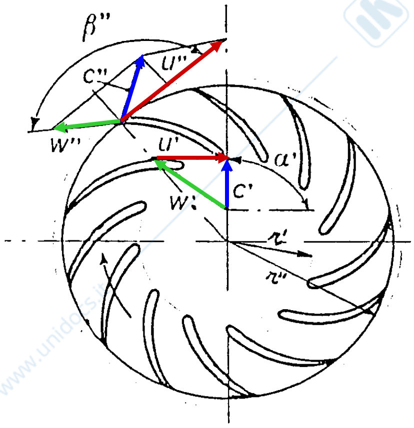
notice that in case of radial blades, the specific work can be expressed as:

$$L_i = (u'')^2$$

i.e. it does not depend on nothing but the blade's tangential velocity.

By applying the energy conservation to the impeller:

$$L_i = h'' - h' + \frac{c''^2 - c'^2}{2} \quad (17)$$



Centrifugal compressor

Velocity triangles

The change in enthalpy between section " and ' can be worked out from the energy conservation applied to the relative flow in the impeller:

$$h'' - h' = \frac{w'^2 - w''^2}{2} - \frac{u'^2 - u''^2}{2} \quad (18)$$

By combining eqs. (17) and (18) :

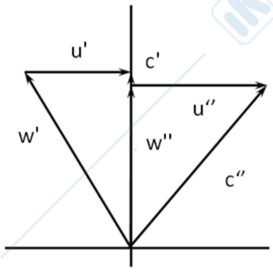
$$L_i = \frac{c''^2 - c'^2}{2} - \frac{w''^2 - w'^2}{2} + \frac{u''^2 - u'^2}{2} \quad (19)$$

Similar considerations to those made about the turbines can be done:

- a centrifugal stage can absorb a greater specific work than an equivalent axial or centripetal one;
- the relative velocity term is subtracted; it must be arranged so that w" < w' so that this term makes a net positive contribution to the work absorbed;
- the absolute velocity term is additive. The impeller should therefore be designed to accelerate the flow, thus resulting in c" > c'.

Centrifugal compressor

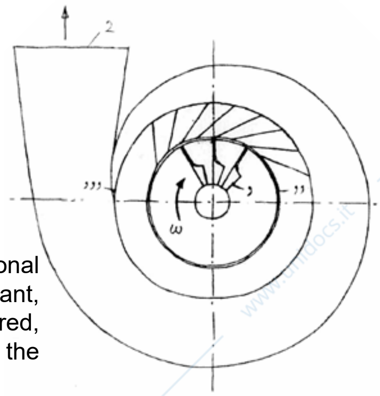
Compressor layout

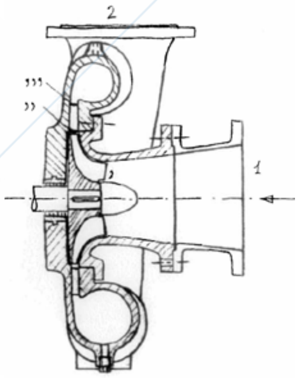


From the conservation of mass:

$$\xi \pi d' l' \rho' w'_a = \xi \pi d'' l'' \rho'' w''_r$$

$$w''_r = \frac{d' l' \rho'}{d'' l'' \rho''} w'_a$$

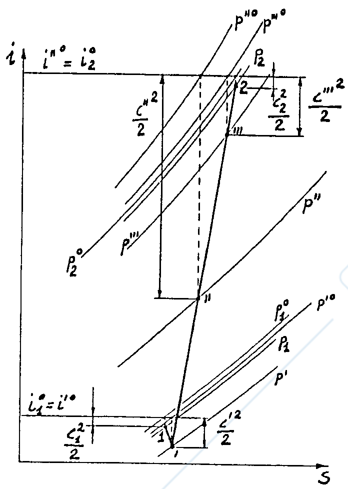




To keep the meridional component nearly constant, the rotor blades are tapered, in order to counteract the increase in density.

Centrifugal compressor

Thermodynamic evolution



$$h' - h_1 = \frac{c_1^2 - c'^2}{2} \qquad h'' - h' = \frac{w'^2 - w''^2}{2} - \frac{u'^2 - u''^2}{2}$$

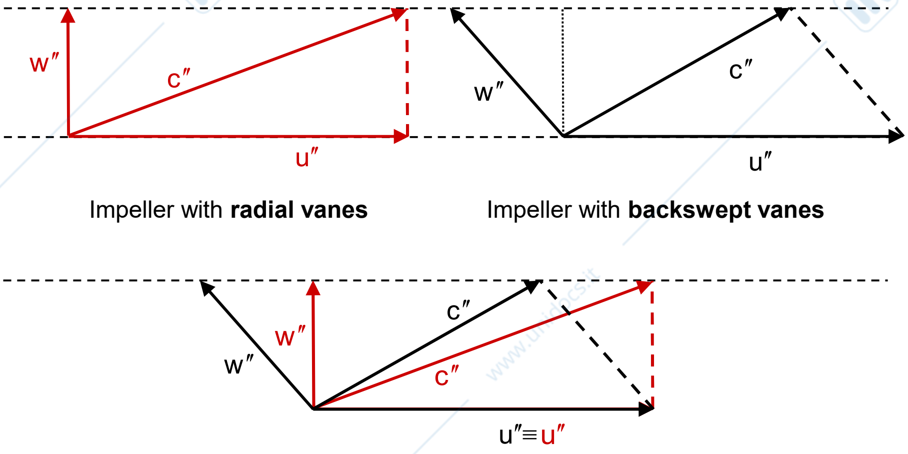
$$h''' - h'' = \frac{c''^2 - c'''^2}{2} \qquad h_2 - h''' = \frac{c'''^2 - c_2^2}{2}$$

$$L_i = h_2 - h_1 + \frac{c_2^2 - c_1^2}{2}$$

If $c_1^2 \approx c'^2 \approx c'''^2 \approx c_2^2$ then $L_i \approx h''' - h'$

Centrifugal compressor

Radial vs backswept vanes



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Centrifugal compressor

Radial vs backswept vanes

The velocity triangles at the tip section (") for radial and backswept impellers are shown in the previous slide for the same blade speed u".

Assuming the radial component of the velocity to be the same, implying roughly the same mass flow rate, it can be seen that in the backswept impeller the velocity relative to the tip (w") is increased while the absolute velocity of the fluid (c") is reduced.

This means that:

- a **radial impeller** should be coupled with a **vaned diffuser**, to recover the high amount of kinetic energy that is produced at the rotor output;
- the **backswept architecture** implies less stringent diffusion requirements in both the impeller and diffuser and, in particular, a **vaneless diffuser** could be used.

The backsweep angle may be in the region of 30-40 deg. The work-absorbing capacity of the rotor is reduced, but this effect is counteracted by the increased efficiency of the components. It must be taken in mind that the ultimate goal is combining a high pressure ratio and good efficiency.

Centrifugal compressor

Vaneless diffuser

The **vaneless diffuser** is frequently employed in process compressors, refrigeration compressors, and turbocharger compressors.

A reasonable level of static pressure recovery can be achieved with a properly designed vaneless diffuser, which is also inherently inexpensive. It essentially features two parallel walls forming an open radial annular passage from the impeller tip to some limiting outer radius. It is most frequently followed by a collecting scroll which accumulates flow and delivers it to a single exit port.

The vaneless diffuser allows the **input velocity to change consistently** during off design operations, since the incidence has no effect on its performance.
Another characteristic of the vaneless diffuser is the **absence of a throat**. There is, therefore, no realistic possibility of choking the diffuser, and this leads to the possibility of a wide range of operation.

Outline

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Characteristic diagram

Fluid-dynamic similitude

To correlate performances of similar turbomachines working in different conditions, it is useful to refer to the "**fluid-dynamic similitude**" theory.

Fluid-dynamic similitude can be related to different machines or different working conditions of the same machine, and comprises:

- Geometric similarity (similitude of shapes and dimensions)
- Kinematic similarity (similitude of velocity fields)
- Dynamic similarity (similitude of force fields)
- Thermal similarity (similitude of temperature fields)

A full similitude condition is almost impossible to be achieved; anyway it is possible to satisfy at least some of the requests.
In the 1-D theory context, the fluid-dynamic similitude is considered to be achieved when the geometric similarity of the machines and of velocity triangles is verified, along with the equality of some **nondimensional parameters**.

Characteristic diagram

Nondimensional parameters

The most important **nondimensional parameters** in the turbocompressors field are:

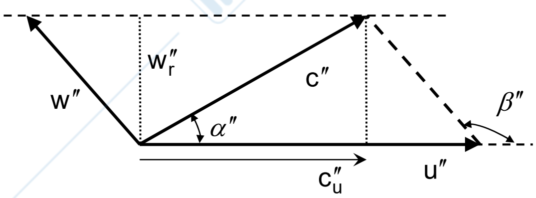
$\Phi = \frac{w_r''}{u''}$	Flow coefficient	$\Psi = \frac{L_i}{u''^2/2}$	Load factor
$\zeta = \frac{L_w}{u''^2/2}$	Loss coefficient	$C_{ru} = \frac{u''}{\sqrt{2h^0}}$	Crocco number

If these parameters are the same for different machines, they can be considered to work under fluid-dynamic similitude conditions, with satisfactory accuracy. Under these conditions, it can be shown that the machines have the same efficiency η_y.

Nondimensional or "corrected" parameters are used to **define the characteristic diagram**. This allows:
- to include the effects of all operating conditions in the same diagram;
- to extend the characteristic diagram to all turbomachines which are similar (i.e. a "family" of turbomachines).

Characteristic diagram

Nondimensional parameters



$$\frac{w''_r}{u''-c''_u} = tg(\pi - \beta'') = -tg\beta''$$

$$u''-c''_u = -\frac{w''_r}{tg\beta''} = -w''_r cotg\beta''$$

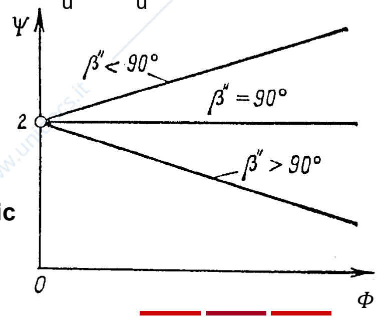
$$1-\frac{c''_u}{u''} = -\frac{w''_r}{u''}cotg\beta''$$

$$\frac{c''_u}{u''} = 1+\frac{w''_r}{u''}cotg\beta''$$

$$\Psi = \frac{L_i}{\frac{u''^2}{2}} = 2\frac{u''c''_u}{u''^2} = 2\frac{c''_u}{u''} = 2\left(1+\frac{w''_r}{u''}cotg\beta''\right)$$

$$\Psi = 2(1+\Phi cotg\beta'') \qquad (21)$$

The **nondimensional $\Psi - \Phi$ characteristic** explicitly depends on the blade outlet angle β''.



Characteristic diagram

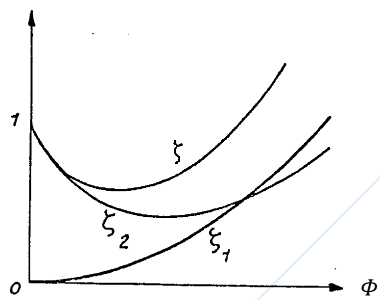
Loss coefficient

The loss coefficient can be also expressed as a **function of** Φ.
ζ is made up of two contributions. For a turbocompressor with vaned diffuser:

- ζ_1 indicates the profile losses due to skin friction on both impeller and diffuser vane surface;
- ζ_2 indicates the losses due to incidence when the flow passes from rotating to fixed blades (and from fixed to rotating blades in the case of a fixed stator upstream from the impeller).

For a turbocompressor featuring a **diffuser without vanes**:

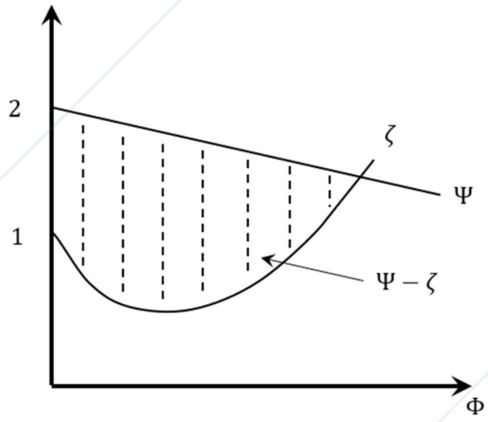
- ζ_1 refers to losses in the impeller;
- ζ_2 refers to losses in the diffuser.



Characteristic diagram

Nondimensional map

To get the compressor characteristic map, it is necessary to **plot $\Psi - \zeta$ as a function of Φ**.



The chart reported in this slide is the first characteristic diagram example, which holds within the 1-D theory with some simplifying hypotheses.
Still, the final map should be expressed with reference to some engineering parameters, like β and $\dot{m}$.

Moreover, the 1-D theory is not sufficient to exhaustively describe the behaviour of a compressor, while a 3-D theory accompanied by experimental results should be considered.

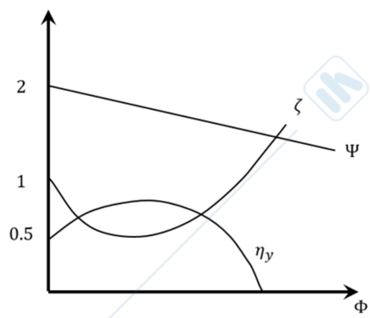
Characteristic diagram

Nondimensional map

Based on eq. (10) and on the definitions of load factor and loss coefficient it can be easily shown that:

$$\eta_{yc} = \frac{L_i - L_w}{L_i} = \frac{\Psi - \zeta}{\Psi} \qquad (22)$$

Then it is possible to plot this curve of **η_{yc} as a function of Φ**.



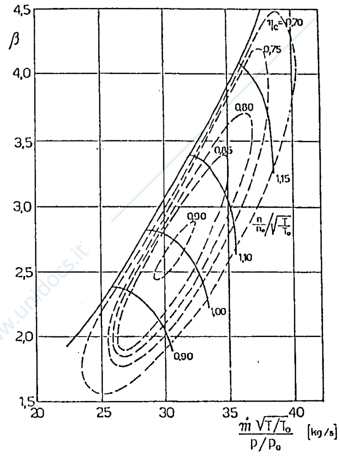
This **simplified analysis**, that implies geometric similitude ($\frac{l}{D} = const$), clearly shows that, when the velocity triangles are similar ($\Phi = const.$), the considered turbomachines not only have the same efficiency ($\eta_{yc} = const.$), but also are in dynamic similitude ($\Psi = const$).

Characteristic diagram

Dimensional map

The real interesting map for **engineering applications** is presented below; it plots the compression ratio of the machine β as a function of the corrected mass flow rate $\frac{\dot{m}\sqrt{T/T_0}}{p/p_0}$.

T_0, p_0 and n_0 represent <u>reference values</u> for temperature, pressure and angular speed.
Dashed lines report the values of the isentropic efficiency.



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Characteristic diagram

Dimensional map: proof

The **characteristic diagram** of a centrifugal machine can be easily obtained within this frame:

$$L_i = \int_1^2 vdp + \cancel{\Delta E_c} + L_w \qquad \Psi \frac{u''^2}{2} = \int_1^2 vdp + \zeta \frac{u''^2}{2}$$

$$(\Psi - \zeta)\frac{u''^2}{2} = \frac{m}{m-1} RT_1 \left(\beta_c^{\frac{m-1}{m}} - 1\right) \qquad \frac{1}{2}(\Psi - \zeta)\left(\frac{u''}{\sqrt{RT_1}}\right)^2 = \frac{m}{m-1}\left(\beta_c^{\frac{m-1}{m}} - 1\right)$$

That is a **relationship between Ψ, ζ (as a function of Φ) and β_c** parameterized with m (i.e. η_{yc}) and u'' (i.e. n).

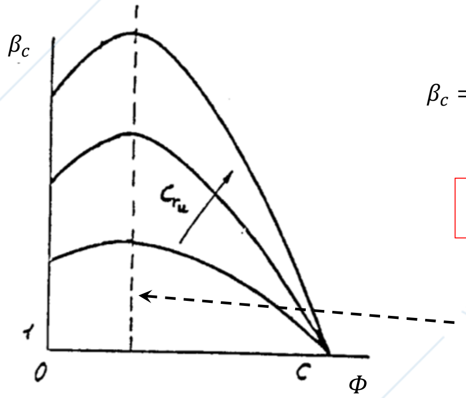
$$\frac{m}{m-1} = \eta_{yc} \frac{k}{k-1} = \frac{\Psi - \zeta}{\Psi} \frac{k}{k-1}$$

$$\boxed{\frac{1}{2}(\Psi - \zeta)\left(\frac{u''}{\sqrt{RT_1}}\right)^2 = \frac{\Psi - \zeta}{\Psi} \frac{k}{k-1}\left(\beta_c^{\frac{m-1}{m}} - 1\right)} \qquad (23)$$

Characteristic diagram

Dimensional map: proof

$$\frac{1}{2}(\Psi-\zeta)\left(\frac{u^{"}}{\sqrt{RT_1}}\right)^2=\frac{\Psi-\zeta}{\Psi}\frac{k}{k-1}\left(\beta_c^{\frac{m-1}{m}}-1\right)$$



$$\beta_c=\left(1+const\cdot\Psi\left(\frac{u^{"}}{\sqrt{2c_pT_1}}\right)^2\right)^{\eta_{yc}\frac{k}{k-1}}$$

$$\beta_c=\left(1+const\cdot\Psi {C_{ru}}^2\right)^{\eta_{yc}\frac{k}{k-1}} \quad (24)$$

$iso-efficiency\ line$

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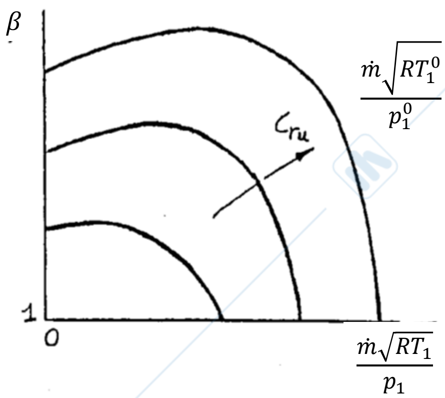
Characteristic diagram

Dimensional map: proof

$$\dot m=\frac{\xi^{"}\pi d^{"}l^{"}\Phi u^{"}}{v^{"}}\approx const.\frac{\Phi u^{"}}{\left(\frac{RT_1^0}{p_1^0}\right)}$$



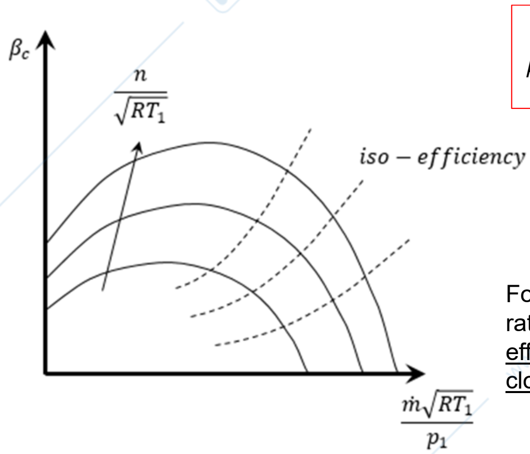
$$\frac{\dot m\sqrt{RT_1^0}}{p_1^0}\approx\frac{\dot m\sqrt{RT_1}}{p_1}\approx const\cdot\frac{u^{"}}{\sqrt{RT_1^0}}\cdot\Phi\approx const\cdot C_{ru}\cdot\Phi$$

$$\frac{\dot m\sqrt{RT_1}}{p_1}=const\cdot C_{ru}\cdot\Phi \quad (25)$$

Characteristic diagram

Dimensional map

Further working on the previous expressions, we get:



$$\beta_c = \left(1 + const \cdot \Psi \left(\frac{n}{\sqrt{RT_1}}\right)^2\right)^{\eta_{yc}\frac{k}{k-1}}$$

$$\frac{\dot{m}\sqrt{RT_1}}{p_1} = const.\frac{n}{\sqrt{RT_1}} \cdot \Phi$$

For very low and very high mass flow rates, the 1-D theory fails and the iso-efficiency lines usually collapse into closed lines.

