sessagesimale (° ' ") → sessadecimale (°) : minuti + $\frac{\text{primi}}{60}$ + $\frac{\text{secondi}}{3600}$

sessadecimale (°) → sessagesimale (° ' ") : $\alpha - \alpha^{\circ} = \dots \cdot 60 \Rightarrow \alpha'$ $\quad$ $\alpha' - \alpha = \dots \cdot 60 \Rightarrow \alpha''$

parte intera

$\alpha^{RAD} = \frac{\pi}{180}\alpha^{\circ}$ $\quad$ $\alpha^{\circ} = \frac{180}{\pi}\alpha^{RAD}$ $\quad$ $\alpha^{GON} = \frac{10}{9}\alpha^{\circ}$ $\quad$ $\alpha^{\circ} = \frac{9}{10}\alpha^{GON}$

distanza euclidea $= \|P-P'\| = \sqrt{(x-x')^2+(y-y')^2+(z-z')^2}$ $\quad$ distanza reale $= \frac{\text{distanza orizzontale}}{\sin\theta_z}$

Posizione 2 punti dato azimut, zenit : $P = R_A(\theta_A) R_z(\theta_z) U(d)$ $\quad$ $P' = R'_A(\theta_A') R'_z(\theta_z') U(d')$

$$R_z(\theta_z) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_z & \sin\theta_z \\ 0 & -\sin\theta_z & \cos\theta_z \end{pmatrix} \qquad R_A(\theta_A) = \begin{pmatrix} \cos\theta_A & \sin\theta_A & 0 \\ -\sin\theta_A & \cos\theta_A & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

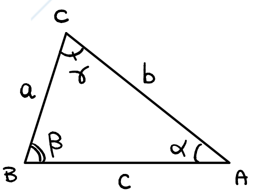
Spezzata noto primo angolo e misure progressive : (5 vertici)

$\theta_1 = \alpha_1^{gon}$ $\quad$ $\theta_2 = (\theta_1 + \alpha_2 \pm 200)^{gon}$ $\quad$ $\theta_3 = (\theta_2 + \alpha_3 \pm 200)^{gon}$ $\quad$ $\theta_4 = (\theta_3 + \alpha_4 \pm 200)^{gon}$

$x_1 = d_{01}\sin\theta_1$ $\quad$ $x_2 = x_1 + d_{12}\sin\theta_2$ $\quad$ $x_3 = x_2 + d_{23}\sin\theta_3$ $\quad$ $x_4 = x_3 + d_{34}\sin\theta_4$

$y_1 = d_{01}\cos\theta_1$ $\quad$ $y_2 = y_1 + d_{12}\cos\theta_2$ $\quad$ $y_3 = y_2 + d_{23}\cos\theta_3$ $\quad$ $y_4 = y_3 + d_{34}\cos\theta_4$

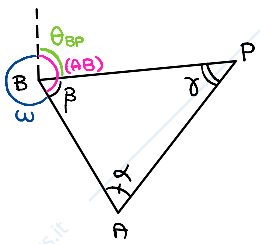
se somma > 200 devo −200
se somma negativa devo +200



Teo. seni : $\frac{BC}{\sin\alpha} = \frac{AC}{\sin\beta} = \frac{AB}{\sin\gamma}$ $\quad$ Teo. Carnot : $AB^2 = BC^2 + AC^2 - 2\,BC \cdot AC \cdot \cos\gamma$

Semiperiodo : $p = \frac{AB+BC+AC}{2}$ $\quad$ Area : $S = \frac{BC \cdot AC \cdot \sin\gamma}{2}$ $\quad$ Erone : $S = \sqrt{p(p-a)(p-b)(p-c)}$

TRILATERAZIONE :



Conosco A, B, α, ω, posizione di P ?

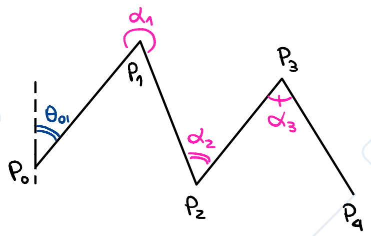
$\overline{AB} = \sqrt{(x_B-x_A)^2+(y_B-y_A)^2}$ $\quad$ $\beta = 2\pi - \omega$ $\quad$ $(AB) = \arctan\left(\frac{x_A-x_B}{y_A-y_B}\right)$ Trovo angolo tra segm. AB e la retta verticale.

