

V.M QUESTIONS COLLECTING

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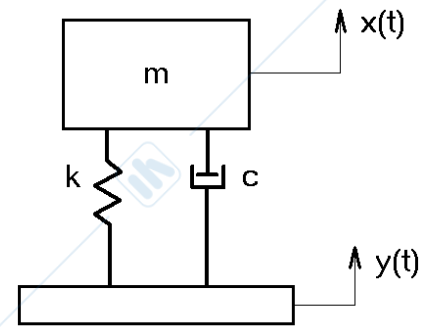
We stand along together

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QUESTIONS COLLECTING---1

- The base of the SDOF system in the figure is moving with $y(t) = y_0 \cdot \cos \omega t$ and $y_0 = \text{const}$
Find the amplitude and phase of the force transmitted from the mass m to the base.



- For an “ n ” degrees of freedom system with proportional viscous damping you are given the mode shapes $\{\psi_r\}$, the natural frequencies ω_r and the damping factors $\zeta_r (r = 1, 2 \dots, n)$.
Calculate the expression of the receptance $\alpha_{ij}(\omega)$.
- For an “ n ” degrees of freedom system with non-proportional damping proof that the mode shapes are orthogonal (Duncan method).
- For a given signal $x(t)$ with a total duration T_{tot} explain how to compute the spectrum $S_{xy}(\Omega)$ according to the Welch’s periodogram method.
- Explain the “-N dB” method, with its pros and cons, to determine the loss factor of a SDOF system with hysteretic damping.
- Explain the origin of aliasing and leakage and how to limit their effects.
- MDOF: Hysteretic proportional damping - derive the expression of receptance
- EXTRACTION METHODS: Fractional polynomial method
- Given an input and output random signal: derive their relations in frequency and time domain based on system responses h and H .
- Derive the coherence function.
- 3db method
- Discrete Fourier series
- hysteretic damping
- Build an even function (e) in Fourier Series
- Question related to signal and noise ratio....something like that!
- Orthogonality property of MDOF system

QUESTIONS COLLECTING---2

1. A GENERAL SDOF HYSTERETIC DAMPED SYSTEM

Given the function: $m\ddot{x} + x(1 + i\beta)k = F_0^{i\Omega t}$,

- 1) use the -3dB method and calculate the receptance $\alpha(\Omega)$;
- 2) plot $\alpha(\Omega)$ showing the influence of β ;
- 3) proof that resonance ω_n is at natural frequency;
- 4) Proof that $\beta = \frac{\Omega b^2 - \Omega a^2}{2\omega_n^2}$;

2. NDOF PROPORTIONAL VISCOUS DAMPED SYSTEM

Given the function: $[m]\{\ddot{x}\} + [c]\{\dot{x}\} + [k]\{x\} = \{0\}$

- 1) Define the eigenproblem;
- 2) Proof the $[m]$ $[k]$ $[c]$ orthogonality;
- 3) given $\{x_0\}$ and $\{v_0\}$ calculate the free response;

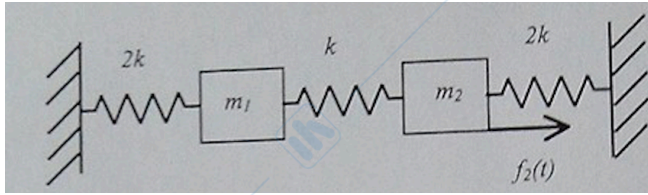
3. The FOURIER TRANSFORM is

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\Omega)e^{i\Omega t} d\Omega \leftrightarrow F(\Omega) = \int_{-\infty}^{\infty} f(t)e^{-i\Omega t} dt$$

- 1) proof the Fourier Transform of a real an even function is also a real and even function
- 2) $x(t)$ is a rectangular function on $[-T/2, T/2]$ with amplitude equal to 1 calculate its Fourier Transform
- 3) proof the Parseval's theorem

QUESTIONS COLLECTING---3

1. FORCED VIBRATIONS



For the two degrees of freedom system sketched above:

- 1) Write the equations of motion as a function of m_1 , m_2 and k ;
- 2) With $m_1=m_2=m$ compute the eigenvalues and the eigenvectors;
- 3) With $m_1=m_2=m$ compute the modal mass and modal stiffness matrices;
- 4) With $f_2(t) = F \sin(\Omega t)$ compute the amplitude of vibration of mass m_1 when $m_1=m_2=m=2$ kg, $k=1000$ N/m, $F=20$ N, $\Omega=40$ rad/s.

2. EXTRACTION OF MODAL PARAMETERS

The modulus of the receptance $\alpha(\Omega)$ of a SDOF system with hysterical damping may be

$$\text{written in the form: } |\alpha(\Omega)| = \frac{A}{\sqrt{(\omega_n^2 - \Omega^2)^2 + (\beta \omega_n^2)^2}}$$

Where ω_n is the natural frequency, β is the loss factor and A is a constant.