Outline

- Introduction
- Basic operation
- Centrifugal compressors
- Characteristic diagram
- **Surge and stall line**
- Axial compressor
- Control of turbocompressors

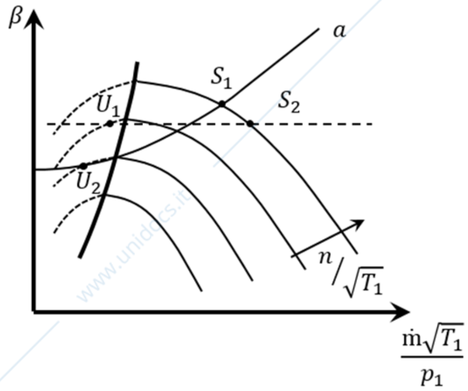
Surge and stall line

Operating point of a turbocompressor

The **operating point** is represented by the intersection between turbocompressor and external characteristic diagrams. In many turbocompressor applications, the user requirement is a virtually constant pressure level, which is independent from the mass flow rate.

This user requirement can be represented by the horizontal dashed line (external characteristic) on the characteristic diagram of the machine.

However, in some applications, different external characteristics should be considered (for instance, the plot labelled with "a").



Surge and stall line

Definition of stability

Stability is related to the **response of the compressor to a disturbance** which perturbs compressor operation from a steady operating point.

If the disturbance is transient, the performance is considered stable if the system returns to the original point of operational equilibrium. If the response is to drive operation away from the original point, the performance is unstable.

Stability can be classified into **operational and aero-dynamic**:

- operational stability is concerned with the matching of performance characteristics of the compressor with a downstream flow device such as a tank, a turbine, or a nozzle;
- aerodynamic stability is concerned with the limitation of steady state operation due to stall and surge.

Surge and stall line

Operational stability

The system is in equilibrium at point S_1 (S_2).

If an **external perturbation** is imposed on the system such that mass flow is reduced, the pressure request will stay constant (will diminish) and the pressure rise across the compressor will increase. In the next instant of time, the compressor pressure rise will be higher than requested pressure, thus imposing an increase of mass-flow rate and equilibrium will be restored. If the perturbation causes an increase in mass flow, the pressure request will stay constant (will increase) and the pressure rise across the compressor will decrease. The diminished pressure with respect to the requested value will impose a decrease in mass flow and equilibrium will be restored.

Point U_1 (U_2) is an **unstable point** on the compressor characteristic. If an external perturbation is imposed on the system such that mass flow is reduced, the pressure request will stay constant (will diminish) and the pressure rise across the compressor will diminish. In the next instant, the pressure rise across the compressor is less than that requested. Mass flow will continue to decrease, and equilibrium will not be restored.

Surge and stall line

Operational stability

These examples show that system operation is stable when the **slope of the compressor characteristic** is negative or zero or when the slope is 'less positive' than the slope of the user characteristic. Stated mathematically, system stability is established when the rate of change of compressor pressure rise with mass flow is algebraically less than the rate of change of pressure requested by the user.

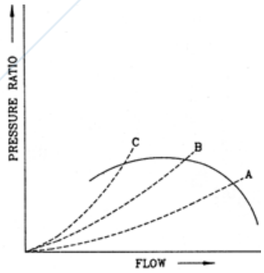
This criterion has limited application. Only low pressure ratio, single-stage axial compressors or blowers have steady state operation with a positively sloped performance characteristic.

Practically speaking, in gas turbines or engine applications, compressor operation is always **limited to the negative sloped** performance characteristic. This is meant to define operational stability in an independent way from the user tasks. Moreover, dynamical behavior can be commonly modeled as a perturbation with constant pressure.

Surge and stall line

Aerodynamic stability

Compressor **aerodynamic stability** is the ability of the entire system (guide vanes, rotors and stators or diffusers) to maintain or increase the delivery pressure to a downstream reservoir when the compressor operation has been perturbed to a lower flow.



The figure shows the **pressure ratio/flow characteristic** of a compressor intersected by three throttle characteristics.
It could illustrate three positions of a throttle valve as data are taken on a compressor rig, for instance.
As shown in the preceding section, each of the throttle intersections is a stable operating point.

However, the part of the characteristic with a **positive slope** is usually a region of the compressor map containing subsystem stall or complete instability resulting in surge. As compressor operation moves into this region, the static pressure recovery of the system may be smaller than at a previous operating point at a slightly higher mass flow.

Surge and stall line

Surge

In this case, when operation is perturbed to the lower mass flow, the compressor **fails to match the pressure** established in the downstream reservoir at the higher mass flow, and the reservoir can discharge mass flow through the compressor in surge or in transition to rotating stall.

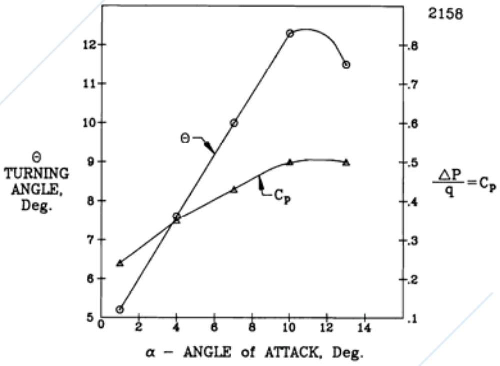
In **surge**, mass flow fluctuates during this unstable operation as the compressor attempts to produce a pressure rise when mass flow is taken back in, only to stall out again and permit mass flow to pass out again. Consequently, even though operational stability is theoretically possible with a positive slope on the compressor characteristic, the operation is typically aerodynamically unstable due to blade row stall initiation and system surge.

The occurrence of surge is preceded by the stalling of some subsystem element. Stall can be regarded as the cessation of a continued rise in lift on an airfoil or in static pressure recovery in a diffuser or cascade.

Surge and stall line

Stall

In **external aerodynamics**, stall refers to a sharp drop in lift coefficient just after maximum lift coefficient has been achieved as angle of attack increases from low values to high values. The maximum value of lift coefficient is limited by the ability of the airfoil to maintain conditions on the suction surface to keep the flow attached.



The same phenomenon occurs in **cascade aerodynamics**. In this case, however, performance is characterized by fluid deflection and the rise in static pressure coefficient. Beyond a certain value of angle of attack, turning angle and static pressure rise cease to increase because of suction surface separation.

Surge and stall line

Stall

In a compressor, a blade row element can be completely or partially stalled, but the compressor system can still be stable. Surge is the result of the **stalling of some system components** or combination of components such that continued reduction in mass flow makes the entire system unstable and causes audible flow fluctuations.

Typically, when a blade row stalls, all the airfoils **do not stall uniformly**. Instead, groups of airfoils stall to form a zone or zones of stalled flow around the annulus. Mass flow is reduced during the transient formation of the stall zones. When the zones are established, mass flow ceases to change and stable operation is restored but at a reduced level of mass flow relative to unstalled operation.

It is possible to operate under this condition, but it is structurally risky because of vibratory stresses set up by the rotating stall zone.

Surge and stall line

Types of stall

The **onset of stall** is brought about by the limited ability of the boundary layer to sustain an adverse pressure gradient. Because of this, limitations are reached in circulation for isolated airfoils and cascades and in pressure recovery for cascades and diffusers. In compressors, therefore, the ability to provide a pressure rise is limited by blading and diffuser stall.

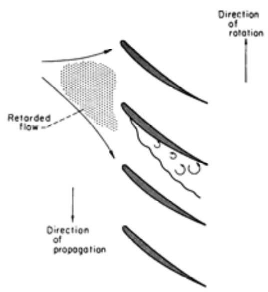
There are two types of stall that have been experimentally identified for compressors. One is individual blade stall, and the other is rotating stall:

- **Individual blade stall** occurs when the entire blade row stalls at once. This is the type of stall that would be expected from airfoil or cascade data. If all the airfoils are identical and if they all experience the stall incidence angle or angle of attack simultaneously, then the complete blade row annulus should stall as a unit.
- The most common type of stall, however, is **rotating stall**. This stall consists of zones of stalled passages covering a small number of blades and rotating at some fraction of the rotor speed in the absolute direction of rotor rotation. It occurs in rotating and stationary blade rows and in both axial and centrifugal compressors.

Surge and stall line

Rotating stall

The figure shows an **axial compressor rotor** with an example of rotating stall. The annulus has two cells of stalled flow concentrated near the blade tips. These cells extend upstream and downstream of the blade row. The cells have total pressure depressions and contain little or no flow. Relative to the rotor, the cells rotate opposite rotor direction. The speed of rotation is from 10% to 90% of the rotor speed and in the direction of rotor rotation as viewed from the absolute frame of a stationary observer.



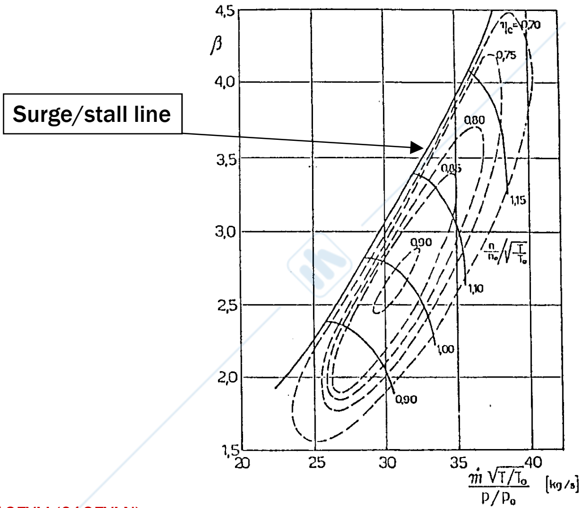
Surge and stall line

Rotating stall

The **rotation of the stall cells** can be explained by viewing the blade row from the top of the annulus as shown in the right-hand side of previous slide. Stall is initiated on some blade: it is not known for sure why all the blades do not stall simultaneously.
As pointed out in Schulze, et al. (1950), dimensional tolerances could be one reason. One blade could be manufactured with a slightly higher stagger angle, or the profile may be slightly different. The inlet air angle may approach a blade or group of blades slightly differently to instigate stall.
When stall begins on a blade, flow is deflected toward passages on either side of the affected blade. Stalling intensifies, and the channel becomes blocked. The streamline deflection at the blade row inlet decreases the incidence angle of the forward blade and increases incidence on the rearward blade (opposite rotor rotation). The rearward blade becomes the next stalled blade.
The stall cell is induced to **propagate opposite rotor rotation** in the relative frame. As it does so, the original blade comes under the influence of the low incidence side of the cell and becomes unstalled.
As observed from the absolute frame, the cell appears to be moving in the direction of rotor rotation but at much reduced speed.

Surge and stall line

Surge/stall line



Outline

- Introduction
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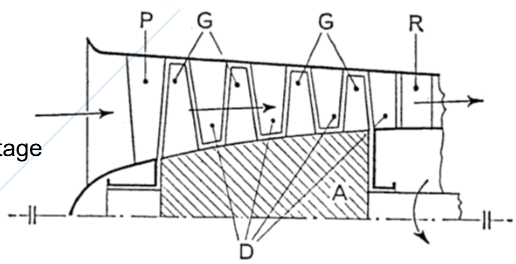
Axial compressor

Schematic of the machine

G: blade row rotors
D: fixed blade rows of diffusers
P: fixed blade row before 1st stage
R: fixed blade row downstream of last stage



While centrifugal compressors are usually single stage turbomachines, axial compressors are always **multistage**; this happens because the pressure ratio obtainable with an axial single stage is quite lower than the ratio a centrifugal one can reach.

$$\beta_{stage,ax} \sim 1.2 \div 1.4$$

$$\beta_{stage,centr} \sim 2 \div 5$$

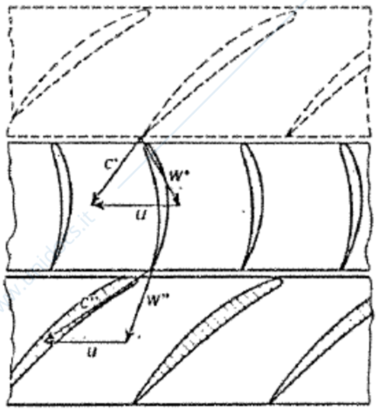
Axial compressor

Deflection and velocity triangles

This substantial distinction between axial and centrifugal machines is basically due to their **different working condition**, since axial compressors usually deal with much bigger mass flow rates than centrifugal ones.

This must be considered together with the general difficulty of deviating a flow which is being compressed, since the gas is pushed against the pressure gradient; hence an axial compressor is particularly at risk with respect to flow detachment and recirculation.

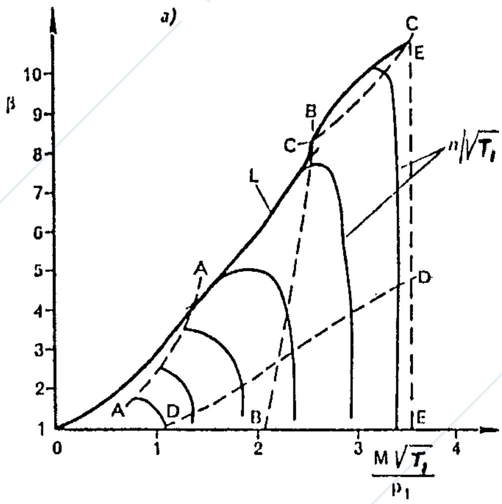
To avoid this harmful phenomenon, a lower deflection of the blades must be introduced and this consequentially means a lower compression ratio and a specific work absorption.



Axial compressor

Characteristic diagram

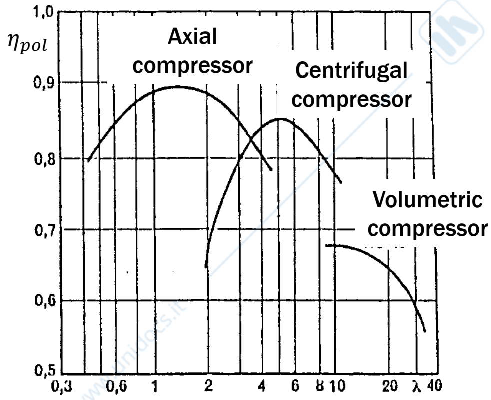
The **characteristic diagram** reflects the difference between centrifugal and axial machines; it clearly appears to be much steeper than the typical centrifugal one, proving that axial turbocompressors are much less convenient to be adjusted, since their performance abruptly decays moving away from design conditions.



Axial compressor

Efficiency

On the other hand, axial compressors turn out to be **more efficient** than centrifugal ones.



$$Load\ factor:\ \lambda = \frac{(2L_{is})^{3/4}}{2n\sqrt{\pi\,\dot{m}/\rho_1}}$$

In the development of the highly efficient modern axial flow compressor, the study of the 2D flow through cascades of airfoils has played an important part, since blades are carefully twisted along the length to accommodate the radial variations in flow.

Outline

- Introduction
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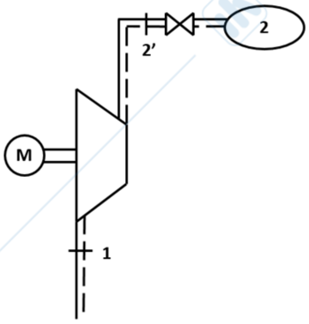
Off-design operations

Introduction



A compressor is controlled in order to satisfy the circuit requirement in terms of delivered mass-flow rate.
Considering the **infinite capacity** as a downstream environment, we would need to achieve variable mass flow rate complying with the constant pressure constraint ($p_2 = const$).

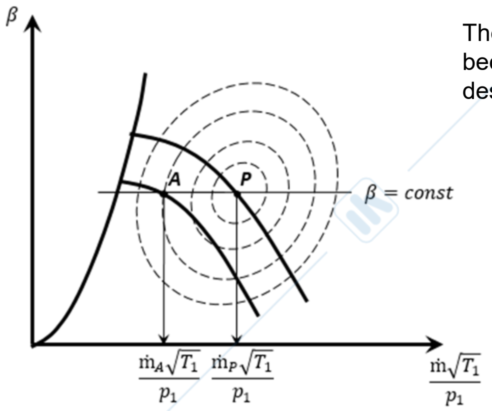
A compressor can be governed according to the following **methods**:

- change in the angular speed
- throttling at the compressor delivery
- throttling at the compressor suction
- geometry variation

Off-design operations

Change in the angular speed

We could easily change the mass flow rate by changing the angular speed: use of an **inverter** connected to a feedback control system.



The **efficiency** (P→A) anyhow decreases, because we are getting farther from the design point.

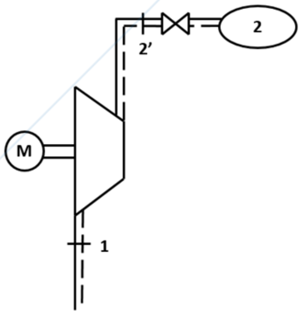
The **absorbed power** usually varies according to the mass flow rate; an increase of the first one determines an increase of the second one.

If we reduce the mass flow rate, we cannot go below a **minimum value** that corresponds to the surge limit.

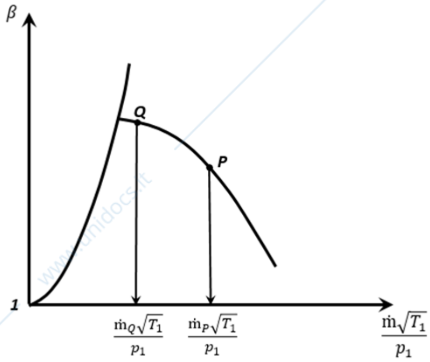
Off-design operations

Throttling at the compressor delivery

If we want to reduce the mass flow rate sticking to the compressor angular speed, we have to comply with an increase of the compression ratio.



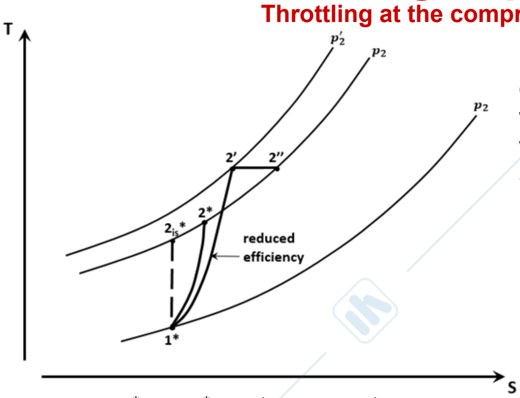
Hence a **throttling valve** is needed to reestablish the p_2 pressure.



Moving from P to Q, the efficiency decreases and so does the curve steepness on the T-S diagram.