$\theta_{BP} = (AB) - \beta$ angolo tra BP e la retta verticale

$x_P = x_B + \overline{BP} \cdot \sin\theta_{BP}$

$y_P = y_B + \overline{BP} \cdot \cos\theta_{BP}$

POLIGONALE APERTA



Conosco P_0, P_1, P_3, P_4, d_{12}, d_{23}, $\alpha_1, \alpha_2, \alpha_3$

① Trovare angoli di direzione : $\theta_{01} = \arctan\left(\frac{x_1-x_0}{y_1-y_0}\right) \longrightarrow \theta_{01}^{gon} = \frac{10}{9}\theta_{01}$

$\theta_{12} = (\theta_{01} + \alpha_1 - 200)^{gon}$ $\quad$ $\theta_{23} = (\theta_{12} + \alpha_2 - 200)^{gon}$ $\quad$ $\theta_{34} = (\theta_{23} + \alpha_3 - 200)^{gon}$

Calcolo l'ultimo angolo anche in coord. cartesiane : $\theta'_{34} = \arctan\left(\frac{x_4-x_3}{y_4-y_3}\right)$ $\quad$ ② compens. emp. : $\delta\alpha = \frac{\theta_{34}-\theta'_{34}}{m}$ n° di vertici escludendo gli estremi (3)

③ Angoli corretti : $\theta_{i-1,i} = \theta_{i-1,i} + i\frac{\delta\alpha}{3}$ $\quad$ $\theta_{01}^c = \theta_{01} + \frac{\delta\alpha}{3}$ $\quad$ $\theta_{12}^c = \theta_{12} + 2\frac{\delta\alpha}{3}$ $\quad$ $\theta_{23}^c = \theta_{23} + \delta\alpha$ $\quad$ $\theta_{34}^c = \theta_{34} + 4\frac{\delta\alpha}{3}$

④ Coord. vertici : $x_2' = x_1 + d_{12}\sin\theta_{12}^{CORR.}$ $\quad$ $x_3' = x_2 + d_{23}\sin\theta_{23}^{CORR.}$ $\quad$ $\delta x = x_3 - x_3'$ $\quad$ $x_2^{CORRETTO} = x_2' + \delta x \cdot \frac{d_{12}+d_{23}+d_{34}}{d_{12}+d_{23}}$

$y_2' = y_1 + d_{12}\cos\theta_{12}^{CORR.}$ $\quad$ $y_3' = y_2 + d_{23}\cos\theta_{23}^{CORR.}$ $\quad$ $\delta y = y_3 - y_3'$ $\quad$ $y_2^{CORRETTO} = y_2' + \delta y \cdot \frac{d_{12}+d_{23}+d_{34}}{d_{12}+d_{23}}$

Coordinate cartesiane ⟶ geografiche

$e^2 = \frac{a^2-b^2}{a^2}$ $e'^2 = \frac{a^2-b^2}{b^2}$ $p = \sqrt{x^2+y^2}$

$\theta = \arctan\left(\frac{z \cdot a}{p \cdot b}\right)$ $f = \frac{a-b}{a} \longrightarrow b = a(1-f)$

$$\begin{cases} \phi = \arctan\left(\dfrac{z + e'^2 b (\sin\theta)^3}{p - e^2 a (\cos\theta)^3}\right) & \text{LATITUDINE} \\ \lambda = \arctan\left(\dfrac{y}{x}\right) & \text{LONGITUDINE} \\ h = \dfrac{p}{\cos\phi} - N_\phi & \text{QUOTA ELLISSOIDICA} \end{cases}$$

Ellissoide Hayford: $a = 6378388$ $b = 6356912$ $f = 0{,}003367$

Ellissoide WGS84: $a = 6378137$ $b = 6356752$ $f = 3{,}35 \cdot 10^{-6}$

RAGGIO CURVATURA sez. merid.: $\rho_\phi = \dfrac{a(1-e^2)}{(1-e^2(\sin\phi)^2)^{3/2}}$

RAGGIO DELLA GRAN NORMALE: $N_\phi = \dfrac{a}{\sqrt{1-e^2(\sin\phi)^2}}$

RAGGIO DELLA SFERA MEDIA LOCALE: $R = \sqrt{\rho_\phi N_\phi}$

RAGGIO del PARALLELO: $r_{(\phi)} = N_\phi \cos\phi_P$

LEGGE EULERO: $\dfrac{1}{R_\alpha} = \dfrac{\cos^2\alpha}{\rho} + \dfrac{\sin^2\alpha}{N_\phi}$

raggio di curv. sezione normale non principale di azimut α

TEO. MEUSNIER: $R_\theta = R_\alpha \cos\theta$

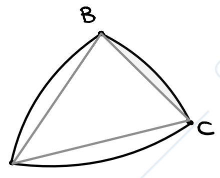
raggio di curv. sezione obliqua che forma θ con la normale