- 1) Proof that the resonance of the function occurs at $\Omega_{res} = \omega_n$;
- 2) Proof that with the “-3dB method” (i.e., half power points method), the loss factor is

$$\beta = \frac{\Omega_b + \Omega_a}{2\omega_n} \frac{\Omega_b - \Omega_a}{\omega_n} \text{ And define } \Omega_b \text{ and } \Omega_a \text{ accordingly;}$$

- 3) Discuss the pros and cons of the method.

3. FOURIER TRANSFORM

A rectangular function $f(t)$ is defined as follows:

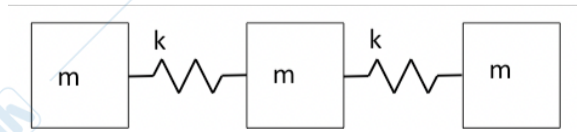
$$f(t) = F_0 \text{ with } -\frac{T}{2} < t < \frac{T}{2}$$

$$f(t) = 0 \text{ elsewhere}$$

- 1) Compute its Fourier Transform;
- 2) Plot the transform and find its zeros;
- 3) Show how the result can be used to define the Fourier Transform of a constant.

QUESTIONS COLLECTING---4 (18.09.2014)

1. OSCILLATIONS OF A MDOF SYSTEM



The three degrees of freedom system sketched in the figure is free to oscillate in a horizontal plane.

- 1) Plot the free body diagram of the three masses;
- 2) Write the equations of motion;
- 3) Express the equations in matrix notation;
- 4) Compute the eigenvalues $\omega_1^2 < \omega_2^2 < \omega_3^2$ as a function of m and k ;
- 5) Compute the mode shapes $\{\psi_1\}$, $\{\psi_2\}$ and $\{\psi_3\}$, assuming that $\psi_{1r} = 1$, $r = 1, 2, 3$;
- 6) Show that $\{\psi_1\}^T [m] \{\psi_2\} = 0$.

2. DUNCAN METHOD

Consider a n degrees of freedom system with **non-proportional** viscous damping; symmetric matrices $[m]$, $[k]$ and $[c]$ are given.

- 1) Express the equations of motion in the state space according to the Duncan's method;
- 2) Derive the corresponding eigenvalue problem;
- 3) Proof that the eigenvectors are orthogonal to the resulting matrices.

3. Extraction of modal parameters

Assume that the FRF $\alpha_{jk}(\Omega) = H(\Omega)$ of a structure is given, for example it is the result of an experimental test. Also assume that the FRF may be expressed in the following form

$$\alpha_{jk}(\Omega) = \sum_{r=1}^n \frac{a_r + i\Omega b_r}{\omega_r^2 - \Omega^2 + i2\zeta_r \omega_r \Omega}$$

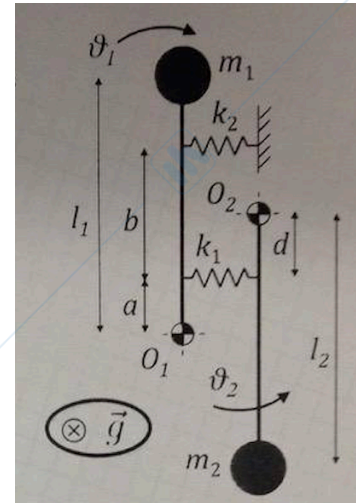
By considering a single mode S and using the above expression

- 1) Derive the necessary equations to determine the unknown modal parameters a_s , b_s , ω_s , ζ_s ;
- 2) Discuss the pros and cons of the method.

QUESTIONS COLLECTING---5 (21.02.2015)

1. OSCILLATION OF A 2 DOFS SYSTEM

Two masses m_1 and m_2 are connected to their hinges O_1 and O_2 by rigid beams. Mass and moment of inertia of each beam are negligible. Gravity is perpendicular to the plane so that it can be neglected. Consider SMALL oscillations only, $\vartheta_1 \ll 1$ and ϑ_2 are both $\ll 1$



According to the sketch:

- 1) Plot the free body diagram of the two beams
- 2) Write their equations of motion
- 3) express the equations in matrix notation

Under the hypothesis that $m_1=m_2=m$, $I_1=I_2=I$,
 $a=b=d$, $k_2=0$

- 4) Compute the eigenvalues as a function of m, k_1, I, a
- 5) Compute the second mode shape
- 6) Compute the second modal mass

2. FOURIER SERIES

The periodic function $f(t)$, period T_0 can be expressed in

The form:

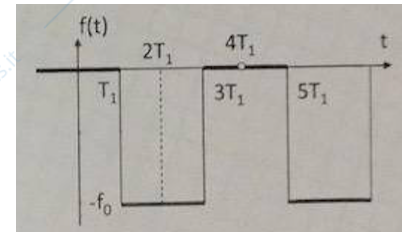
$$f(t) = a_0 + \sum_{k=1}^n (a_k \cos(k\Omega_0 t) + b_k \sin(k\Omega_0 t))$$

Where:

$$a_0 = \frac{1}{T_0} \int_0^{T_0} f(t) dt$$

$$a_k = \frac{2}{T_0} \int_0^{T_0} f(t) \cos(k\Omega_0 t) dt$$

$$b_k = \frac{2}{T_0} \int_0^{T_0} f(t) \sin(k\Omega_0 t) dt$$



- 1) Express the fundamental frequency Ω_0 as a function of T_1 ;
- 2) Compute a_0 ;
- 3) Compute b_k ;
- 4) Evaluate a_k with $k=1$ and $k=2$.