Off-design operations

Throttling at the compressor delivery



If we merely refer to the **compression efficiency**, we should compare the work performed at Q to the ideal work we would perform to achieve p_2 starting from p_1:

$$L_{i,is} = c_p T_1 \left(\beta^{*\frac{R}{c_p}} - 1\right) = L_{i,is}^*$$

$$L_i^* = \frac{1}{\eta_{is,c}^*} \cdot c_p T_1 \left(\beta^{*\frac{R}{c_p}} - 1\right)$$

$$L_i = \frac{1}{\eta_{is,c}} \cdot c_p T_1 \left(\beta^{\frac{R}{c_p}} - 1\right)$$

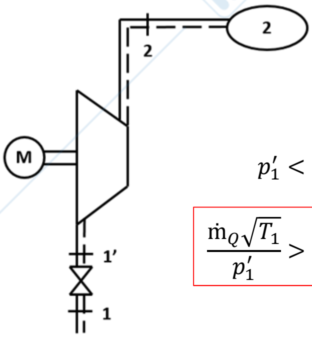
$$\eta_{comp} = \frac{L_{i,is}^*}{L_i} = \frac{L_{i,is}^*}{L_i^*} \cdot \frac{L_i^*}{L_i} = \eta_{is,c}^* \cdot \frac{L_i^*}{L_i} < \eta_{is,c}^*$$

As expected, the system efficiency more than decreases.

Off-design operations

Throttling at the compressor suction

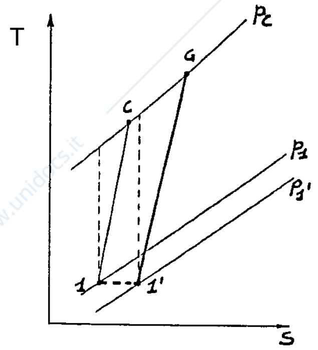
The effect on the characteristic diagram cannot be obtained as easily as before.



Considering the $\dot{m}_Q$ mass flow rate reduction produced by the previously described technique, the reduced mass flow rate factor would result to be **bigger for the throttling at the suction**.

$$p_1' < p_1$$

$$\frac{\dot{m}_Q\sqrt{T_1}}{p_1'} > \frac{\dot{m}_Q\sqrt{T_1}}{p_1}$$

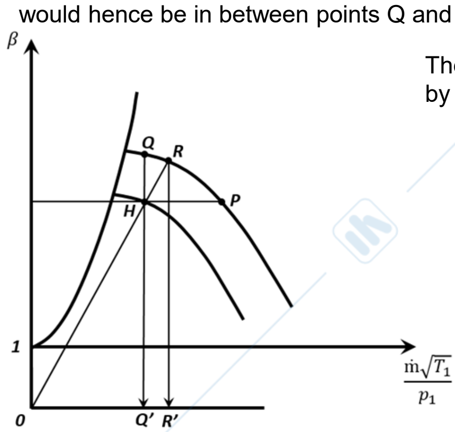


β increases due to the reduction of the p_1 pressure, for the given p_2 value.

Off-design operations

Throttling at the compressor suction

Point R, representing the operating point for a throttling at the compressor suction, would hence be in between points Q and P, since $\beta_R > \beta_P$ and $\frac{\dot{m}_Q\sqrt{T_1}}{p_1'} > \frac{\dot{m}_Q\sqrt{T_1}}{p_1}$



The **graphical construction** can be verified by considering the OHQ' and ORR' triangles.

$$\frac{\overline{OQ'}}{\overline{OR'}} = \frac{\overline{HQ'}}{\overline{RR'}} \qquad \text{geometric similitude}$$

$$\frac{\dfrac{\dot{m}_Q\sqrt{T_1}}{p_1}}{\dfrac{\dot{m}_Q\sqrt{T_1}}{p_1'}} = \frac{p_2/p_1}{p_2/p_1'}$$

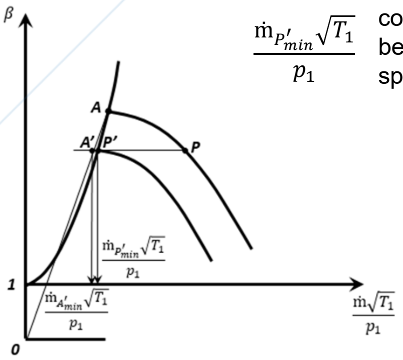
The identity is satisfied, hence confirming and assessing for the method validity.
The **efficiency** of point R is reduced with respect to point P but is anyhow greater than that of point Q.

Off-design operations

Throttling at the compressor suction

$$\eta_{comp} = \frac{\left(\beta^{*\frac{R}{c_p}} - 1\right)}{\left(\beta_R^{\frac{R}{c_p}} - 1\right)} \cdot \eta_{is,cR}$$

$\beta_R < \beta_Q$ and $\eta_{is,cR} > \eta_{is,cQ}$ thus opposing to the compression efficiency reduction.
Moreover, by throttling at the compression suction, we can possibly cover a wider region.



$\frac{\dot m_{P'_{min}}\sqrt{T_1}}{p_1}$ corresponds to the **minimum mass flow rate** to be achieved by changing the compressor angular speed ($\beta = \text{const.}$)

$$\dot m_{P',min} = \left[\frac{\dot m\sqrt{T_1}}{p_1}\right]_{P'} \cdot \frac{p_1}{\sqrt{T_1}}$$

Let's now **compare** the last two governing techniques.
Moving along the P angular speed curve, the extreme point would be point A on the surge line.

Off-design operations

Minimum mass-flow rate

If we consider point A as the equivalent of point Q (throttling at the **delivery**)

$$\dot m_{A,min} = \left[\frac{\dot m\sqrt{T_1}}{p_1}\right]_{A} \cdot \frac{p_1}{\sqrt{T_1}}$$

If we now consider point A as the equivalent of point R (throttling at the **suction**)

$$\dot m_{R,min} = \left[\frac{\dot m\sqrt{T_1}}{p_1}\right]_{R} \cdot \frac{p_1'}{\sqrt{T_1}} < \dot m_{Q,min}$$

In order to properly detect $\dot m_{R,min}$, we can go through a **reverse construction**, where A is the result of the previously described technique

$$\dot m_{R,min} = \dot m_{A',min} = \left[\frac{\dot m\sqrt{T_1}}{p_1}\right]_{R} \cdot \frac{p_1^*}{\sqrt{T_1^*}}$$

Gas Turbine Power Plants

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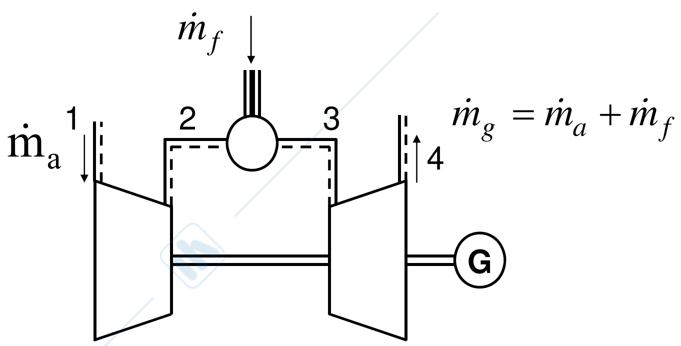
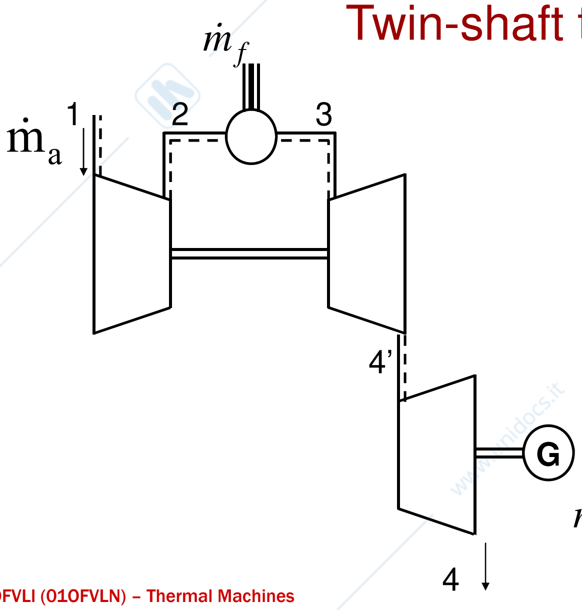
BSc in
Automotive Engineering

Single shaft gas turbine

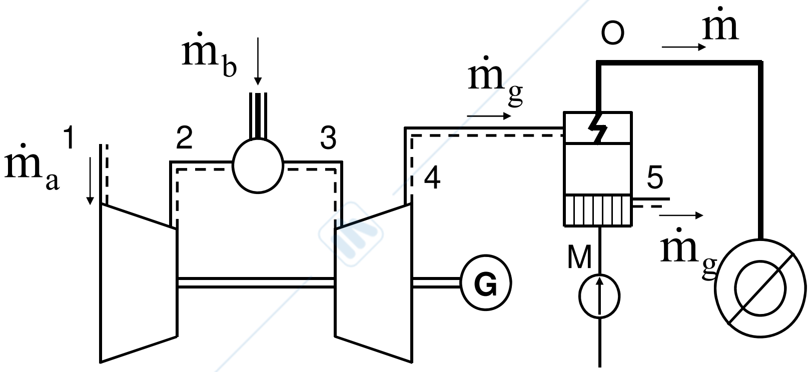


Twin-shaft turbine

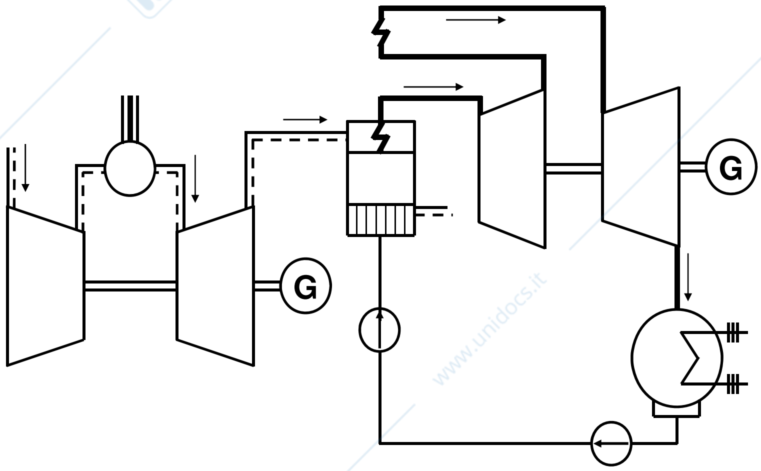


$$\dot{m}_g = \dot{m}_a + \dot{m}_f$$

Cogenerative plant with gas turbine



Gas-steam combined power plant



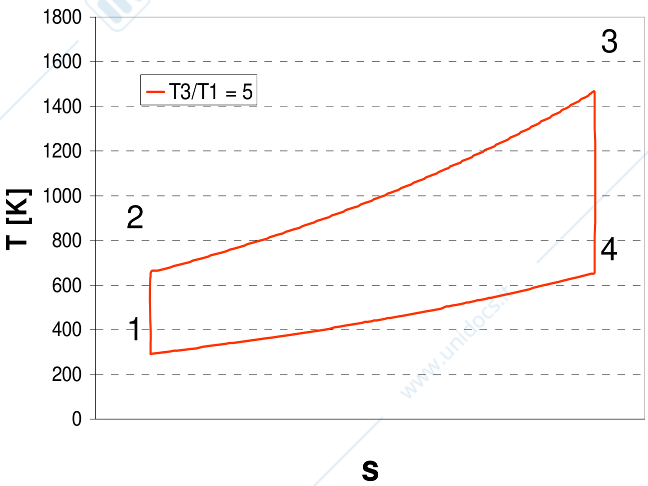
Ideal cycle

Thermodynamic cycle without losses; the working fluid is an ideal gas.

↓

JOULE CYCLE

Joule cycle



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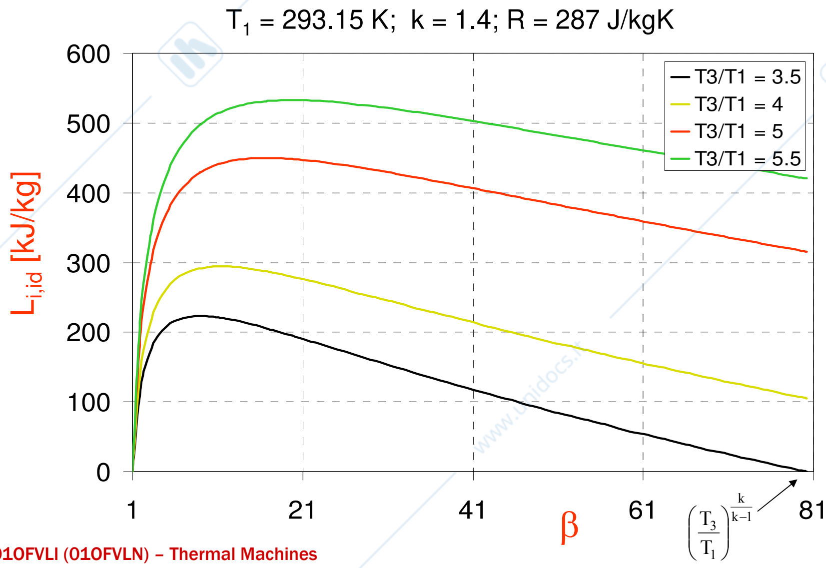
Specific work of ideal cycle

$$L_{i,id} = c_p\left(T_3 - T_4\right) - c_p\left(T_2 - T_1\right) =$$

$$= c_p T_3\left(1 - \frac{1}{T_3/T_4}\right) - c_p T_1\left(\frac{T_2}{T_1} - 1\right) =$$

$$= c_p T_3\left(1 - \frac{1}{\beta^{\frac{k-1}{k}}}\right) - c_p T_1\left(\beta^{\frac{k-1}{k}} - 1\right)$$

Specific work of ideal cycle



Max. specific power output of ideal cycle

$$L_{i,id} = c_p(T_3 - T_4) - c_p(T_2 - T_1) =$$
$$= c_p(T_3 + T_1) - c_p(T_2 + T_4)$$

T_3 e T_1 fixed: $L_{i,id} = L_{i,id,max} \Rightarrow (T_2 + T_4)_{min}$

Ideal cycle: $T_2 T_4 = T_1 T_3 = const$

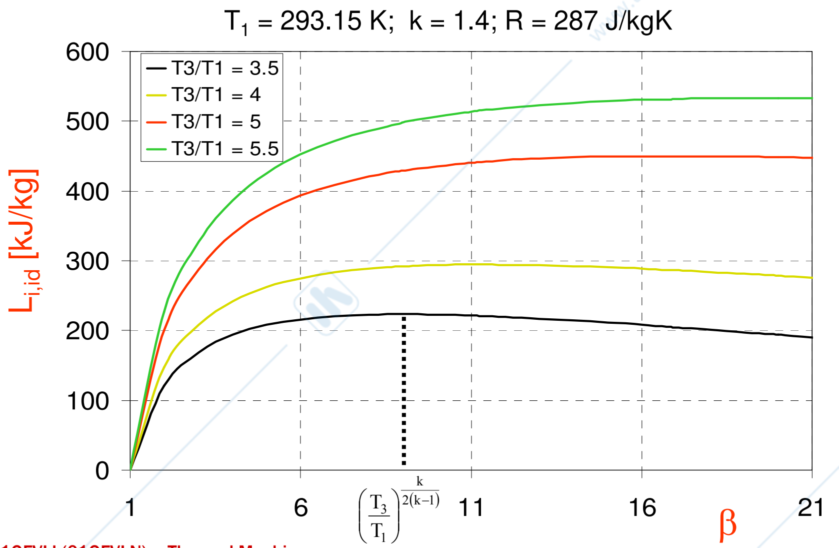
$$L_{i,id,max} \rightarrow T_2 = T_4 = \sqrt{T_2 T_4} = \sqrt{T_1 T_3}$$

Max. specific power output of ideal cycle

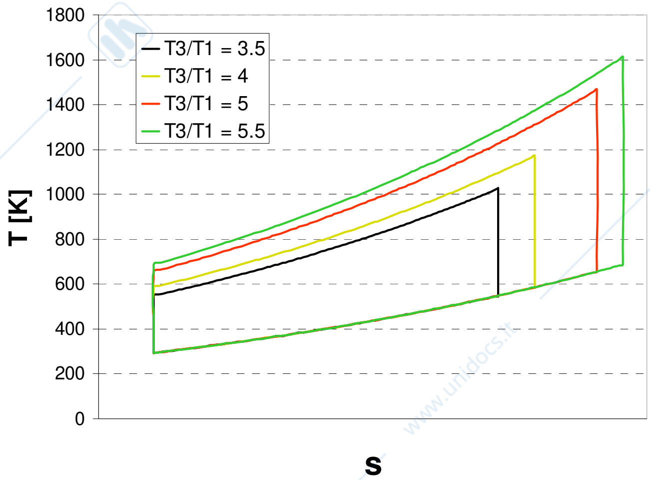
$$L_{i,id,max} \rightarrow T_2 = T_4 = \sqrt{T_2 T_4} = \sqrt{T_1 T_3}$$

$$\beta_{L_{i,id,max}} = \left(\frac{\sqrt{T_1 T_3}}{T_1} \right)^{\frac{k}{k-1}} = \left(\frac{T_3}{T_1} \right)^{\frac{k}{2(k-1)}}$$

Max. specific power output of ideal cycle



Max. specific power output of ideal cycle



Efficiency of ideal cycle

$$\eta_{id} = \frac{L_{i,id}}{Q_{1,id}} = \frac{L_{i,id}}{c_p\left(T_3 - T_2\right)}$$

$$\eta_{id} = \frac{c_p(T_3 - T_4) - c_p(T_2 - T_1)}{c_p(T_3 - T_2)} = \frac{c_p(T_3 - T_2) - c_p(T_4 - T_1)}{c_p(T_3 - T_2)} = 1 - \frac{c_p(T_4 - T_1)}{c_p(T_3 - T_2)} =$$

$$= 1 - \frac{T_3 / \beta^{\frac{k-1}{k}} - T_1}{T_3 - T_1 \beta^{\frac{k-1}{k}}} = \quad 1 - \frac{1}{\beta^{\frac{k-1}{k}}}\left(\frac{T_3 / \beta^{\frac{k-1}{k}} - T_1}{T_3 / \beta^{\frac{k-1}{k}} - T_1}\right) = \quad 1 - \frac{1}{\beta^{\frac{k-1}{k}}}$$

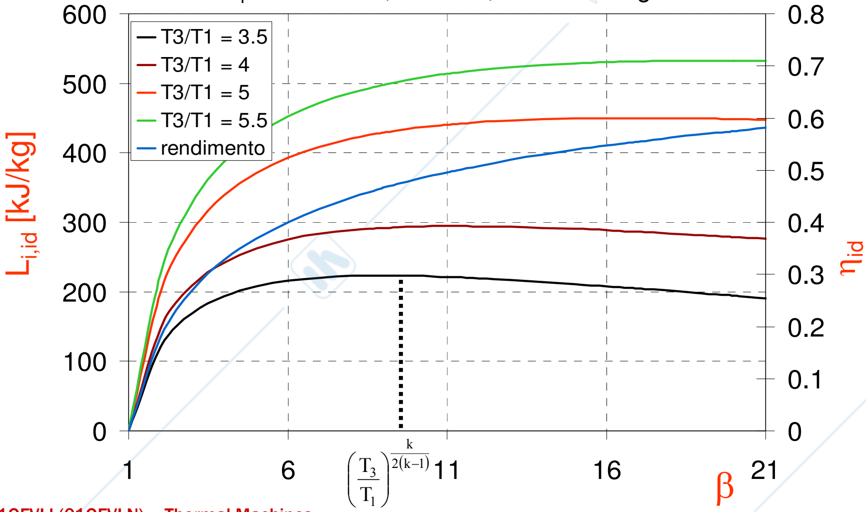
Efficiency of ideal cycle

$$\eta_{id} = \frac{L_{i,id}}{Q_{1,id}} = \frac{L_{i,id}}{c_p\left(T_3 - T_2\right)}$$