Coord. geografiche ⟶ cartesiane:

$$\begin{cases} x = (N_\phi + h)\cos\phi\cos\lambda \\ y = (N_\phi + h)\cos\phi\sin\lambda \\ z = [N_\phi(1-e^2) + h]\sin\phi \end{cases}$$

Quota ortometrica: $H = h - N_\phi$

TRIANGOLO SFERICO:



Hp: $\begin{cases} \text{lati} < 200\,\text{km} \\ \text{Teo. Legendre: } 3\varepsilon = \alpha + \beta + \gamma - \pi \end{cases}$

Lo approssimiamo ad un triangolo piano e calcoliamo lati e angoli.

$S = \dfrac{AB \cdot AC \cdot \sin\alpha}{2}$ $R = \sqrt{\rho_\phi N_\phi}$

$R = \dfrac{a\sqrt{1-e^2}}{1-e^2(\sin\phi)^2}$

Teo. Cavalieri: $3\varepsilon = \dfrac{S}{R^2}$ trovo l'eccesso sferico:

$\widehat{AB} = \overline{AB} + \varepsilon$ $\widehat{\alpha} = \overline{\alpha} + \varepsilon$

$\widehat{BC} = \overline{BC} + \varepsilon$ $\widehat{\beta} = \overline{\beta} + \varepsilon$

$\widehat{AC} = \overline{AC} + \varepsilon$ $\widehat{\gamma} = \overline{\gamma} + \varepsilon$

Calcolare azimut di una geodetica in P: Teo. Clairaut: $r_{(\phi)} = \dfrac{cost}{\sin\alpha}$ $\sin\alpha = \dfrac{cost}{N_\phi \cos\phi_P}$

$\phi_P = \phi_{P_1}$ hanno stessa latitudine

⟶ sono sullo stesso parallelo: **Distanza lungo l'ellissoide** $= N_\phi \cos\phi\,(\lambda_P - \lambda_{P_1})$

media $m = \dfrac{\sum x_i}{n}$ **deviaz. standard:** $\sigma = \sqrt{\dfrac{\sum(x_i - m)^2}{n-1}}$ $\sigma = \sqrt{\dfrac{\sum(x_i-\mu)^2}{n}}$

incertezza / stima sulla media: $\sigma_m = \dfrac{\sigma}{\sqrt{n}}$ **outlier:** $|e_i| = |x_i - m| > 3\sigma$ **errore relativo** $= \dfrac{\sigma_m}{m}$

Misure divise in classi con frequenze:
1- punto medio ogni intervallo
2- freq. relat.: $f_i = \dfrac{F}{n_{TOT,freq.}}$
3- $m = \sum f_i x_i$
4- $\sigma = \sum (x_i - m)^2 f_i$

errore distanza: $\sigma_d = \sqrt{\sigma_x^2 + \sigma_y^2}$

Stimatore Blue: tante misure con incert. indipend. della stessa cosa

$$w_i = \frac{\frac{1}{\sigma_i^2}}{\sum 1/\sigma_j^2} \quad \begin{cases} \frac{1/\sigma_1^2}{1/\sigma_1^2+1/\sigma_2^2+1/\sigma_3^2} \\ \frac{1/\sigma_2^2}{1/\sigma_1^2+1/\sigma_2^2+1/\sigma_3^2} \\ \dots \end{cases} \qquad \hat{\alpha} = \sum w_i \alpha_i \qquad \sigma_{\hat{\alpha}} = \sqrt{\sum w_i^2 \sigma_i^2} \qquad \text{errore rel.} = \frac{\sigma_{\hat{\alpha}}}{\hat{\alpha}}$$

Propagazione errore:

- **Area triangolo**: $S = \sqrt{p(p-a)(p-b)(p-c)}$ $\qquad \sigma_S = \sqrt{\left(\frac{\partial S}{\partial a}\right)^2 \sigma_a^2 + \left(\frac{\partial S}{\partial b}\right)^2 \sigma_b^2 + \left(\frac{\partial S}{\partial c}\right)^2 \sigma_c^2}$

Chiamo $L = \sqrt{-a^4 - b^4 - c^4 + 2a^2b^2 + 2a^2c^2 + 2b^2c^2}$

$$\left|\frac{\partial S}{\partial a}\right| = \frac{1}{8L}\left|-4a^3 + 4a(b^2+c^2)\right| \qquad \left|\frac{\partial S}{\partial c}\right| = \frac{1}{8L}\left|-4c^3 + 4c(a^2+b^2)\right|$$

$$\left|\frac{\partial S}{\partial b}\right| = \frac{1}{8L}\left|-4b^3 + 4b(a^2+c^2)\right|$$