3. EIGENPROBLEM

Consider a n degree of freedom system with non proportiona viscous damping and its equation of motion (Duncan;s formulation) $[A]\{\dot{y}\} + [B]\{y\} = \{F_0\}e^{i\Omega t}$ With given symmetric matrices $[A]$ and $[B]$.

Define the eigenprobelm associated to the system;

By assuming that the resulting eigenvectors $\{\vartheta\}_r$, are orhogonal to $[A]$ and $[B]$, proof that the receptance $\alpha_{pq}(\Omega) = \frac{X_{0p}}{F_{0q}}$ maybe wrieten as a function of the r -th eigenvector $\{\vartheta\}_r$

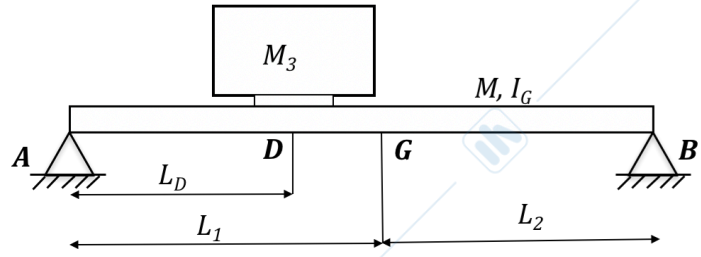
And the constant α_r and b_r .

Notice that X_{0p} is the amplitude of the reponse at point p , F_{0q} is the amlitude of the input at point q .

QUESTIONS COLLECTING---6 (02.02.2017)

1. OSCILLATIONS OF A THREE DEGREES OF FREEDOM SYSTEM

A motor (with mass M_3) is fixed in D to an infinitely rigid beam AB (with mass M and moment of inertia I_G) and its unbalance generates a periodic vertical force $f(t) = m\epsilon\Omega^2 e^{i\Omega t}$. Supports A, B and link D are modelled by springs and dampers, respectively named $k_1, c_1, k_2, c_2, k_3, c_3$.



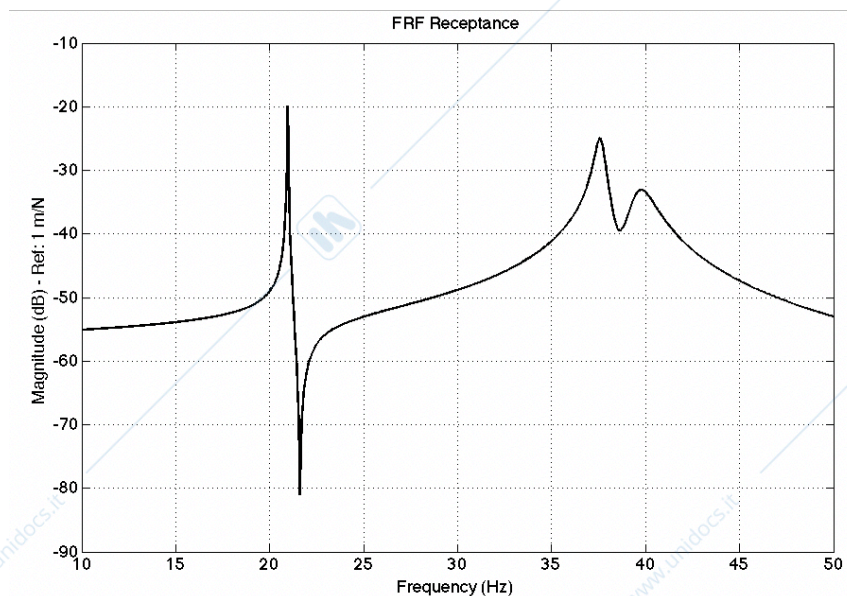
Assumed degrees of freedom: vertical displacement of mass M , rotation (small) of beam AB and vertical displacement of M_3 .

$$[m] = \begin{bmatrix} 200 & 0 & 0 \\ 0 & 37.5 & 0 \\ 0 & 0 & 20 \end{bmatrix} \quad [k] = 10^6 \begin{bmatrix} 5.0 & -0.075 & -1.0 \\ -0.075 & 2.2556 & 0.075 \\ -1.0 & 0.075 & 1.0 \end{bmatrix}$$

Natural frequencies: $f_1 = 20.96 \text{ Hz}$, $f_2 = 37.60 \text{ Hz}$, $f_3 = 30.96 \text{ Hz}$

Model shapes: $\{\psi_1\}^T = [1.0 \quad -0.025 \quad 1.534]$; $\{\psi_2\}^T = [1.0 \quad 3.41 \quad -6.42]$

- 1) Numerically verify the orthogonality of $\{\psi_1\}$ and $\{\psi_2\}$ with respect to $[m]$.
- 2) Compute the third eigenvector $\{\psi_3\}$
- 3) Given the generic matrices $[m]$, $[k]$, $[\Psi]$ and the eigenvalues ω_n^2 , $r=1, \dots, n$, determine the formal expression of the receptance $\alpha_{pq}(\Omega)$
- 4) Given the following figure $\alpha_{33}(\Omega)$, plot on the same graph the contribution of the second mode.