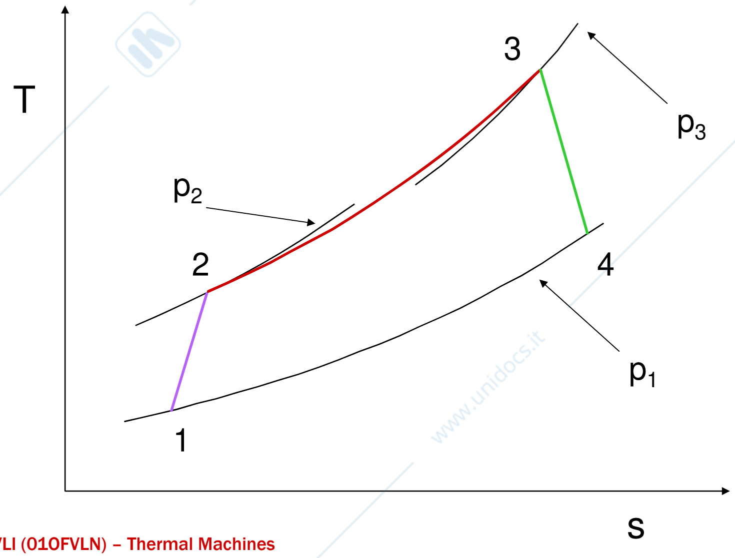
$$\eta_{id} = 1 - \frac{1}{\beta^{\frac{k-1}{k}}}$$

η_{id} and $L_{i,id}$ patterns

T_1 = 293.15 K; k = 1.4; R = 287 J/kgK



Real cycle ("open" cycle)



Compression work in real cycle

$$L_{i,c} = c_p (T_2 - T_1) = \frac{1}{\eta_c} c_p T_1 \left(\beta^{R/c_p} - 1\right) =$$

$$= c_p T_1 \left(\beta^{\frac{1}{\eta_{y,c}} \frac{R}{c_p}} - 1\right) \qquad \beta = \frac{p_2}{p_1}$$

Turbocompressor efficiencies:
- η_c isentropic
- $\eta_{y,c}$ polytropic

Expansion work in real cycle

$$L_{i,t} = c'_p (T_3 - T_4) = \eta_t c'_p T_3 \left(1 - \frac{1}{\beta_t^{R'/c'_p}} \right) =$$

$$= c'_p T_3 \left(1 - \frac{1}{\beta_t^{\eta_{y,t} \frac{R'}{c'_p}}} \right) \qquad \beta_t = \frac{p_3}{p_4} = \eta_{\pi b} \beta$$

Turbine efficiencies

- η_t isentropic
- $\eta_{y,t}$ polytropic

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Expansion work in real cycle

$$L_{i,t} = c'_p (T_3 - T_4) = \eta_t c'_p T_3 \left(1 - \frac{1}{\beta_t^{R'/c'_p}} \right) =$$

$$= c'_p T_3 \left(1 - \frac{1}{\beta_t^{\eta_{y,t} \frac{R'}{c'_p}}} \right) \qquad \beta_t = \frac{p_3}{p_4} = \eta_{\pi b} \beta$$

Pneumatic efficiency of the burner

$$\eta_{\pi b} = \frac{p_3}{p_2}$$

Power of the gas turbine plant

$$\dot{\mathcal{L}}_{plant} = P_{plant} = \eta_m \left(P_{i,t} - P_{i,c} \right)$$

$$P_{i,t} = \left(\dot{m}_a + \dot{m}_f \right) L_{i,t} = \dot{m}_a \left(\frac{1+\alpha}{\alpha} \right) L_{i,t} \qquad P_{i,c} = \dot{m}_a L_{i,c}$$

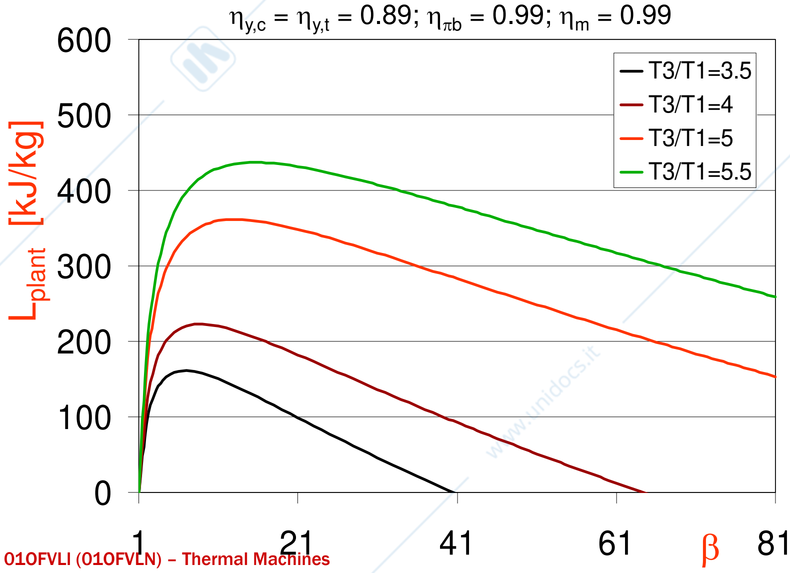
$$P_{plant} = \eta_m \dot{m}_a \left(\frac{1+\alpha}{\alpha} L_{i,t} - L_{i,c} \right) \qquad \alpha = \frac{\dot{m}_a}{\dot{m}_f}$$

Specific power of the gas turbine plant

$$P_{plant} = \eta_m \dot{m}_a \left(\frac{1+\alpha}{\alpha} L_{i,t} - L_{i,c} \right)$$

$$L_{plant} = \frac{P_{plant}}{\dot{m}_a} = \eta_m \left(\frac{1+\alpha}{\alpha} L_{i,t} - L_{i,c} \right)$$

T_1 = 293.15 K; k = 1.4; R = 287 J/kgK; k' = 1.333; R'= 286.8 J/kgK;
$\eta_{y,c} = \eta_{y,t}$ = 0.89; $\eta_{\pi b}$ = 0.99; η_m = 0.99



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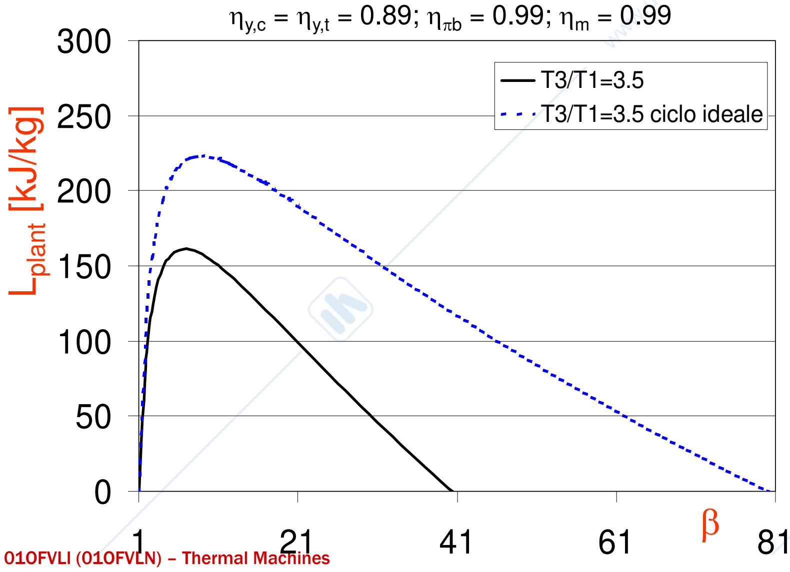
T_1 = 293.15 K; k = 1.4; R = 287 J/kgK; k' = 1.333; R'= 286.8 J/kgK;
$\eta_{y,c} = \eta_{y,t}$ = 0.89; $\eta_{\pi b}$ = 0.99; η_m = 0.99



Efficiency of the gas turbine plant

$$\eta_{plant} = \frac{P_{plant}}{\dot{m}_f H_{Lp}}$$

$$\eta_b \dot{m}_f H_{Lp} = (\dot{m}_a + \dot{m}_f) c'_p (T_3 - T_2)$$

η_b → Burner efficiency

$$\dot{m}_f H_{Lp} = \frac{(\dot{m}_a + \dot{m}_f) c'_p (T_3 - T_2)}{\eta_b}$$

$$\eta_{plant} = \eta_b \frac{P_{plant}}{(\dot{m}_a + \dot{m}_f) c'_p (T_3 - T_2)}$$

Efficiency of the gas turbine plant

$$\eta_{plant} = \eta_b \frac{P_{plant}}{(\dot{m}_a + \dot{m}_f) c'_p (T_3 - T_2)}$$

$$P_{plant} = \dot{m}_a L_{plant}$$

$$\eta_{plant} = \eta_b \frac{\dot{m}_a L_{plant}}{\dot{m}_a \left(\frac{1+\alpha}{\alpha}\right) c'_p (T_3 - T_2)}$$

$$L_{plant} = \eta_m \left(\frac{1+\alpha}{\alpha} L_{i,t} - L_{i,c}\right)$$

$$\eta_{plant} = \eta_b \eta_m \frac{\frac{1+\alpha}{\alpha} L_{i,t} - L_{i,c}}{\left(\frac{1+\alpha}{\alpha}\right) c'_p (T_3 - T_2)}$$

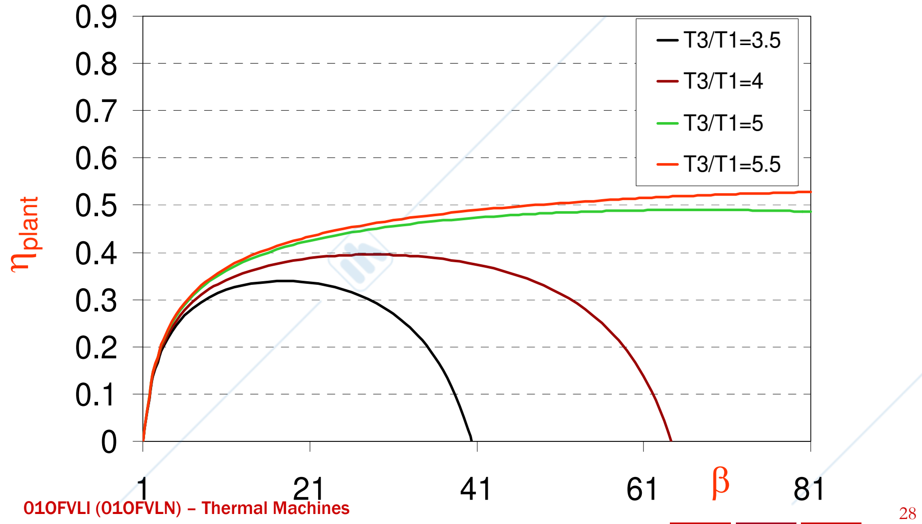
Efficiency of the gas turbine plant

$$\eta_{plant} = \eta_b \eta_m \frac{\frac{1+\alpha}{\alpha} L_{i,t} - L_{i,c}}{\left(\frac{1+\alpha}{\alpha}\right) c'_p \left(T_3 - T_2\right)}$$

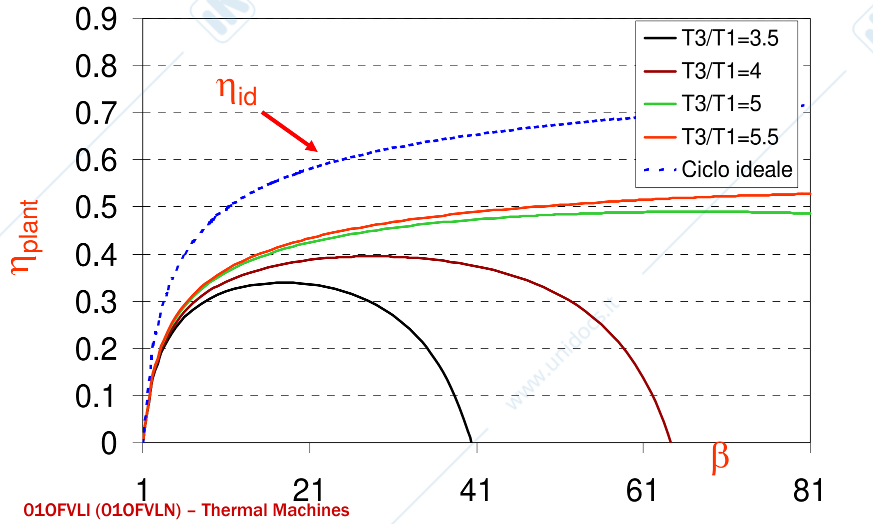
$$L_{i,t} = \eta_t c'_p T_3 \left(1 - \frac{1}{\beta_t^{\frac{R'}{c'_p}}}\right) = c'_p T_3 \left(1 - \frac{1}{\beta_t^{\eta_{y,t}\frac{R'}{c'_p}}}\right) \qquad \beta_t = \frac{p_3}{p_4} = \eta_{\pi b}\beta$$

$$L_{i,c} = \frac{1}{\eta_c} c_p T_1 \left(\beta^{R/c_p} - 1\right) = c_p T_1 \left(\beta^{\frac{1}{\eta_{y,c}}\frac{R}{c_p}} - 1\right) \qquad T_2 = T_1 + \frac{L_{i,c}}{c_p}$$

T_1 = 293.15 K; k = 1.4; R = 287 J/kgK; k' = 1.333; R'= 286.8 J/kgK;
$\eta_{y,c} = \eta_{y,t} = 0.89$; $\eta_{\pi b} = 0.99$; $\eta_m = 0.99$



T_1 = 293.15 K; k = 1.4; R = 287 J/kgK; k' = 1.333; R'= 286.8 J/kgK;
$\eta_{y,c} = \eta_{y,t}$ = 0.89; $\eta_{\pi b}$ = 0.99; η_m = 0.99



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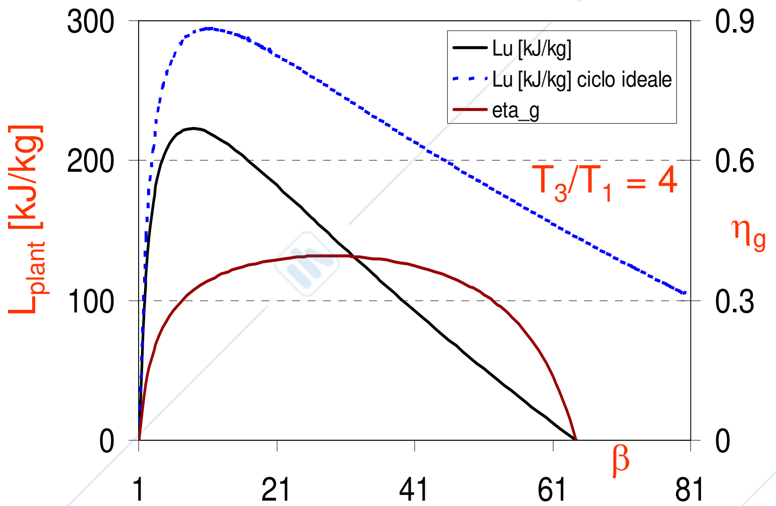

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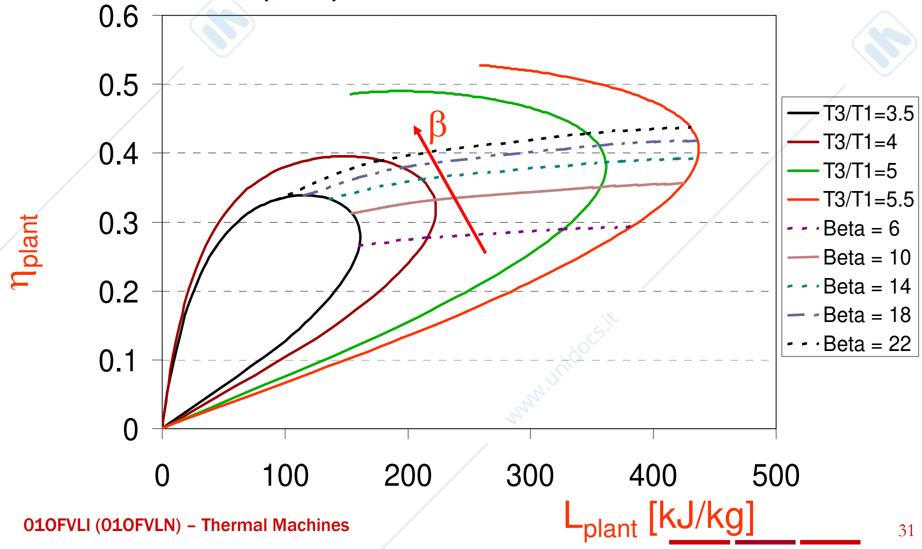
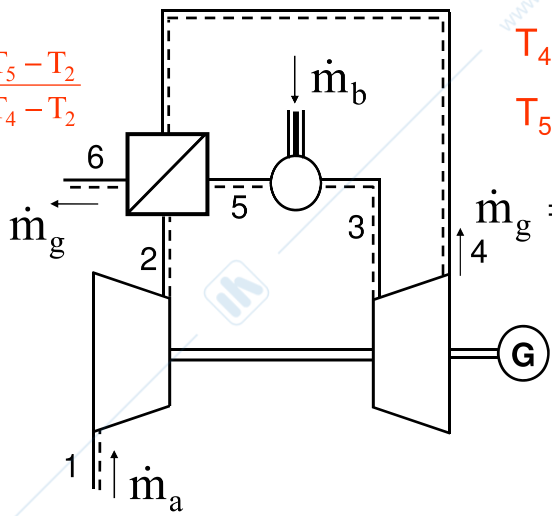


Regeneration in gas turbines

$$R_s = \frac{T_5 - T_2}{T_4 - T_2}$$

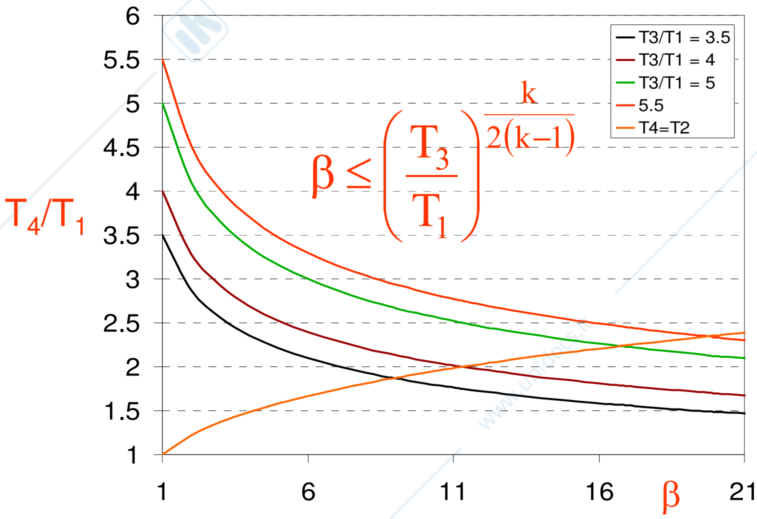
$T_4 > T_2$

$T_5 \leq T_4$



$$\dot{m}_g = \dot{m}_a + \dot{m}_b$$

Ideal cycle



Ideal cycle: efficiency of regenerative cycle

$$\eta_{id,R_s} = \frac{L_{i,id}}{Q_{1,id,Rs}} = \frac{L_{i,id}}{c_p(T_3 - T_5)}$$