- **Area rettangolo**: $S = a \cdot b$ $\qquad \sigma_S = \sqrt{\left(\frac{\partial S}{\partial a}\right)^2 \sigma_a^2 + \left(\frac{\partial S}{\partial b}\right)^2 \sigma_b^2} = \sqrt{b^2\sigma_a^2 + a^2\sigma_b^2}$ $\qquad$ err. rel. $= \frac{\sigma_S}{S}$

Matrice covarianza: $\Sigma = \begin{pmatrix} \frac{\partial S}{\partial a} & \frac{\partial S}{\partial b} \\ \frac{\partial P}{\partial a} & \frac{\partial P}{\partial b} \end{pmatrix} \begin{pmatrix} \sigma_a^2 & 0 \\ 0 & \sigma_b^2 \end{pmatrix} \begin{pmatrix} \frac{\partial S}{\partial a} & \frac{\partial P}{\partial a} \\ \frac{\partial S}{\partial b} & \frac{\partial P}{\partial b} \end{pmatrix}$

Covarianza tra area e perimetro (rettangolo): $\sigma_{S_P} = \begin{pmatrix} \frac{\partial S}{\partial a} & \frac{\partial S}{\partial b} \end{pmatrix} \begin{pmatrix} \sigma_a^2 & 0 \\ 0 & \sigma_b^2 \end{pmatrix} \begin{pmatrix} \frac{\partial P}{\partial a} \\ \frac{\partial P}{\partial b} \end{pmatrix}$

- **Area rettangolo date serie misure**: $\bar{a} = \frac{\sum a_i}{m}$ $\quad \bar{b} = \frac{\sum b_i}{m}$ $\quad S = \bar{a} \cdot \bar{b}$ $\quad \sigma_a = \sqrt{\frac{\sum (a_i - \bar{a})^2}{m-1}}$ $\quad \sigma_b = \sqrt{\frac{\sum (b_i - \bar{b})^2}{m-1}}$

$$\sigma_{MA} = \frac{\sigma_a}{\sqrt{m}} \qquad \sigma_{Mb} = \frac{\sigma_b}{\sqrt{m}} \qquad \sigma_S = \sqrt{\left(\frac{\partial S}{\partial a}\right)^2 \sigma_{aM}^2 + \left(\frac{\partial S}{\partial b}\right)^2 \sigma_{bM}^2}$$

- **Distribuire equamente l'errore**: $\sigma_S = \sqrt{\left(\frac{\partial S}{\partial a}\right)^2 \frac{\sigma_a^2}{N_a} + \left(\frac{\partial S}{\partial b}\right)^2 \frac{\sigma_b^2}{N_b}}$ $\quad \begin{cases} \left(\frac{\partial S}{\partial a}\right)^2 \frac{\sigma_a^2}{N_a} \geq \frac{\sigma_S^2}{2} \\ \left(\frac{\partial S}{\partial b}\right)^2 \frac{\sigma_b^2}{N_b} \geq \frac{\sigma_S^2}{2} \end{cases}$

⚠
- usare formula Erone per area
- Mettere tutti e la calcolatrice in RAD
- coerenza tra cifre significative dell'incertezza e della misura

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Poligonale aperta:

① Calcolo angoli di direzione: $\theta_{ij} = (\theta_{i-1,j-1} + \alpha_{j-1} \pm 200)^{gon}$

② Si calcola $\delta\alpha$ come $\theta_{34} - \theta'_{34}$ dove $\theta_{34} = \arctan\left(\frac{x_3 - x_4}{y_3 - y_4}\right)$

③ Angoli corretti: $\alpha_i^c = \alpha_i + \frac{\delta\alpha}{3}$

Angoli di direzione corretti: $\theta_{i-1,i}^c = \theta_{i-1,i} + i\,\frac{\delta\alpha}{3}$

④ Trovo le coord. vertici: $x'_i = x_{i-1} + d_{i-1,i} \sin\theta_{i-1,i}^c$ $\qquad y'_i = y_{i-1,i} + d_{i-1,i} \cos\theta_{i-1,i}^c$

⑤ Trovo $\delta x = x_3 - x'_3$ e $\delta y = y_3 - y'_3$

⑥ Correggo tutte le misure dei vertici: $x_i^{corretto} = x'_i + \delta x \cdot \frac{d_{12} + d_{23} + d_{34}}{d_{12} + d_{23}}$

$$y_i^{corretto} = y'_i + \delta y \cdot \frac{d_{12} + d_{23} + d_{34}}{d_{12} + d_{23}}$$