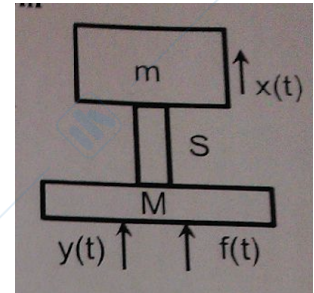


QUESTIONS COLLECTING---7

1. OSCILLATIONS OF A SINGLE DEGREE OF FREEDOM SYSTEM

in a fatigue test machine, a specimen S is positioned between a moving table with mass M and an upper body with mass m. The moving table undergoes the harmonic motion $y(t) = y_0 e^{i\Omega t}$ whilst $x(t) = x_0 e^{i\Omega t}$ is the absolute displacement of mass m. the specimen can be modelled as a spring with stiffness

$k = \frac{EA}{L}$ (E is the Young's modulus of the material, A is the area of the cross section and L is its length) in parallel, with a viscous damper (damping factor $\zeta = 1\%$).



$$E = 2.0 \times 10^{11} \frac{N}{m^2} \quad A = 100 mm^2 \quad L = 10 cm \quad m = 2 kg$$

- 1) Write the equation of motion of mass m;
- 2) Compute the stiffness k;
- 3) Compute the nature frequency ω_n ;
- 4) Write the expression of the amplitude A and the phase α of the relative motion

$$z(t) = x(t) - y(t) = z_0 e^{i\Omega t} = A e^{-i\alpha} e^{i\Omega t} \text{ with } |z_0| = A$$

- 5) Compute the value of A when $\Omega = \omega_n$ and $y_0 = 0.1$ mm.
- 6) Write the expression of the force $f(t)$ to be applied at mass M in order to obtain the displacement $y(t)$

2. FOURIER SERIES

Any periodic function $f(t)$, with period T_0 , can be expressed in the form

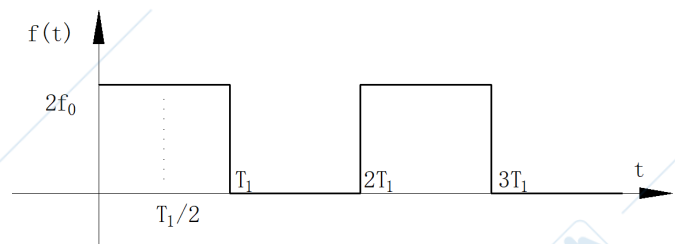
$$f(t) = a_0 + \sum_{k=1}^n (a_k \cos(k\Omega_0 t) + b_k \sin(k\Omega_0 t))$$

Where:

$$a_0 = \frac{1}{T_0} \int_0^{T_0} f(t) dt \quad a_k = \frac{2}{T_0} \int_0^{T_0} f(t) \cos(k\Omega_0 t) dt$$

$$b_k = \frac{2}{T_0} \int_0^{T_0} f(t) \sin(k\Omega_0 t) dt$$

- 1) Express the fundamental frequency Ω_0 as a function of T_1 ;
- 2) Compute a_0 ;
- 3) Compute b_k ;



- 4) Evaluate b_k with $k=1$ and $k=2$;
- 5) Plot $g(t) = a_0 + b_1 \sin(\Omega_0 t)$ on the same graph of $f(t)$

3. EIGENPROBLEM

Consider a n degree of freedom system with non proportional viscous damping and its equation of motion $[m]\{\ddot{x}\} + [c]\{\dot{x}\} + [k]\{x\} = \{F_0\}e^{i\Omega t}$

Given symmetric matrices $[m]$ $[k]$ $[c]$

- 1) Write the equation of motion in the state space according to Duncan's formulation
- 2) Define the eigenproblem associated to the resulting system of equations;
- 3) Proof the property of orthogonality of the eigenvectors;

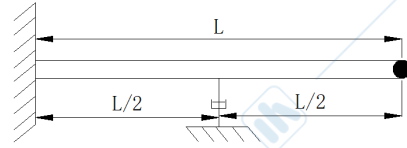
QUESTIONS COLLECTING---8

1. In the figure, determine the natural frequency of the system and damping factor based on LOG DECREMENT METHOD.

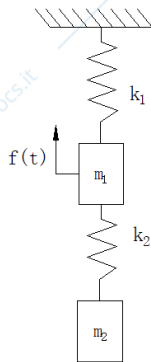
Data:

beam length L ;

body with mass m is connected to the beam;



2. Determine the amplitude and force transmitted from the mass m_1 to the whole system.



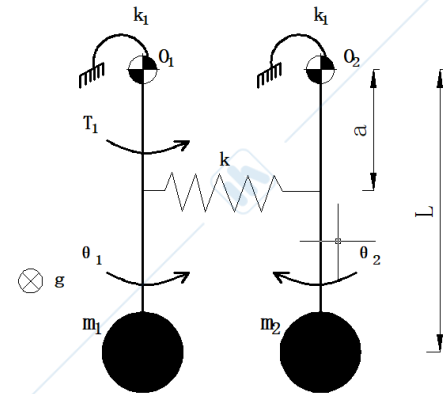
3. Determine the equations of WIENER-KHINCHIN and express its effect on power spectral density. (show the passages)

QUESTIONS COLLECTING---9(31.01.2018)

1. Oscillations of a two degrees of freedom system

Two rigid beams, with length l , rotate about hinges O_1 and O_2 . and carry two point masses m_1 and m_2 . k is the stiffness of a linear spring while k_1 and k_2 , are those of the torsional springs. T is the external torque. Gravity is orthogonal to the plane containing the oscillating beams.