$R_s = 1$: $T_5 = T_4$

$$\eta_{id,R_s=1} = \frac{L_{i,id}}{c_p(T_3 - T_4)} = \frac{c_p(T_3 - T_4) - c_p(T_2 - T_1)}{c_p(T_3 - T_4)} = 1 - \frac{c_p(T_2 - T_1)}{c_p(T_3 - T_4)}$$

Ideal cycle: efficiency of regenerative cycle

$$\eta_{id,R_s=1} = 1 - \frac{c_p(T_2 - T_1)}{c_p(T_3 - T_4)} = 1 - \frac{T_1}{T_3} \frac{\beta^{\frac{k-1}{k}} - 1}{1 - \frac{1}{\beta^{\frac{k-1}{k}}}} = 1 - \frac{1}{T_3/T_1} \beta^{\frac{k-1}{k}}$$

$$\eta_{id,R_s=1} = 1 - \frac{1}{T_3/T_1} \beta^{\frac{k-1}{k}}$$

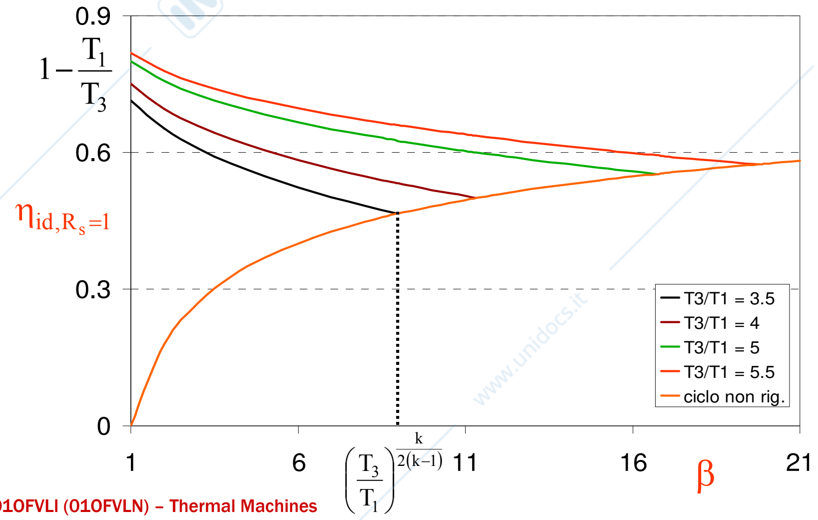
Ideal cycle: efficiency of regenerative cycle

$$\eta_{id,R_s=1} = 1 - \frac{1}{T_3/T_1} \beta^{\frac{k-1}{k}}$$

when $\beta = \left(\frac{T_3}{T_1}\right)^{\frac{k}{2(k-1)}}$ → $\eta_{id,R_s=1} = \eta_{id}$

when $\beta \to 1$ → $\eta_{id,R_s=1} \to 1 - \frac{T_1}{T_3}$

Ideal cycle: efficiency of regenerative cycle



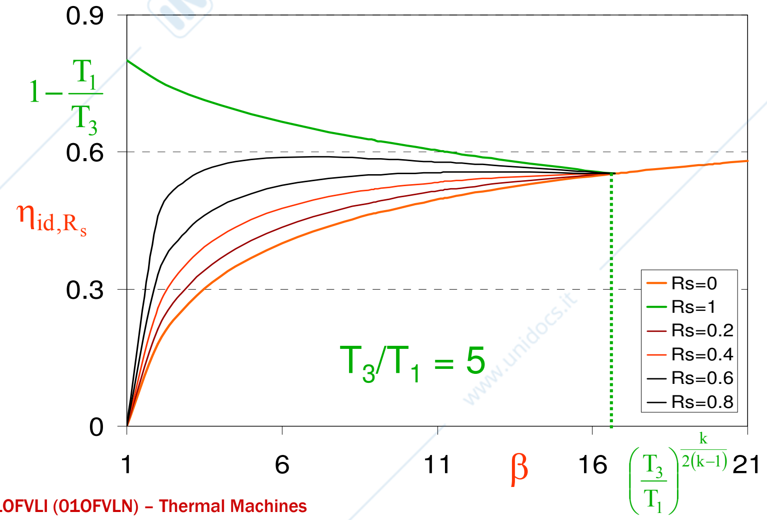
Ideal cycle: efficiency of regenerative cycle

$$\eta_{id,R_s} = \frac{L_{i,id}}{Q_{1,id,Rs}} = \frac{L_{i,id}}{c_p\left(T_3 - T_5\right)}$$

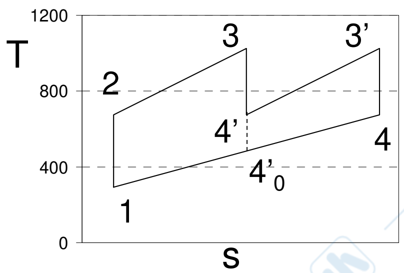
$R_s < 1$: $\quad T_5 = T_2 + R_s\left(T_4 - T_2\right)$

$$\frac{1}{\eta_{id,R_s}} = \left(1 - R_s\right)\frac{1}{\eta_{id}} + R_s\frac{1}{\eta_{id,R_s=1}}$$

Ideal cycle: efficiency of regenerative cycle



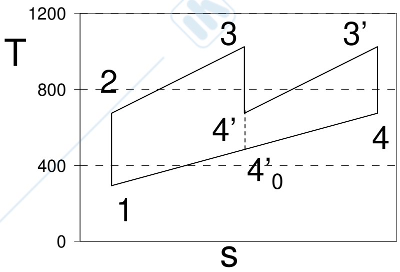
Re-burning in ideal cycle



- Increase of $L_{i,id}$

- Decrease of η_{id} (the additional cycle $4'_0 4'3'4$ features β lower than reference cycle)

Re-heating in ideal cycle



$$\beta' = \frac{p_3}{p'_4} \qquad \beta'' = \frac{p'_3}{p_4}$$

$$\beta'\beta'' = \beta = \frac{p_2}{p_1}$$

For $T'_3 = T_3$, the increase of specific work peaks when:

$$\beta' = \beta'' = \sqrt{\beta}$$

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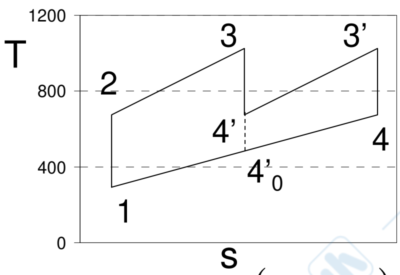
Re-heating in ideal cycle



$$T_3 = T'_3 \qquad \beta' = \beta'' = \sqrt{\beta}$$

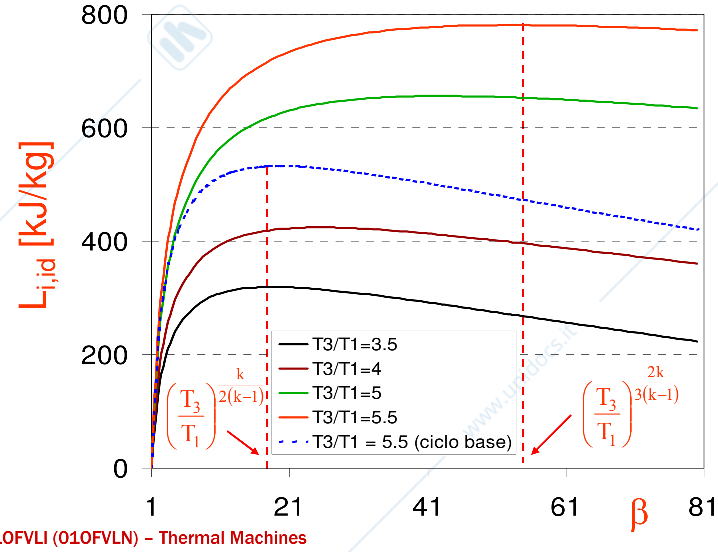
$$\frac{T_3}{T'_4} = \beta^{\frac{k-1}{2k}} \qquad \frac{T'_3}{T_4} = \beta^{\frac{k-1}{2k}}$$

$$T_4 = T'_4$$

$$L_{i,id} = \underbrace{c_p\left(T_3 - T'_4\right) + c_p\left(T'_3 - T_4\right)} - c_p\left(T_2 - T_1\right)$$

$$L_{i,id} = 2c_p T_3\left(1 - \frac{1}{\beta^{\frac{k-1}{2k}}}\right) - c_p T_1\left(\beta^{\frac{k-1}{k}}\right)$$

Re-heating in ideal cycle



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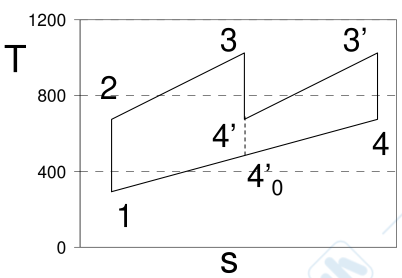
Re-heating in ideal cycle



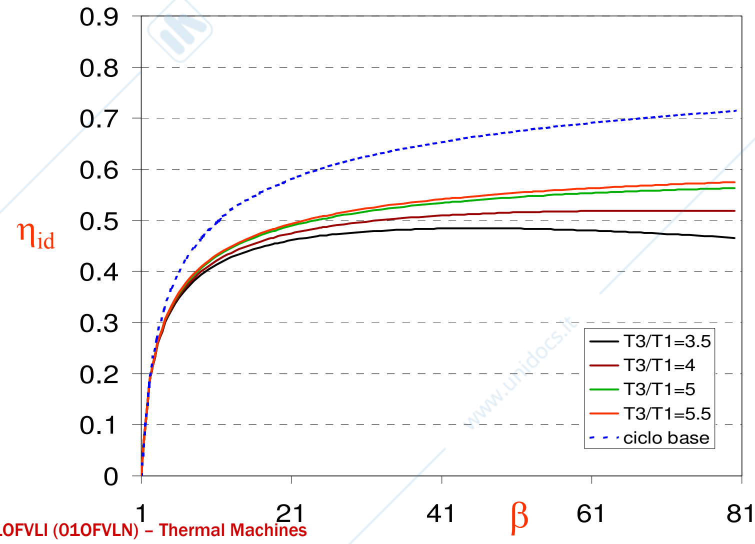
$$T_3 = T_3' \qquad \beta' = \beta'' = \sqrt{\beta}$$

$$\frac{T_3}{T_4'} = \beta^{\frac{k-1}{2k}} \qquad \frac{T_3'}{T_4} = \beta^{\frac{k-1}{2k}}$$

$$T_4 = T_4'$$

$$\eta_{id} = \frac{L_{i,id}}{Q_{1,id}} = \frac{L_{i,id}}{c_p(T_3 - T_2) + c_p(T_3' - T_4')}$$

Re-heating in ideal cycle



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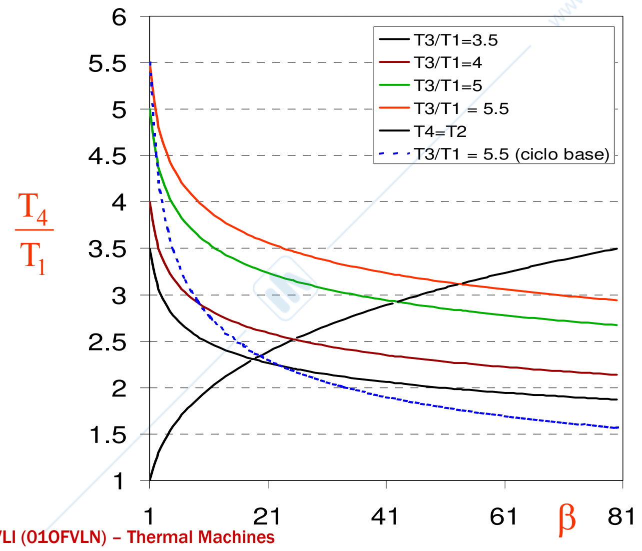
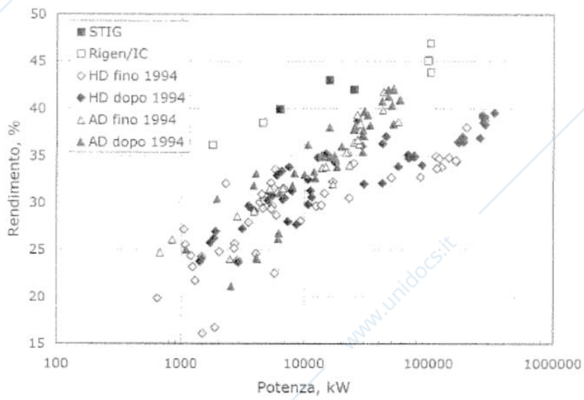
Re-heating in ideal cycle



Perfomance of existing plants

STIG = steam-injection cycles – Rigen/IC = regenerative and/or intercooled compression cycles – HD = heavy-duty – AD = derived from aircraft applications



Source: G. Lozza, "Turbine a gas e cicli combinati", 2° ed., Esculapio Eds., Bologna, 2006.

Gas/Steam Combined Power-Plants and Cogeneration

Heat rejected by a gas turbine plant can be used to raise steam temperature in a "Heat Recovery Steam Generator" (HRSG) in order to:

- produce power in a lower steam cycle;

↓

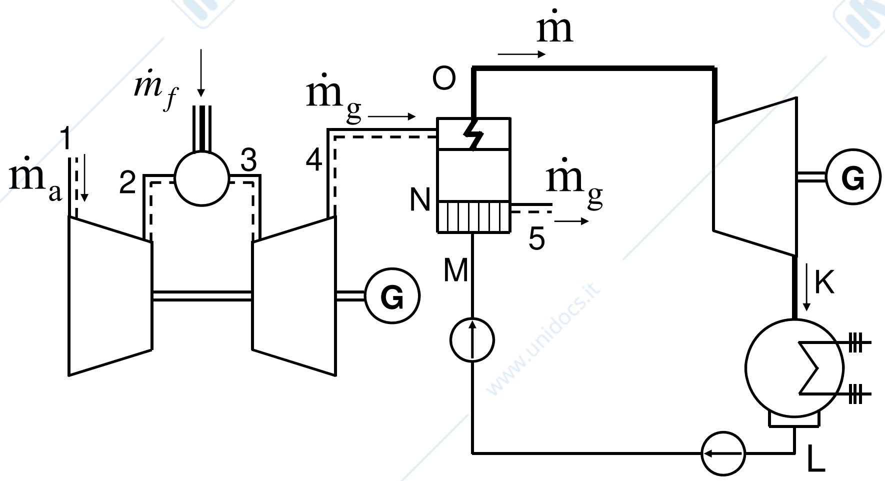
Gas/Steam Combined Power-Plant (CPP)

- deliver steam to a thermal user

↓

Cogeneration (CG) Gas-Turbine Plant

Gas/Steam Combined Power-Plants



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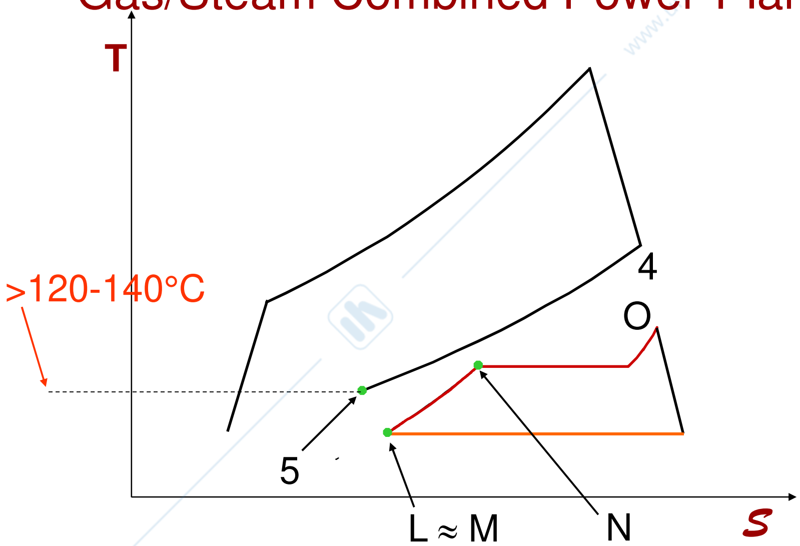
Gas/Steam Combined Power-Plants

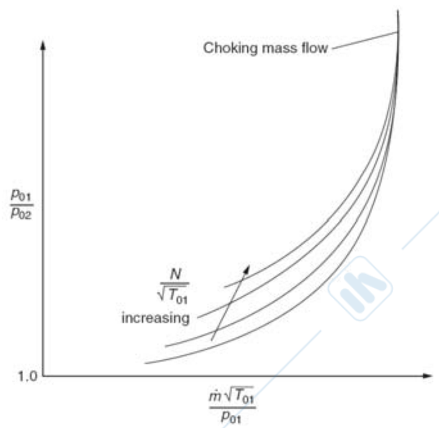


Characteristic curves of Turbines

Prof. Mirko Baratta

Dipartimento Energia
Politecnico di Torino

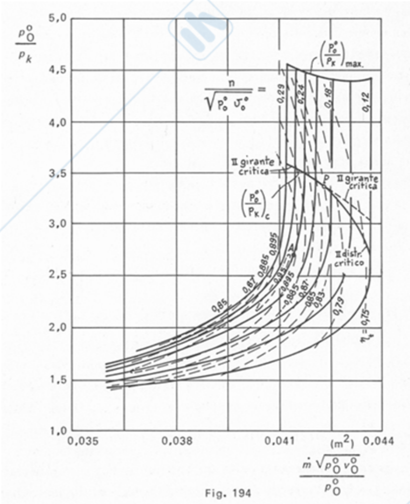
Characteristic of a turbine



Turbine characteristics are plotted in the same way as compressor characteristics but the behaviour is very different. Turbines are able to operate with a high pressure ratio across each stage because the boundary layers on the surfaces of the turbine blades are accelerating and therefore stable. Once one or more turbine stators are fully choked, the operating point is independent of $N/\sqrt{T_{01}}$ because the rotation of the blades has virtually no influence on either the pressure ratio or the nondimensional mass flow rate.

Source: S.L. Dixon, C.A. Hall, "Fluid Mechanics and Thermodynamics of Turbomachinery", 6th ed., Butterworth-Heinemann, Elsevier.

Characteristic of a turbine

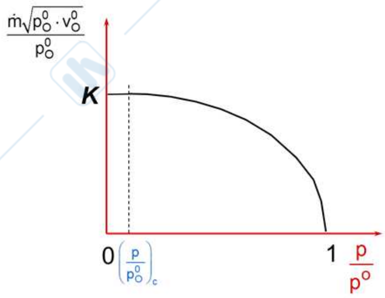


Two-stage steam reaction turbine characteristic

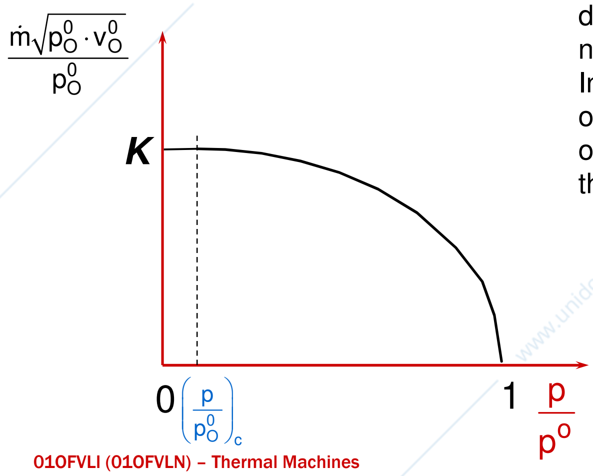
Source: A.E. Catania, «Complementi di Macchine», Levrotto&Bella, Torino.

Characteristic of a turbine

The effect of the speed on the turbine performance can be neglected in turbines for energy production systems. As a matter of fact, such sistem work under fixed engine-speed conditions.
From a practical point of view, the turbine characteristic can be approximated with the one of a nozzle.



Characteristic of a turbine



The critical pressure ratio depends mainly on the number of stages.
In a multistage turbine, the operation point falls most ofter under or very close to the chocking conditions.

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Characteristic of a turbine

The following expressions can be used:

$$\frac{\dot{m}\sqrt{p_O^0 \cdot v_O^0}}{p_O^0} = K$$ (Chocked turbine)

$$\left(\frac{\frac{\dot{m}\sqrt{p_O^0 \cdot v_O^0}}{p_O^0}}{K}\right)^2 + \left(\frac{\frac{p}{p_0{}^\circ} - \left(\frac{p}{p_0{}^\circ}\right)_c}{1 - \left(\frac{p}{p_0{}^\circ}\right)_c}\right)^2 = 1$$ (Subcritical turbine)

The gas turbine as a compressor 'user'

Hp: turbine under chocking conditions

$$\frac{\dot{m}_g\sqrt{p_3 \cdot v_3}}{p_3} = K \quad \dot{m}_g = \frac{1+\alpha}{\alpha}\cdot\dot{m}$$

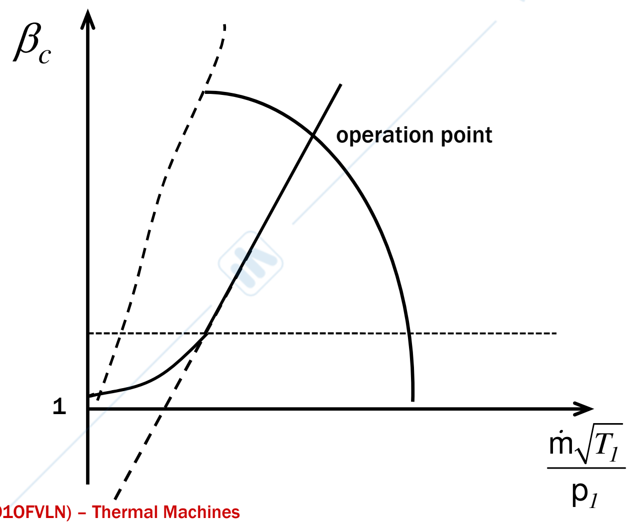
$$\dot{m} = \frac{\alpha}{1+\alpha}K\frac{p_3}{\sqrt{R \cdot T_3}} = \frac{\alpha}{1+\alpha}K\frac{\beta_c \cdot \eta_{\pi b} \cdot p_1}{\sqrt{R \cdot T_3}}$$

$$\frac{\dot{m}}{p_1} = \frac{\alpha}{1+\alpha}K\frac{\beta_c \cdot \eta_{\pi b}}{\sqrt{R \cdot T_3}}$$

$$\frac{\dot{m}\sqrt{T_1}}{p_1} = \frac{\alpha}{1+\alpha}K\frac{\beta_c \cdot \eta_{\pi b}\sqrt{T_1}}{\sqrt{R \cdot T_3}}$$

$$\beta_c = const. \cdot \sqrt{T_3} \cdot \frac{\dot{m}\sqrt{T_1}}{p_1} \qquad \left(\frac{1+\alpha}{\alpha} \approx const.\right)$$

The gas turbine as a compressor 'user'



Volumetric compressors

Proff. Mirko Baratta, Daniela Misul

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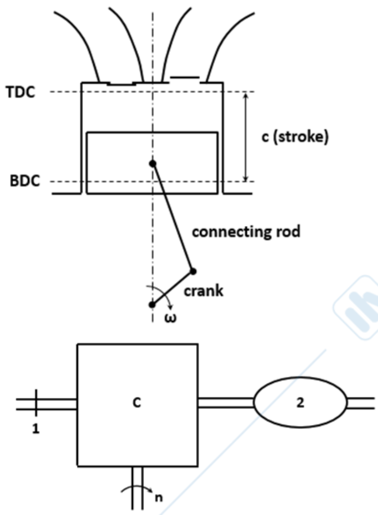
Reciprocating compressors (RC)



The piston slides within a cylinder moving in between the Top Dead Center (**TDC**) and the Bottom Dead Center (**BDC**) position. TDC corresponds to the minimum chamber volume (clearances) whereas BDC corresponds to the maximum swept volume.

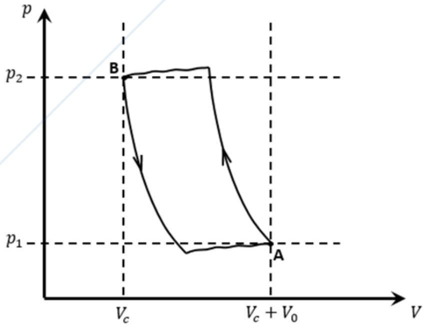
V_0 = displacement = $V_{max} - V_{min}$

μ = clearance coefficient = $\frac{V_{min}}{V_0}$

In order to understand the compressor functioning we can refer to the **working cycle** performed by the machine. The working cycle is to be represented on a p-V diagram and depicts the in-cylinder pressure as a function of the displaced volume.

RC - Real cycle

The compressor is normally equipped with **automatic valves** which automatically open and close depending on the pressure difference experienced by the valve itself. Considering point A, a slight increase in pressure is sufficient to close the inlet valve, thus confining the environment within the cylinder. The piston displacement hence produces a gradual **compression** of the trapped fluid as long as $p_{cyl} < p_2$. Considering the viscous losses through the valve, a pressure higher than p_2 is needed to force the delivery valve to open.



$$\cancel{L_t} = \int vdp + \Delta E_k + \cancel{\Delta E_g} + L_w \qquad L_w \propto \frac{w^2}{2}$$

$$\frac{\Delta p}{\rho} = (p_{downst} - p_{upst}) \cdot \frac{1}{\rho} = -(\Delta E_k + L_w)$$

$$p_{upst} - p_{downst} = \rho \cdot (\Delta E_k + L_w) \longrightarrow p_{upst} - p_{downst} \propto w^2$$

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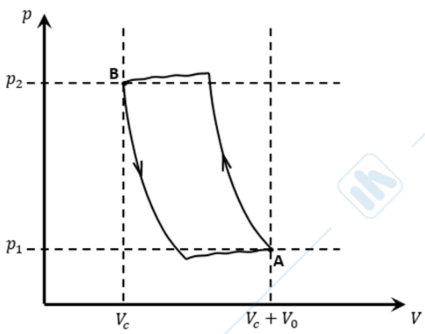
RC - Real cycle

The **pressure drop** across the valve hence reduces as the piston approaches the TDC, i.e. as the piston speed and, consequently, the fluid speed, diminishes. Consistently, point B holds the pressure of the delivery environment. As the piston reverts its motion, the delivery valve closes and the in-cylinder pressure diminishes. The fluid entrapped in the clearances expands down to the suction pressure. A pressure lower than p_1 is needed to produce the inlet valve to open. If we consider the **work performed by the piston** on the fluid to be positive (operating machine convention):

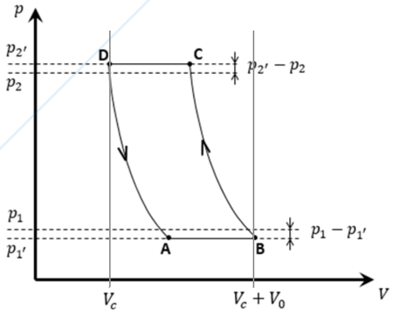


$$d\mathcal{L} = -pdV \qquad \mathcal{L}_c = \oint -pdV$$

$$P_i = \mathcal{L}_c n_c \qquad n_c = n \qquad P_{abs} = \frac{1}{\eta_m} P_i = \frac{1}{\eta_m} \mathcal{L}_c n$$

RC - Conventional cycle

The working cycle area is not easily to be computed unless the actual in-cylinder pressure time-history is available. We hence normally refer to a **conventional cycle** featuring constant pressure during the intake and the delivery and performing the actual machine work.



$$\delta_1 = \frac{p_1 - p_{1'}}{p_1} \longrightarrow p_{1'} = p_1(1-\delta_1)$$

$$\delta_2 = \frac{p_{2'} - p_2}{p_2} \longrightarrow p_{2'} = p_2(1+\delta_2)$$

The compression and the expansion phases are described by means of equivalent indexes ($m^*, m^{*\prime}$):

$$p_A V_A^{m^{*\prime}} = p_D V_D^{m^{*\prime}} \qquad p_B V_B^{m^*} = p_C V_C^{m^*}$$

The two curves differ from actual polytropic evolutions given that the mass is not constant (leakages). Should the mass be constant, m^* and $m^{*\prime}$ would anyhow differ from the isentropic k index due to the heat exchanges.

RC - Conventional cycle

$$\mathcal{L}_c = \oint -p dV = \oint V dp \qquad \oint d(pV) = 0$$

$$\mathcal{L}_c = \cancel{\int_A^B V dp} + \int_B^C V dp + \cancel{\int_C^D V dp} + \int_D^A V dp$$

$$\mathcal{L}_c = \frac{m^*}{m^*-1} p_B V_B \left[\left(\frac{p_C}{p_B}\right)^{\frac{m^*-1}{m^*}} - 1 \right] - \frac{m^{*\prime}}{m^{*\prime}-1} p_A V_A \left[\left(\frac{p_D}{p_A}\right)^{\frac{m^{*\prime}-1}{m^{*\prime}}} - 1 \right]$$

$$\beta_i = \frac{p_C}{p_B} = \frac{p_2(1+\delta_2)}{p_1(1-\delta_1)} = \beta \frac{(1+\delta_2)}{(1-\delta_1)}$$

The **losses** are mainly to be connected to the L_w viscous losses during the intake and the delivery phases. During the expansion and the compression phases, L_w is negligible with respect to the heat exchanges contribution.

$$P_i = \mathcal{L}_c n = \dot{m} L_i \qquad L_i = \frac{\mathcal{L}_c n}{\dot{m}} \qquad \eta_{compression} = \frac{L_{i,is}}{L_{i_{compressor}}}$$

RC - Volumetric efficiency

In order to evaluate the mass flow rate, we introduce the **volumetric efficiency λ_V**:

$$\lambda_V = \frac{delivered\ mass}{theoretical\ mass} = \frac{M}{\varrho_1 V_0}$$

The theoretical mass is the mass that would fill the displacement volume at the environmental pressure and temperature. The **induced mass is limited** due to different factors.

a. The fluid sucked by the compressor normally holds a temperature lower than the wall temperature. Thus, the fluid will experience heat exchanges which produce an increase in its temperature and a decrease in the density.

b. A similar effect on the density is produced by the **pressure drop** induced by the throttling on the intake valve.

$$\frac{p}{\varrho} = RT \longrightarrow \varrho = \frac{p}{RT}$$

c. The throttling on the delivery valve together with the clearances also affect the volumetric efficiency.

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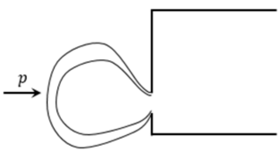
RC - Volumetric efficiency

$$\mathcal{Q}_e + \mathcal{L}_e = \Delta\mathcal{U} + \Delta\mathcal{E}_k + \cancel{\Delta\mathcal{E}_g} + \cancel{\Delta\mathcal{E}_\omega}$$

$\mathcal{L}_e$ is the **external work** performed by the pressure forces on the displacing boundaries of the system.



$$dF = pdA$$

$$d^2\mathcal{L} = -pdA \cdot ds$$

$$d\mathcal{L} = \int_A -pdA \cdot ds = \int_A -pd^2V = -pdV$$

$$\mathcal{L}_{e,ext} = \int_i^f -pdV = p_1(V_f - V_i) = -p_1(0 - V_1) = p_1V_1$$

$$\mathcal{L}_e = p_1V_1 - \int_i^f p_{cyl}dV = p_1V_1 - p_B(V_B - V_A) = p_1V_1 - p_BV_B + p_AV_A$$

$$\mathcal{Q}_e + p_1V_1 - p_BV_B + p_BV_A = m_BU_B - m_AV_A - m_1U_1$$

$$\mathcal{Q}_e = m_B(U_B + p_Bv_B) - m_A(U_A + p_Av_A) - m_1(U_1 + p_1v_1)$$

RC - Volumetric efficiency

$$Q_e = m_B h_B - m_A h_A - m_1 h_1 = m_B c_p T_B - m_A c_p T_A - m_1 c_p T_1$$

$$\frac{Q_e}{c_p} = m_B T_B - m_A T_A - m_1 T_1 \longrightarrow T_B = \frac{m_A T_A + m_1 T_1 + {}^{Q_e}/_{c_p}}{m_B}$$

$$\rho_B V_B T_B - \frac{Q_e}{c_p} = \rho_A V_A T_A + \rho_1 V_1 T_1$$

${}^{Q_e}/_{c_p}$ penalises the trapped mass and increases T_B ($Q_e > 0$) and is accounted for in η_τ (**heat exchange coefficient**).

$$\eta_\tau \rho_B V_B T_B = \rho_A V_A T_A + \rho_1 V_1 T_1 \qquad \left(\varrho = \frac{p}{RT}\right) \qquad \eta_\tau p_B V_B = p_A V_A + p_1 V_1$$

$$(\eta_\tau V_B - V_A) p_1 (1 - \delta_1) = p_1 V_1 \longrightarrow (\eta_\tau V_B - V_A)(1 - \delta_1) = V_1$$

$$\left(V_1 \rho_1 = M_{int} = \frac{\lambda_V \varrho_1 V_0}{\eta_\varphi}\right) \longrightarrow \boxed{\lambda_V = \eta_\varphi (1 - \delta_1) \frac{(\eta_\tau V_B - V_A)}{V_0}}$$

RC - Volumetric efficiency

Let's try and consider an **ideal machine**, assuming $\delta_1 = \delta_2 = 0; \eta_\tau = 1; \eta_\varphi = 1$

$$\lambda_V = \frac{V_B - V_A}{V_0}$$

λ_V is anyhow different from 1 due to the **clearance volume**

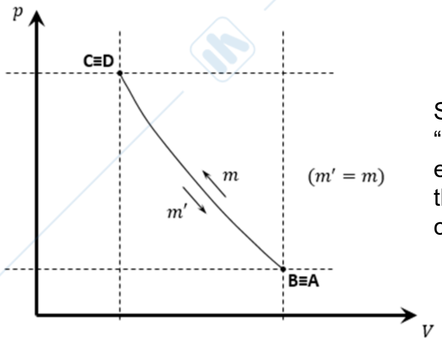
$$V_B = (1 + \mu) V_0 \qquad V_A = V_D \left(\frac{p_D}{p_A}\right)^{\frac{1}{m^{*\prime}}} = \mu V_0 \beta^{1/m^{*\prime}} \qquad (\beta_i = \beta)$$

$$\lambda_V = \frac{V_0 + \mu V_0 (1 - \beta^{1/m^{*\prime}})}{V_0} = 1 - \mu(1 - \beta^{1/m^{*\prime}})$$

Neglecting the leakages ($\eta_\varphi = 1$), m^* and $m^{*\prime}$ would actually correspond to the expansion and to the compression polytropic indexes, respectively. The expression we derived for λ_V allows for estimating an upper β threshold.

$$\lambda_V = 0 \longrightarrow \beta_{lim} = \left(1 + {}^{1}/_{\mu}\right)^{1/m} \qquad (m' = m)$$

RC - Volumetric efficiency



Such a condition would correspond to a "**gas spring**" operation, having coincident expansion and compression and being the trapped mass simply expanded and compressed over and over.

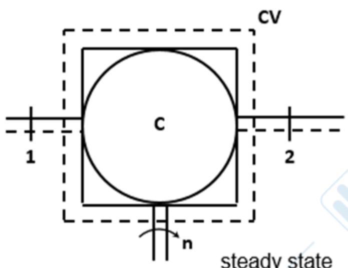
The **delivered mass flow rate** can also be eventually calculated referring to the masses at points C and D:

$$M_{del} = M_C - M_D - M_{leak\ C\to D} = \rho_C V_C - \rho_D V_D - M_{leak\ C\to D} =$$

$$= \frac{p_C}{RT_C}V_C - \frac{p_D}{RT_D}V_D - M_{leak\ C\to D} = p_2(1+\delta_2)\left(\frac{V_C}{RT_C} - \frac{V_D}{RT_D}\right) - M_{leak\ C\to D}$$

RC - Heat exchanges

Let's now evaluate the temperature in the delivery environment T_2 ($\boldsymbol{T_2 \neq T_C, T_D}$)



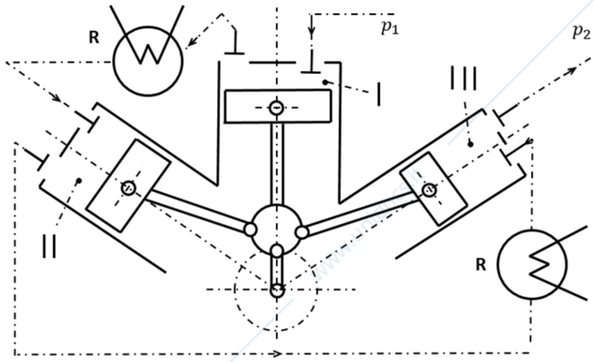
$$\dot{Q} + P_i = \frac{\partial \mathcal{E}}{\partial t} + \mathcal{E}_f$$

$$\dot{Q} + P_i = \dot{m}c_p(T_2 - T_1) \qquad P_i = \dot{m}c_p(T_2 - T_1) - |\dot{Q}|$$

$$L_i = c_p(T_2 - T_1) - |Q| \qquad T_2 = T_1 + \frac{L_i + |Q|}{c_p}$$

Multi-stage compressors

Volumetric compressors are limited in the p_2, T_2 values at the delivery. Such limitation are normally connected to lubrication issues and mechanical resistance constraints on the valves. Thus, we would set an **upper threshold for the compression ratio** ranging around 6-8. If we need to achieve higher compression ratios, we'd better think about multi-stage machines.