Angles $\vartheta_1 \ll 1$ and ϑ_2 are both $\ll 1$.



- 1) Plot the free body diagrams of the two beams, using the given directions of ϑ_1 and ϑ_2 , and T .

The equation of motion of the system are:

$$m_1 l^2 \ddot{\vartheta}_1 + (k_1 + ka^2)\vartheta_1 + ka^2\vartheta_2 = T_1$$

$$m_2 l^2 \ddot{\vartheta}_2 + (k_2 + ka^2)\vartheta_2 + ka^2\vartheta_1 = 0$$

- 2) Write the equation in matrix notation;

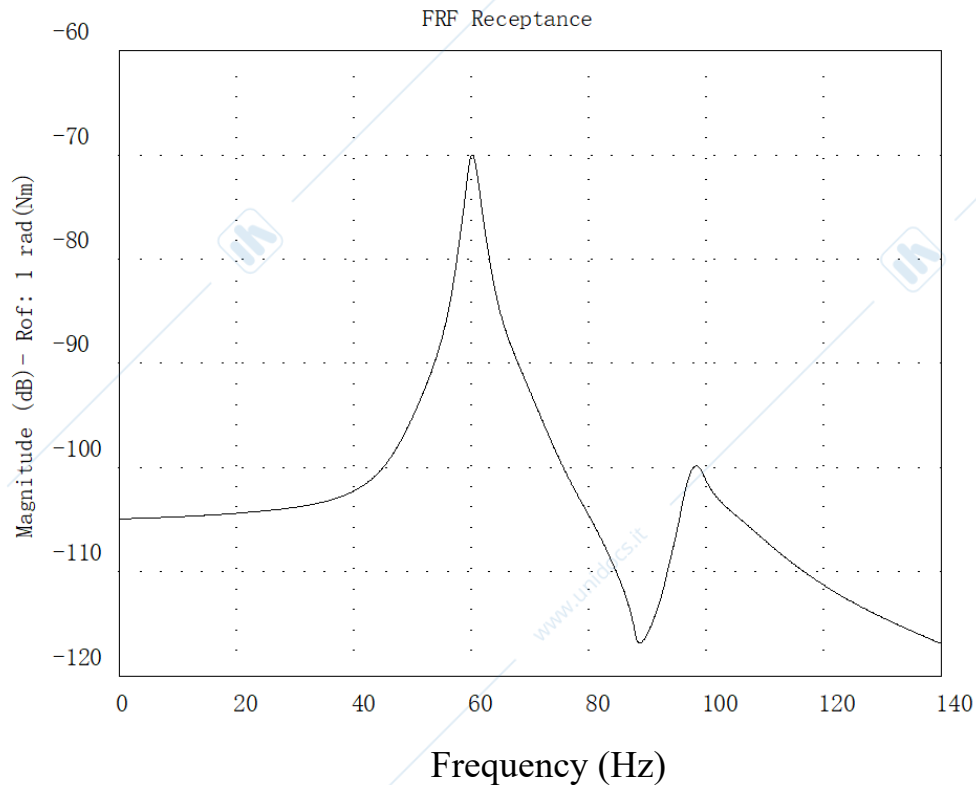
By assuming that $k_1 = k_T$, $k_2 = 2k_T$, $ka^2 = k_T$, $m_1 l^2 = m_2 l^2 = I$, the eigenvalues are:

$$\omega_1^2 = \frac{5-\sqrt{5}}{2} \frac{k_t}{I} \quad \text{and} \quad \omega_2^2 = \frac{5+\sqrt{5}}{2} \frac{k_t}{I}$$

- 3) Determine the corresponding eigenvectors $\{\psi\}_1$ and $\{\psi\}_2$;
 4) With $\psi_{11} = \psi_{12} = 1$ give a pictorial representation of the two mode shapes;
 5) Verify numerically that $\{\psi\}_1^T [m] \{\psi\}_2 = 0$

Use the following plot of a FRF (rotation/torque) to:

- 6) Indicate if it is related to ϑ_1 , or ϑ_2
 7) Determine the maximum oscillation amplitude with $T = 500 \text{ Nm}$