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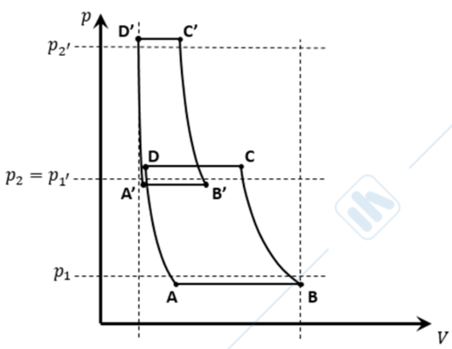
Multi-stage compressors

The **refrigerator**/heat exchanger is needed to reduce the temperature of the fluid delivered by the first stage. Neglecting the leakages between the first and the second stage:

$$\lambda_V \varrho_1 V_0 n = \frac{\lambda'_V \varrho'_1 V'_0 n'}{\eta'_\varphi}$$



Assuming: $\lambda_V \sim \lambda'_V \quad \eta_\tau = 1 \quad \eta_\varphi = 1$

$\mu = \mu' \quad \beta' = \beta'' = \sqrt{\beta} \quad n = n'$

$$\frac{p_1}{T_1} V_0 = \frac{p'_1}{T'_1} V'_0 \;\xrightarrow{(T_1 = T'_1)}\; V'_0 = V_0 \frac{p_1}{p'_1} < V_0$$

$$P_{abs} = \frac{\mathcal{L}_c n}{\eta_m} + \frac{\mathcal{L}'_c n'}{\eta'_m}$$

$$\mathcal{L}_c = \frac{m}{m-1} \lambda_V p_1 V_0 \left(\beta'^{\frac{m-1}{m}} - 1\right) \xrightarrow{(m = k)} \mathcal{L}'_c = \mathcal{L}_c = \frac{k}{k-1} \lambda_V p_1 V_0 \left(\beta^{\frac{k-1}{2k}} - 1\right) \quad \left(\beta = \frac{p'_2}{p_1}\right)$$

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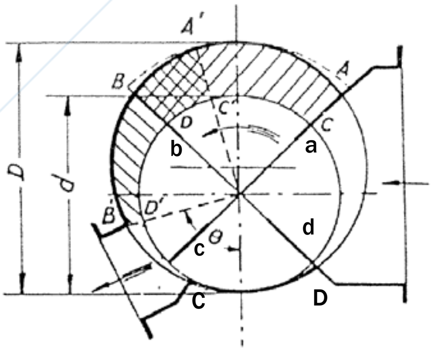
Sliding vane compressors

The compressor is made up of an eccentric rotor carrying **straight blades** forced against the casing by springs. The blades isolate volumes which move about the axes. The inlet and delivery valves are replaced by ports, thus reducing the losses to a minimum ($\delta_1 = \delta_2 = 0$).



For the given geometry (4 blades), the **maximum volume** corresponds to that defined by two blades holding mirrored positions with respect to the line adjoining the rotor and the case centers: (a) occupying the A position and (b) occupying B. The inlet port is designed so as to be closed for the maximum trapped volume.

Sliding vane compressors

$V_0 = V_{max}$ unit displacement

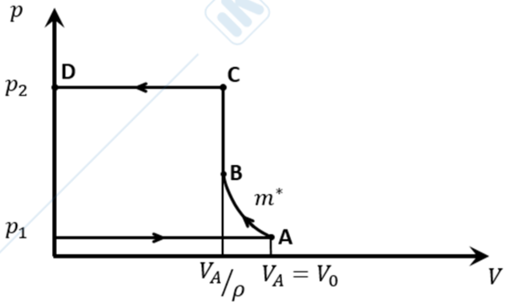
As the rotor rotates, (a) and (b) move to the left and the volume correspondingly decreases. The delivery starts when blade (b) opens the connection to the delivery port (position B'). Such a position corresponds to the **minimum chamber volume**.

$$\rho = volumetric\ comression\ ratio = \frac{V_{max}}{V_{min}}$$

$$V_{0_{tot}} = iV_0$$

The intake port has to be as big as possible to avoids losses during the suction. It is also worth observing that the **compression ratio** is strongly related to the relative position of the ports: we could increase ρ by either reducing the delivery port size or reducing the α angle. The latter has to be carefully considered in order not to induce blow-by. As a matter of fact, the two ports should at any stage be separated by at least one blade. ρ can also be increased by increasing the number of blades/vanes (increased frictions).

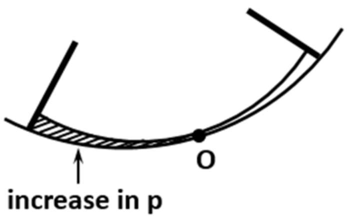
Sliding vane compressors



We witness a **gradual compression from A to B** as the rotor rotates within the casing. Considering $p_2 > p_B$, when the connection is made between the chamber volume and the delivery environment, the difference in pressure drives the fluid from the delivery to **instantaneously compress** the chamber content.

If p_2 was smaller than p_B, we would witness a reverse phenomenon, thus instantaneously relieving the chamber pressure down to p_2. As the compressor rotates from point C on the working cycle, the fluid gets delivered until blade (a) reaches the C position on the sketch.

Sliding vane compressors



If we actually shaped the rotor and the casing so as to produce **one point of contact**, the rotation of the rotor would force the hatched volume to reduce thus highly increasing its pressure: resisting force. Similarly, we would induce vacuum right of point O, reinforcing the resisting force. We should hence trim and smooth the profiles to avoid such a condition.

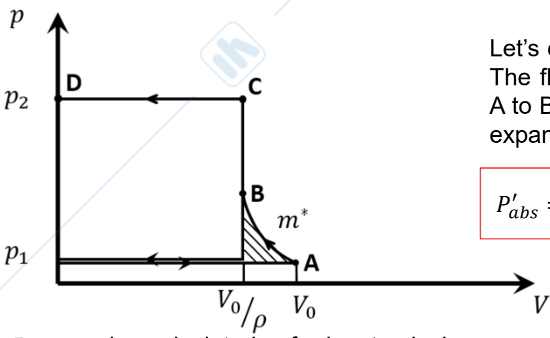
$$\dot{m} = \lambda_V \varrho_1 i V_0 n \qquad \lambda_V = \eta_\varphi (1-\delta_1)\frac{(\eta_\tau V_B - V_A)}{V_0} \sim \eta_\varphi$$

$$\mathcal{L}_c = \oint -pdV = \oint Vdp = \int_A^B Vdp + \int_B^C Vdp = \frac{m^*}{m^*-1} p_1 V_0 \left(\rho^{m^*-1} - 1\right) + \frac{V_0}{\rho}\left(p_2 - p_1 \rho^{m^*}\right)$$

$$P_{abs} = \frac{\mathcal{L}_c n}{\eta_m} \qquad L_i = \frac{\mathcal{L}_c}{M} = \frac{P_i}{\dot{m}}$$

Sliding vane compressors can be conveniently used for $\rho < 6-8$ and $Q \leq 3\ m^3/s$. Still, lubrication will cause the fluid to retain oil particles and **filters at the delivery** might be needed.

Idle operations



Let's consider **idle operations** ($p_2 = p_1$). The fluid gets anyhow compressed from A to B and subsequently instantaneously expands down to pressure p_1.

$$P'_{abs} = i\frac{\mathcal{L}'_c n}{\eta'_m} = i\mathcal{L}'_c n + P_m \qquad (P'_m = P_m)$$

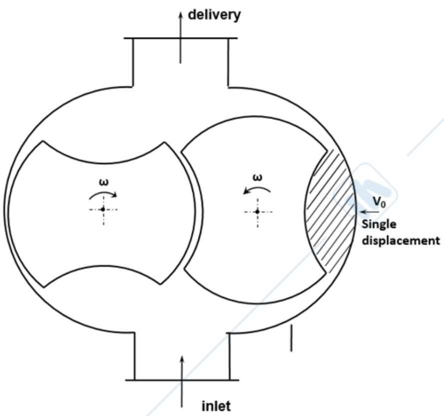
$\boldsymbol{P_m}$ can be calculated referring to design operations and then **considered to be constant** for the given angular speed.

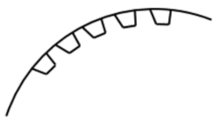
$$\mathcal{L}'_c = \frac{m^*}{m^*-1} p_1 V_0 \left(\rho^{m^*-1} - 1\right) + \frac{V_0}{\rho}\left(p_1 - p_1 \rho^{m^*}\right) =$$

$$= \frac{m^*}{m^*-1} p_1 V_0 \left(\rho^{m^*-1} - 1\right) + p_1 \frac{V_0}{\rho}\left(1 - \rho^{m^*}\right)$$

Roots compressor

The roots compressor is made up of **two matching rotors** which define a confined chamber together with the casing. Once more, the valves are replaced b the ports to reduce the intake ad delivery losses to a minimum.

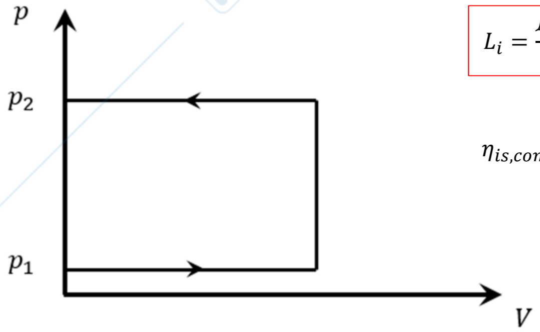




The leakages can be possibly reduced by adopting sealing elements on the lobe periphery (**labyrinth**): carving the profile out, the induced turbulent motion will oppose to the leakages. The fluid is simply displaced form the intake to the delivery and the compression is performed by the gas backflow.

Roots compressor

$$i\mathcal{L}_c = (p_2 - p_1)V_0 \qquad P'_{abs} = i\frac{\mathcal{L}_c n}{\eta_m} \qquad \dot{m} = i\lambda_V \varrho_1 V_0 n$$



$$L_i = \frac{\mathcal{L}_c n}{\dot{m}} = \frac{(p_2 - p_1)\cancel{V_0 n i}}{\lambda_V \varrho_1 \cancel{V_0 n i}} = \frac{RT_1}{\lambda_V}(\beta - 1)$$

$$\eta_{is,comp} = \frac{L_{i,is}}{L_i} = \frac{\frac{k}{k-1}RT_1\left(\beta^{\frac{k-1}{k}} - 1\right)}{\frac{RT_1}{\lambda_V}(\beta - 1)} =$$

$$= \lambda_V \frac{k}{k-1}\frac{\left(\beta^{\frac{k-1}{k}} - 1\right)}{(\beta - 1)}$$

$\eta_{is,comp}$ rapidly decreases as β increases. Let's now thoroughly analyse the volumetric efficiency:

$$\lambda_V = \frac{M}{\varrho_1 V_0} = \frac{M}{M_{int}} \cdot \frac{M_{int}}{\varrho_1 V_0}$$

Roots compressor

$\frac{M}{M_{int}}$ accounts for the **leakages** whereas $\frac{M_{int}}{\varrho_1 V_0}$ accounts for the **losses through the ports**.

$$M = M_{int} - M_{leak} \qquad \frac{M}{M_{int}} = 1 - \frac{M_{leak}}{M_{int}} = 1 - \frac{\dot{m}_{leak}}{\dot{m}_{int}} \sim 1 - \frac{const}{n}$$

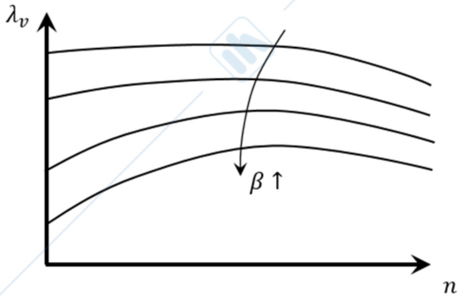
$\dot{m}_{leak}$ is actually connected to the difference in pressure between the two environments. The leaking mass is constant whereas the leaking mass flow rate can only be assumed to be constant.

$$\frac{M_{int}}{\varrho_1 V_0} = \frac{\varrho'_1 V_0}{\varrho_1 V_0} = \frac{p'_1}{p_1} = \frac{p_1 - \Delta p}{p_1} = 1 - \frac{\Delta p}{p_1}$$

$$\Delta p = const \cdot n^2$$

$$\lambda_V = \left(1 - \frac{K}{n}\right)(1 - Hn^2)$$

Roots compressor



For **low β values**, the leakages are negligible and λ_V follows the $(1-Hn^2)$pattern. As β increases, the leakages become more and more evident. The increase in λ_V for average angular speeds at **high β values** is connected to the leakage $(1-{}^{K}/_{n})$ pattern, whereas the drop at high speeds is driven by the losses through the port $(1-Hn^2)$.

Let's now try and evaluate the **delivery temperature**:

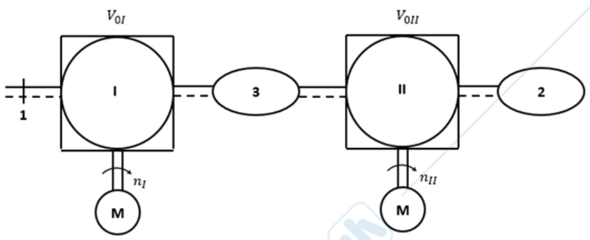
$$\cancel{Q} + L_i = \Delta h + \cancel{\Delta E_g} + \cancel{\Delta E_k} + \cancel{\Delta E_\omega} \qquad \frac{RT_1}{\lambda_V}(\beta-1) = c_p(T_2-T_1) = c_pT_1\left(\frac{T_2}{T_1}-1\right)$$

$$\boxed{\frac{T_2}{T_1} = 1 + \frac{R}{c_p\lambda_V}(\beta-1) = 1 + \frac{k-1}{k}\cdot\frac{1}{\lambda_V}(\beta-1)}$$

Multi-stage compression

Multi-stage Roots compressors are used for **β_s ranging from 1.8 to 2.5-3**.

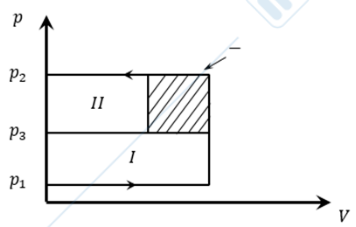
Let's assume overall **steady state** conditions.



$$\dot{m} = const$$

$$\lambda_{VI}\varrho_1 V_{0I} n_I = \lambda_{VII}\varrho_3 V_{0II} n_{II}$$

$$\boxed{V_{0II} = V_{0I}\frac{\varrho_1}{\varrho_3}\cdot\frac{n_I}{n_{II}}\cdot\frac{\lambda_{VI}}{\lambda_{VII}}}$$



Assuming $n_I = n_{II}$ and $\lambda_{VI} = \lambda_{VII}$, having $\varrho_3 > \varrho_1$, the second compressor will feature a **reduced displacement**.

Internal Combustion Engines

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Introduction

The purpose of internal combustion engines (ICE) is the production of mechanical power from the chemical energy contained in the fuel. In internal combustion engines, as distinct from external combustion engines, this energy is released by burning or oxidizing the fuel inside the engine. The fuel-air mixture before combustion and the burned products after combustion are the actual working fluids. The work transfers which provide the desired power output occur directly between these working fluids and the mechanical components of the engine. The internal combustion engines which are the subject of this lecture are spark-ignition engines (sometimes called Otto engines, or gasoline or petrol engines, though other fuels such as CNG and LPG can be used) and compression-ignition or diesel engines. Because of their simplicity, ruggedness and high power/weight ratio, these two types of engine have found wide application in transportation (land, sea, and air) and power generation. It is the fact that combustion takes place inside the work-producing part of these engines that makes their design and operating characteristics fundamentally different from those of other types of engine.

Engine classification

There are many different types of internal combustion engines. They can be classified by:

Application. Automobile, truck, locomotive, light aircraft, marine, portable power system, power generation

Basic engine design. Reciprocating engines (in turn subdivided by arrangement of cylinders: e.g., in-line, V, radial, opposed), rotary engines (Wankel and other geometries)

Working cycle. Four-stroke cycle: naturally aspirated (admitting atmospheric air), supercharged (admitting precompressed fresh mixture), and turbocharged (admitting fresh mixture compressed in a compressor driven by an exhaust turbine); two-stroke cycle: crankcase scavenged, supercharged, and turbocharged

Valve or port design and location. Overhead (or I-head) valves, underhead (or -, L-head) valves, rotary valves, cross-scavenged porting (inlet and exhaust , ports on opposite sides of cylinder at one end), loop-scavenged porting (inlet and exhaust ports on same side of cylinder at one end), through-or uniflow-scavenged (inlet and exhaust ports or valves at different ends of cylinder)

Engine classification

Fuel. Gasoline (or petrol), fuel oil (or diesel fuel), natural gas, liquid petroleum gas, alcohols (methanol, ethanol), hydrogen, dual fuel.

Method of mixture preparation. Carburetion, fuel injection into the intake ports or intake manifold, fuel injection into the engine cylinder

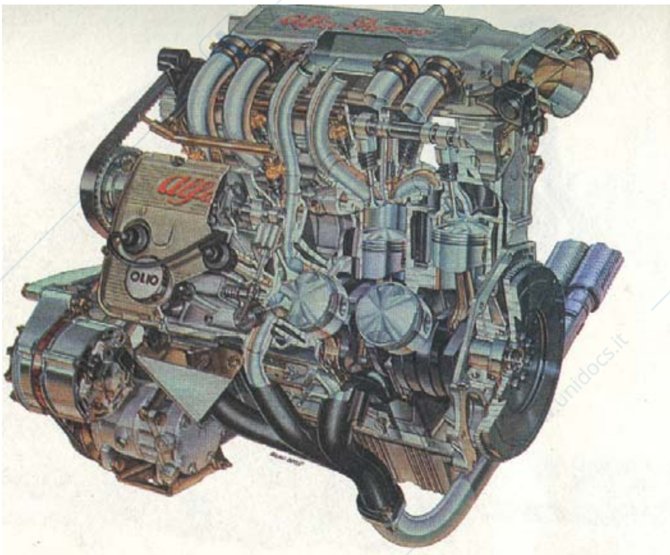
Method of ignition. Spark ignition (in conventional engines where the mixture is uniform and in stratified-charge engines where the mixture is non-uniform), compression ignition (in conventional diesels, as well as ignition in gas engines by pilot injection of fuel oil; in HCCI engines)

Combustion chamber design. Open chamber (many designs: e.g., disc, wedge,hemisphere, bowl-in-piston), divided chamber (small and large auxiliary chambers; many designs: e.g., swirl chambers, prechambers) .

Method of load control. Throttling of fuel and air flow together so mixture composition is essentially unchanged, control of fuel flow alone, a combination of these

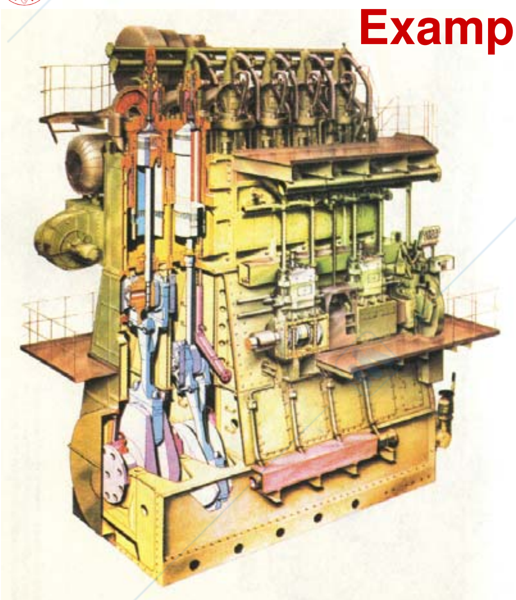
Method of cooling. Water cooled, air cooled, uncooled (other than by natural convection and radiation)

Examples



Alfa-Romeo 6-cylinder 4-stroke NA gasoline engine, 3 liters, 190 HP @ 6000 rpm.
Bore: 93 mm
Stroke: 72.6 mm

Examples

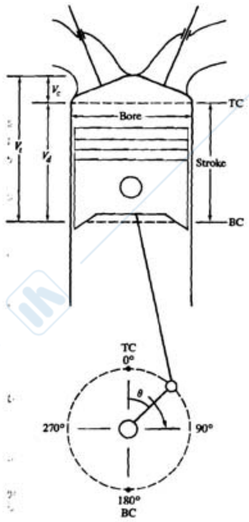


2-stroke Diesel engine for energy production and marine propulsion application. 6 cylinders, total displacement: 2694 liters.
Power: 9,6 MW @ 140 rpm.
Bore: 580 mm
Stroke: 1700 mm

Engine operating cycle

This lecture is about reciprocating engines, where the piston moves back and forth in a cylinder and transmits power through a connecting rod and crank mechanism to the drive shaft as shown in the figure. The steady rotation of the crank produces a cyclical piston motion. The piston comes to rest at the top-dead-center (DTC) crank position and bottom-dead-center (BDC) crank position when the cylinder volume is a minimum or maximum, respectively. The minimum cylinder volume is called the clearance volume V_c. The volume swept out by the piston, the difference between the maximum or total volume V_t and the clearance volume, is called the displaced or swept volume V_0. The ratio of maximum volume to minimum volume is the compression ratio CR. Typical values of CR are 8 to 12 for SI engines and 15 to 20 for CI engines.