2. Fourier series

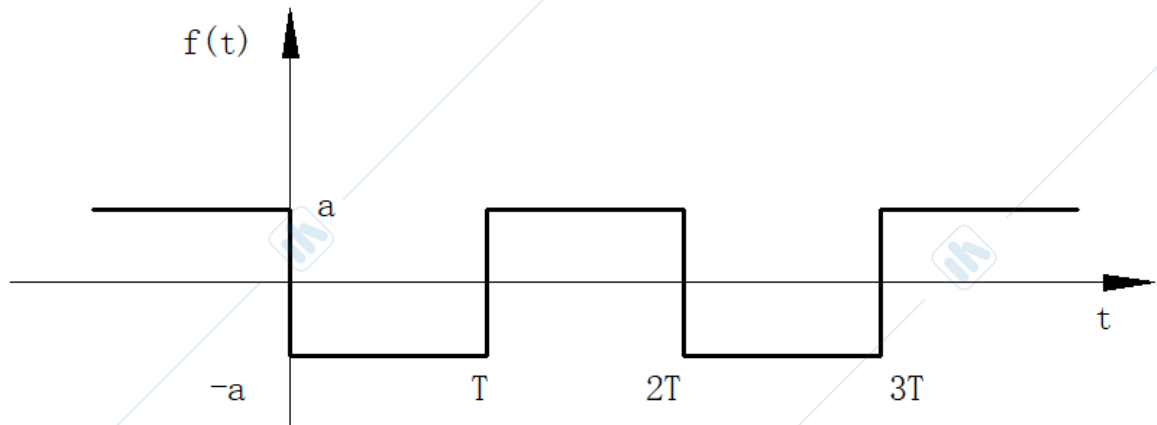
The Fourier series of any periodic function $f(t)$ is expressed by:

$$f(t) = a_0 + \sum_{k=1}^n (a_k \cos(k\Omega_0 t) + b_k \sin(k\Omega_0 t))$$

$$a_0 = \frac{1}{T_0} \int_0^{T_0} f(t) dt = \frac{1}{T_0} \int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} f(t) dt$$

$$a_k = \frac{2}{T_0} \int_0^{T_0} f(t) \cos(k\Omega_0 t) dt = \frac{2}{T_0} \int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} f(t) \cos(k\Omega_0 t) dt$$

$$b_k = \frac{2}{T_0} \int_0^{T_0} f(t) \sin(k\Omega_0 t) dt = \frac{2}{T_0} \int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} f(t) \sin(k\Omega_0 t) dt$$



Given the periodic function in the figure:

- 1) Verify if the function is even, odd or neither even nor odd;
- 2) Determine the fundamental frequency Ω of the function as a function of T;
- 3) Determine a_0 ;
- 4) Determine a_k ;
- 5) Determine b_k ;
- 6) Plot the first 4 spectral lines;
- 7) Plot, on the above figure, the first two non-null harmonics of the series;

3. Modal parameters identification – NbB method

A single degree of freedom system with mass m , stiffness k and loss factor η is forced by harmonic excitation.

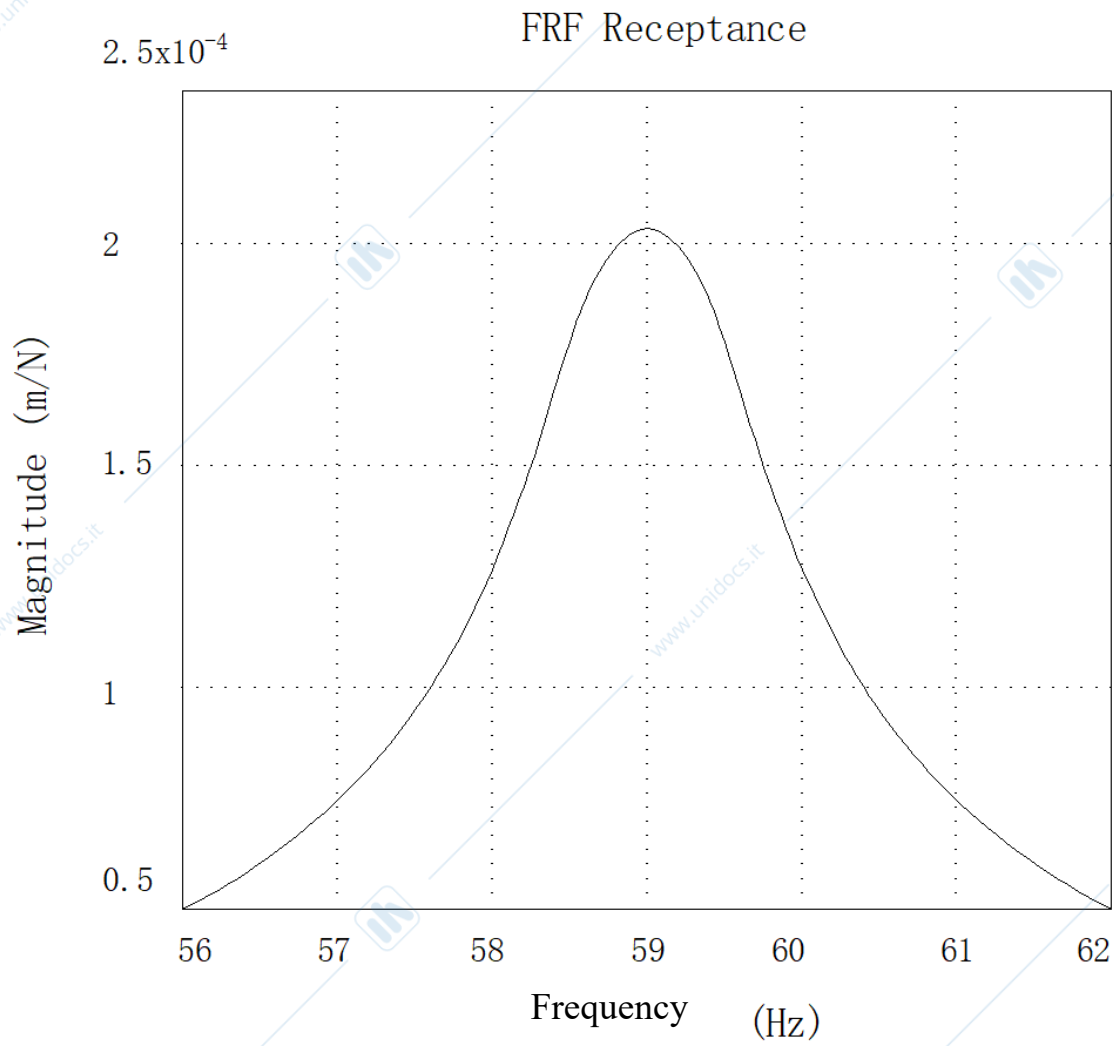
- 1) Write the equation of motion;
- 2) Determine the expression of the receptance;
- 3) Plot the modulus of the receptance;