Engine operating cycle



Engine operating cycle

The majority of reciprocating engines operate on what is known as the four-stroke cycle. Each cylinder requires four strokes of its piston - two revolutions of the crankshaft - to complete the sequence of events which produces one power stroke. Both SI and CI engines use this cycle which comprises:

1. An intake stroke, which starts with the piston at TDC and ends with the piston at BDC, which draws fresh mixture into the cylinder. To increase the mass inducted, the inlet valve opens shortly before the stroke starts and closes after it ends.

2. A compression stroke, when both valves are closed and the mixture inside the cylinder is compressed to a small fraction of its initial volume. Toward the end of the compression stroke, combustion is initiated and the cylinder pressure rises more rapidly.

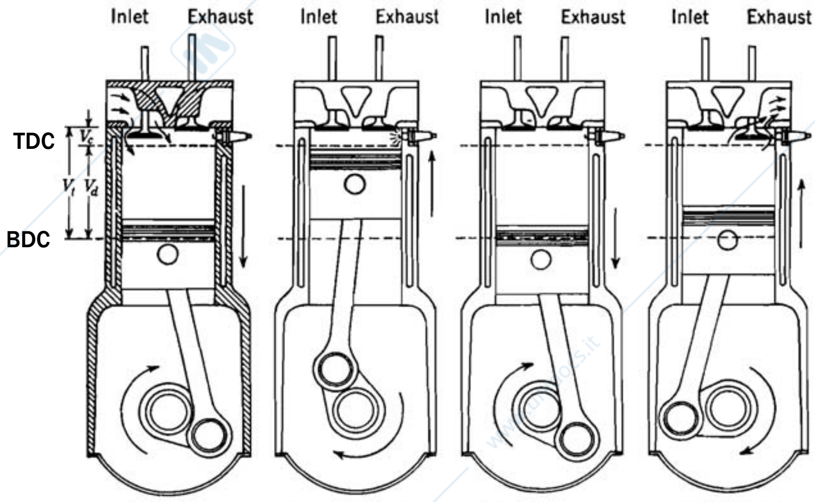
Engine operating cycle

3. A power stroke, or expansion stroke, which starts with the piston at TDC and ends at BDC as the high-temperature, high-pressure, gases push the piston down and force the crank to rotate. About five times as much work is done on the piston during the power stroke as the piston had to do during compression. As the piston approaches BDC the exhaust valve opens to initiate the exhaust process and drop the cylinder pressure to close to the exhaust pressure.

4. An exhaust stroke, where the remaining burned gases exit the cylinder: first, because the cylinder pressure may be substantially higher than the exhaust pressure; then as they are swept out by the piston as it moves toward TDC. As the piston approaches TDC the inlet valve opens. Just after TDC the exhaust valve closes and the cycle starts again.

The four-stroke cycle requires, for each engine cylinder, two crankshaft revolutions for each power stroke.

Engine operating cycle



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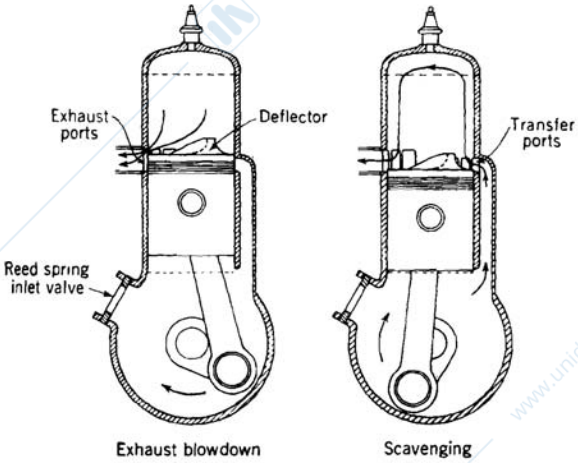
Engine operating cycle

To obtain a higher power output from a given engine size, and a simpler valve design, the two-stroke cycle was developed. The two-stroke cycle is applicable to both SI and CI engines. Next figure shows one of the simplest types of two-stroke engine designs. Ports in the cylinder liner, opened and closed by the piston motion, control the exhaust and inlet flows while the piston is close to BDC. The two strokes are:

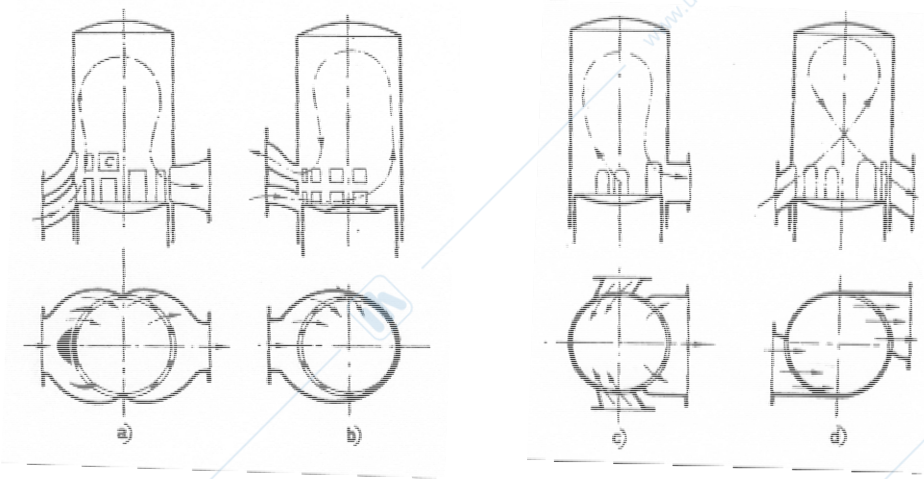
1. A compression stroke, which starts by closing the inlet and exhaust ports, and then compresses the cylinder contents and draws fresh charge into the crankcase. As the piston approaches TDC, combustion is initiated.
2. A power or expansion stroke, similar to that in the four-stroke cycle until the piston approaches BDC, when first the exhaust ports and then the intake ports are uncovered. Most of the burnt gases exit the cylinder in an exhaust blowdown process. When the inlet ports are uncovered, the fresh charge which has been compressed in the crankcase flows into the cylinder. The piston and the ports are generally shaped to deflect the incoming charge from flowing directly into the exhaust ports and to achieve effective scavenging of the residual gases.

Engine operating cycle



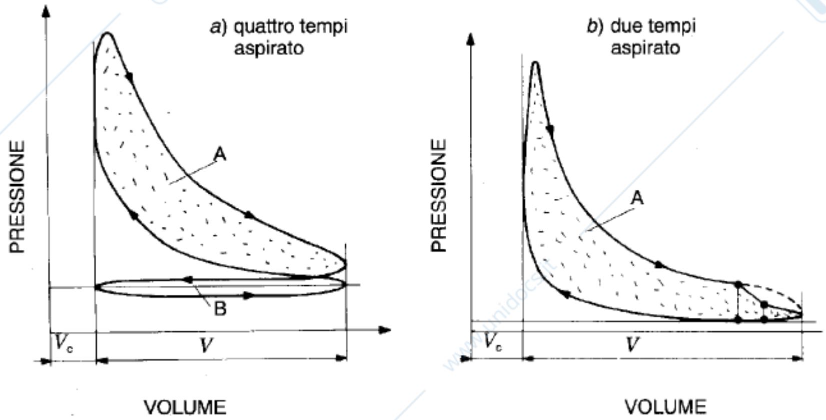
Each engine cycle with one power stroke is completed in one crankshaft revolution. However, it is difficult to fill completely the displaced volume with fresh charge, and some of the fresh mixture flows directly out of the cylinder during the scavenging process. The example shown is a cross-scavenged design; other approaches use loop-scavenging or uniflow systems.

Engine operating cycle

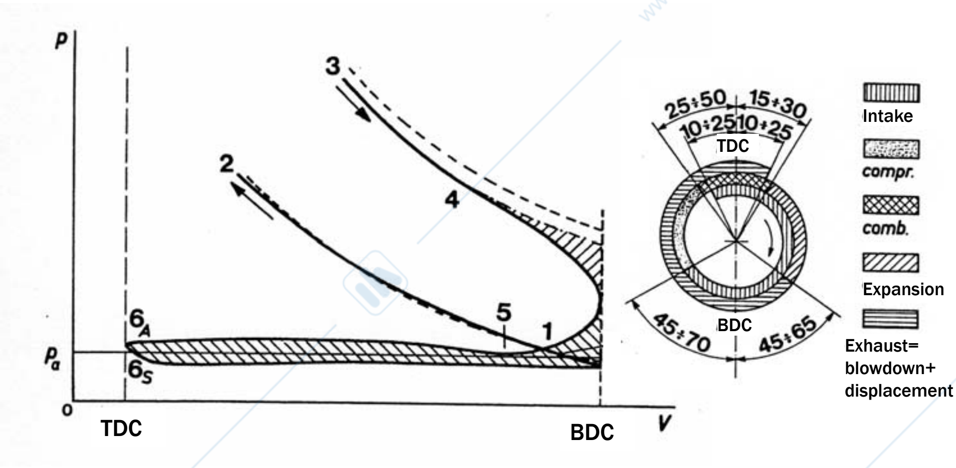


(a,b): cross-scavenged design; (c,d): loop-scavenged design

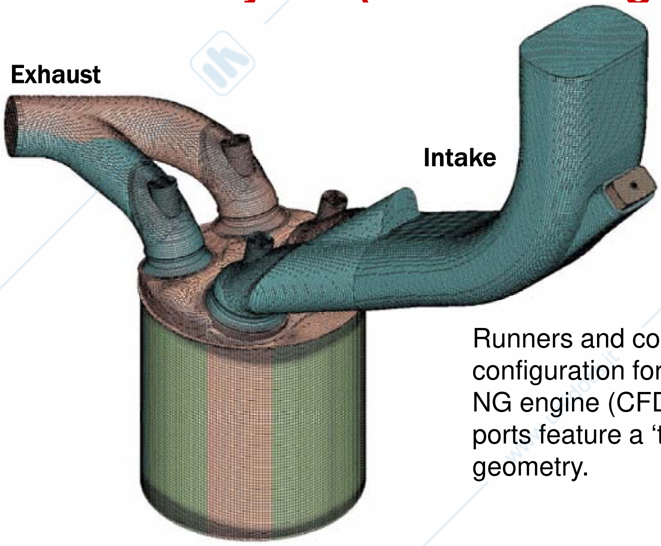
Indicated cycle



Pumping losses (4 stroke engine)

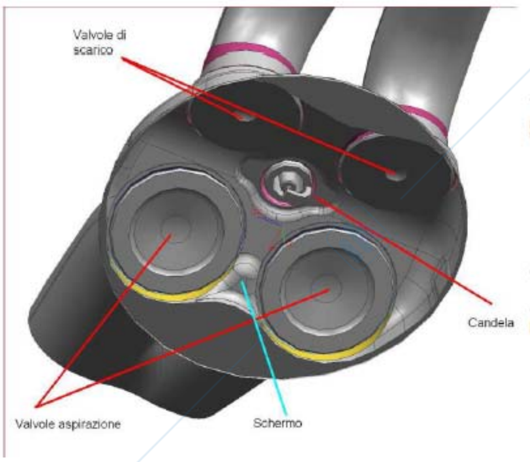


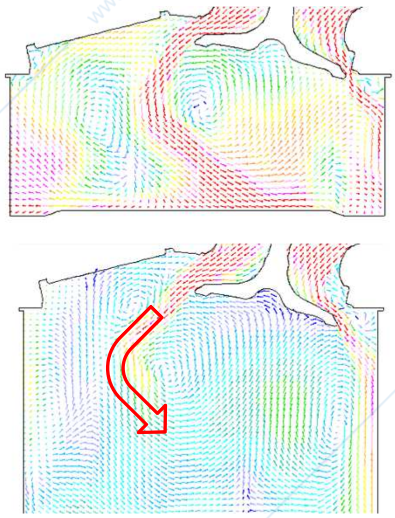
Layout (4 stroke engine)



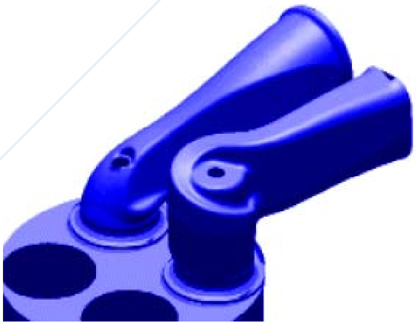
Runners and combustion chamber configuration for a small displacement, NG engine (CFD mesh). The intake ports feature a 'tumble oriented' geometry.

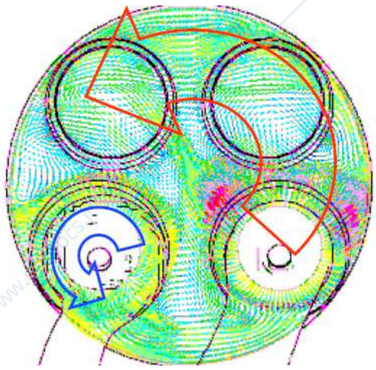
Typical intake system of a SI engine





Typical intake system of a CI engine





Indicated work per cycle

$$\mathcal{L}_{ind} = i \oint_{cycle} pdV = imep \cdot iV_0$$ i: number of cylinders

Indicated power: $$\dot{\mathcal{L}}_{ind} = P_{ind} = \mathcal{L}_{ind}\frac{n}{m}$$

m=2: 4-stroke engines
m=1: 2-stroke engines

Brake power: $$P_{ICE} = \eta_m \cdot P_{ind}$$

$$imep = \frac{\mathcal{L}_{ind}}{iV_0} = \frac{P_{ind}}{iV_0\frac{n}{m}}$$ indicated mean effective pressure

$$bmep = \frac{P_{ICE}}{iV_0 \cdot \frac{n}{m}} = \frac{\mathcal{L}_{ICE}}{iV_0} = \eta_m \frac{\mathcal{L}_{ind}}{iV_0} = \eta_m \cdot imep$$ brake mean effective pressure

Brake torque and bmep

$$T_{ICE} = \frac{P_{ICE}}{\omega} = \frac{bmep\, iV_0 \frac{n}{m}}{\omega} = \frac{bmep\, iV_0 \frac{n}{m}}{2\pi n} = \frac{bmep\, iV_0}{2\pi m}$$

$$bmep = \frac{2\pi m T_{ICE}}{iV_0}$$

ICE efficiencies

$$P_{ICE} = \eta_f \dot{m}_f H_{L,p} = \eta_f m_f \frac{n}{m} H_{L,p}$$

η_f Fuel conversion efficiency

$$bmep = \frac{P_{ICE}}{iV_0 \cdot \frac{n}{m}} = \frac{\eta_f \cdot m_f \cdot H_{L,p}}{iV_0}$$

$$\lambda_v = \frac{m_{air}}{\rho_{a,0} iV_0} = \frac{\dot{m}_{air}}{\rho_{a,0} iV_0 \frac{n}{m}}$$ Volumetric efficiency

$$bmep = \frac{P_{ICE}}{iV_0 \cdot \frac{n}{m}} = \frac{\eta_f \cdot {m_{air}}/{\alpha} \cdot H_{L,p}}{iV_0} = \frac{\eta_f \cdot \lambda_v \cdot \rho_a \cdot {iV_0}/{\alpha} \cdot H_{L,p}}{iV_0}$$

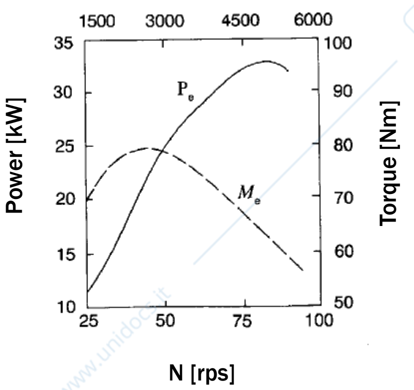
$$bmep = \frac{\eta_f \cdot \lambda_v \cdot \rho_a \cdot H_{L,p}}{\alpha}$$

Engine performance

4-stroke naturally-aspirated SI engine
iV_0=999 cm^3

Maximum torque: 78 Nm @2700 rpm
[bmep=9.8 bar]

Maximum power: 33 kW (45 CV)
@5000 rpm
[bmep=7.9 bar]



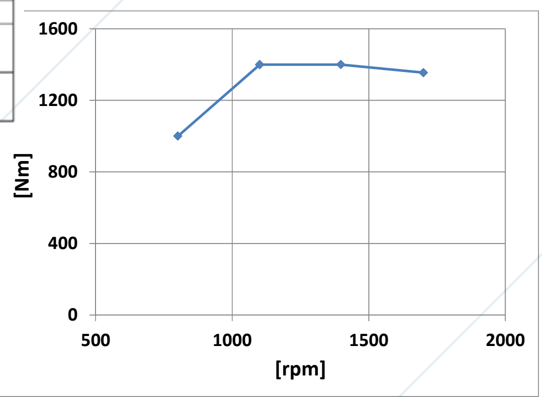
Engine performance

Displacement	8.7 dm^3
Stroke	135 mm
Bore	117 mm
Connecting Rod	200 mm
Compression ratio	16.5:1
Number of cylinders	6
Injection system	Common rail, Bosch CRIN3, 1600 bar
Turbocharger	VGT type

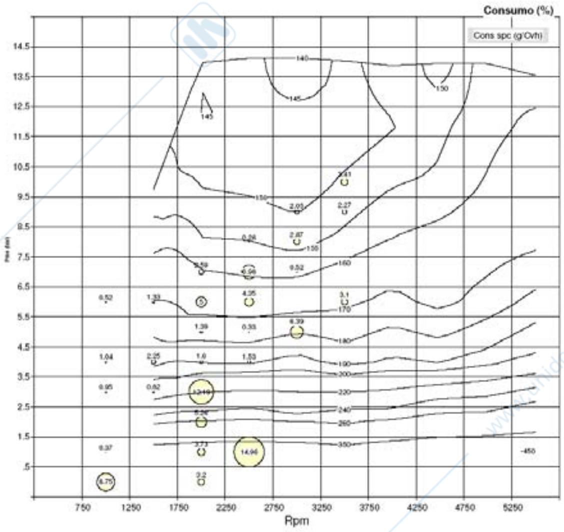
4-stroke turbocharged CI engine

Maximum torque: 1400 Nm @1100-1400 rpm
[bmep=20.2 bar]

Maximum power: 240 kW (325 CV)
@1700 rpm
[bmep=19.5 bar]



Engine performance



Fuel consumption map:
NG engine, 1.4 l