The resonance of the system is at $\omega_n = \sqrt{k/m}$;

- 4a) With $20 \log \sqrt{n} = N$, where N is the number of dBs below the resonance, proof that

$$\eta = \frac{1}{\sqrt{n-1}} \frac{\Omega_b + \Omega_a}{2\omega_n} \frac{\Omega_b - \Omega_a}{\omega_n}$$

- 4b) Indicate Ω_a and Ω_b on the graph of the receptance;
- 5) Use the curve in the following page to estimate the loss factor $N = 3$



QUESTIONS COLLECTING---10

1. Extraction of modal parameters

the Kennedy – Pancu method can be used to extract the loss factor η and the natural frequency f_n of a SDOF forced system with hysteretical damping, whose FRF is

$$\frac{x}{f_0} = \frac{1}{k - m\omega^2 + ik\eta}$$

- 1) Represent the FRF of such a system in the Argand – Gauss plane (complex plane);
- 2) By using the previous graph, proof how the method can be applied to define η and f_n ;
- 3) Is it possible to define many estimates of η by using the same FRF with the K – P method? in case how can this be done?
- 4) Explain how the method can be extended to MDOF systems; which are the limitations?

2. MDOF SYSTEM

A three dof system is defined by the following matrices:

$$[m] = \begin{bmatrix} m & 0 & 0 \\ 0 & 2m & 0 \\ 0 & 0 & m \end{bmatrix} \quad [k] = \begin{bmatrix} k & -k & 0 \\ -k & 2k & -k \\ 0 & -k & k \end{bmatrix} \quad [\omega^2] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & k/m & 0 \\ 0 & 0 & 2k/m \end{bmatrix} = \text{eigenvalues}$$

- 1) Compute the corresponding modal matrix $[\psi]$;
- 2) Verify the orthogonality of $\{\psi_2\}$ and $\{\psi_3\}$ with respect to the mass matrix;
- 3) Under the assumption that numerical eigenvectors $\{\psi_N\}$ have to be compared with experimental mode shapes $\{\psi_l\}$, define the MAC (Modal assurance Criterion) and explain how it can be used;
- 4) Compute the MAC between $\{\psi_2\}$ and the vector $\{v\} = [2 \quad 0 \quad -1.5]^T$;
- 5) What is the MAC between $\{\psi_2\}$ and the vector $\{v\} = [-4 \quad 0 \quad 3.0]^T$;

3. FOURIER TRANSFORM

The Fourier transform pair is defined by the relations:

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\Omega) e^{i\Omega t} d\Omega \leftrightarrow f(\Omega) = \int_{-\infty}^{\infty} F(t) e^{-i\Omega t} d\Omega$$

- 1) Proof that the transform $F(\Omega)$ of a real and even function $e(t)$ is real and even;
- 2) Compute the Fourier transform $F(\Omega)$ of the impulse $f(t) = \delta(t - t_0)$ and its inverse transform;
- 3) Plot $F(\Omega)$;
- 4) Plot the modulus of the Fourier transform of the function $x(t) = A\delta(t - t_0) + A\delta(t + t_0)$

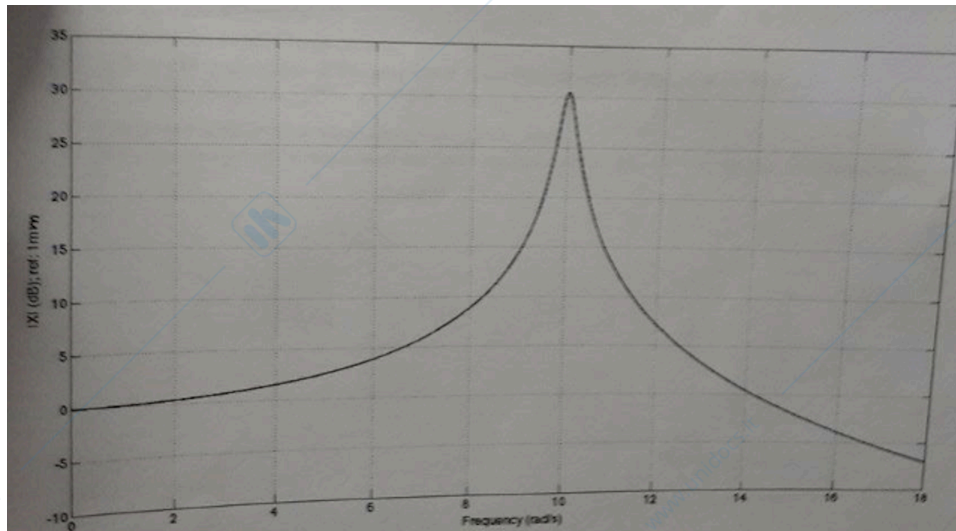
QUESTIONS COLLECTING---11

1. DAMPING AND FREQUENCY IDENTIFICATION

the response of a SDOF system with hysteretical damping may be written in the form of

$$X = \frac{f_0/m}{\omega_n^2 - \Omega^2 + i\omega_n^2\eta}$$

- 1) Compute the modulus of the FRF at the resonant frequency $\omega_R = \sqrt{\frac{k}{m}}$;
- 2) By considering the “- NdB method” ($20\log\sqrt{n} = N$, for example $20\log\sqrt{2} = 3$) and $\eta \ll 1$, derive the expression of the loss factor η ;
- 3) Determine the natural frequency ω_n (rad/s);
- 4) Determine the loss factor η by using both -3dBs and -15 dBs;
- 5) Determine the oscillation amplitude (mm) at $\Omega = 14$ rad/s.



Memo

1. S.D.O.F

- Fres response underdamped system
- Impulse function - Dirac's delta
- Linear (generic) system
 - Convolution intergral;
 - Superposition pricipile;
- Fourier series
 - The Frequency Response Function;
 - modulus and phase;
 - Influence of damping on the resonance;
 - Decibel and dB representation of the gain;
 - Receptance and accelerance;
- Hysteretic damping
 - Hysteretic damping model;
 - Hysteretical damping with complex notation;
 - Nyqyist plot;
- Model of acceleromter
- -3dB method (half-power method)
 - Hysteretical damping model;
- Log, Dec method
 - S.D.O.F time domain;

2. Fourier series

- Single processing;
 - Fourier series;
- Inverse fourier transform;
- Fourier transform pair;
- Fourier transform of delta($t-t_0$);
- Rectangle (window function);
- Fourier transferom of convolution intergral;
- Identification prodedure (inverse problem);
 - Direct parameter estimation(DPE);
 - Singular value decomposition (SVD);
- Steady state solution harmonic
- Parseval's identity;

3. Pendulum

- Dynamic of a compound
 - pendulum in the presence of Coulomb friction;
- Piecewise linear equation;
- Runge-kutta numerical integration;

4. Analogue to digital

- Analogue to digital conversion;
- Nyquist-Shannon sampling theorem;
- Aliasing error;
 - Analogue filter;
 - Digital filter;
 - Band pass filter;
- Analogue – digital transformation;
 - high bit $n = 24$;
 - Alternative coupling/directive coupling;
 - Synchronous Sampling;
 - Multiplexed sampling;
- ADC/Acquisition board;
- Fourier transform;
- Discrete Fourier Transform (DFT);
- Fast Fourier Transform (FFT);
- Leakage;
- Window technique;
- Random process;
 - Autocorrelation function;
 - Cross – correlation function;
- Error process;
- Stationary process;
- Power spectral density (PSD);
- Welch's periodogram;
 - Window;
- Amplitude of the harmonic function from its PSD;
- Dynamic stiffness measurement;

5. M.D.O.F system

- Free body diagram;
- Characteristic equation;
- Orthogonality of modes;
- Expansion theory;
- Proportional damping;
 - Harmonic excitation;
 - Receptance condition;
 - To plot a typical FRF;
- Non – proportional viscous damping;
 - Duncan's method;
 - State space method;
 - The response (Duncan);
 - Receptance;
- Root locus of S.D.O.F system;
 - Root locus of M.D.O.F proportional damping;
 - Root locus of M.D.D.F non-proportional damping;

6. Error analysis

- Estimation of FRF signal input, signal output system SISO;
 - Noise in output;
 - Noise in input;
 - Noise in input and output;
 - Quantity of noise;
- Multiple input Multiple output system MIMO;
- Model parameters extraction methods;
- Time domain - single degree of freedom;
 - Logarithmic decrement
- Frequency domain – single degree of freedom;
 - Hysteretic damping;
 - -3dB method;
 - NdB method;
- Kennedy and Pancu method;
 - The K&P method to S.D.O.F systems;

- - Least square solution
 - S.D.O.F frequency domain;
 - Regenerated FRF;
 - M.D.O.F time domain output only;