V.M QUESTIONS COLLECTING

Andy Lu

We stand along together

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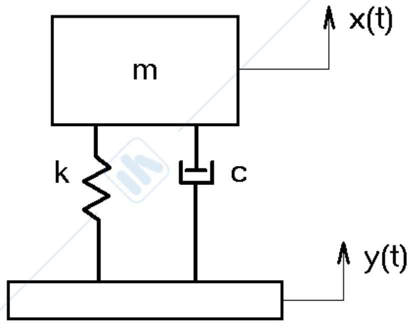
Memo **16**

QUESTIONS COLLECTING---1



1. The base of the SDOF system in the figure is moving with $y(t) = y_0 \cdot \cos \omega t$ and $y_0 = const$
   Find the amplitude and phase of the force transmitted from the mass m to the base.

2. For an "n" degrees of freedom system with proportional viscous damping you are given the mode shapes $\{\psi_r\}$, the natural frequencies $\omega_r$and the damping factors $\zeta_r (r = 1,2 \dots, n)$.
   Calculate the expression of the reacceptance $\alpha_{ij}(\omega)$.

3. For an "n" degrees of freedom system with non-proportional damping proof that the mode shapes are orthogonal (Duncan method).

4. For a given signal x(t) with a total duration T_{tot} explain how to compute the spectrum $S_{xx}(\Omega)$ according to the Welch's periodogram method.

5. Explain the "-N dB" method, with its pros and cons, to determine the loss factor of a SDOF system with hysterical damping.

6. Explain the origin of aliasing and leakage and how to limit their effects.

7. MDOF: Hysteretic proportional damping - derive the expression of receptance

8. EXTRACTION METHODS: Fractional polynomial method

9. Given an input and output random signal: derive their relations in frequency and time domain based on system responses h and H.

10. Derive the coherence function.

11. 3db method

12. Discrete Fourier series

13. hysteretic damping

14. Build an even function (e) in Fourier Series

15. Question related to signal and noise ratio....something like that!

16. Orthogonality property of MDOF system

QUESTIONS COLLECTING---2

1. **A GENERAL SDOF HYSTERETIC DAMPED SYSTEM**

   Given the function: $m\ddot{x} + x(i+\beta)k = F_0^{i\Omega t}$,

   1) use the -3dB method and calculate the receptance $\alpha(\Omega)$;
   2) plot $\alpha(\Omega)$ showing the influence of β;
   3) proof that resonance ω_n is at natural frequency;
   4) Proof that $\beta = \frac{{\Omega_b}^2 - {\Omega_a}^2}{2\omega_n^2}$;

2. **NDOF PROPORTIONAL VISCOUS DAMPED SYSTEM**

   Given the function: $[m]\{\ddot{x}\} + [c]\{\dot{x}\} + [k]\{x\} = \{0\}$

   1) Define the eigenproblem;
   2) Proof the [m] [k] [c] orthogonality;
   3) given $\{x_0\}$ and $\{v_0\}$ calculate the free response;

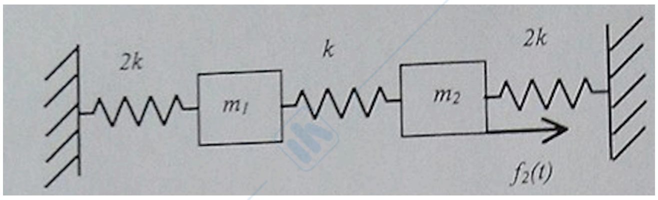
3. The **FOURIER TRANSFORM** is

   $$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\Omega)e^{i\Omega t}d\Omega \leftrightarrow F(\Omega) = \int_{-\infty}^{\infty} f(t)e^{-i\Omega t}dt$$

   1) proof the Fourier Transform of a real an even function is also a real and even function
   2) x(t) is a rectangular function on [-T/2, T/2] with amplitude equal to 1 calculate its Fourier Transform
   3) proof the Parseval's theorem

QUESTIONS COLLECTING---3

1. FORCED VIBRATIONS



For the two degrees of freedom system sketched above:

1) Write the equations of motion as a function of m_1, m_2 and k;
2) With $m_1=m_2=m$ computes the eigenvalues and the eigenvectors;
3) With $m_1=m_2=m$ computes the modal mass and modal stiffness matrices;
4) With $f_2(t) = Fsin(\Omega t)$ compute the amplitude of vibration of mass m_1 when $m_1=m_2=m=2$ kg, k= 1000N/m, F=20 N, Ω=40 rad/s.

2. EXTRACTION OF MODAL PARAMETERS

The modulus of the receptance $\boldsymbol{\alpha(\Omega)}$ of a SDOF system with hysterical damping may be written in the form: $|\boldsymbol{\alpha(\Omega)}| = \frac{A}{\sqrt{(\omega_n{}^2-\Omega^2)^2+(\beta\omega_n^2)^2}}$

Where $\boldsymbol{\omega_n}$ is the natural frequency, $\boldsymbol{\beta}$ is the loss factor and A is a constant.

1) Proof that the resonance of the function occurs at $\boldsymbol{\Omega_{res} = \omega_n}$;
2) Proof that the with the "-3dB method" (i.e., half power points method), the loss factor is $\boldsymbol{\beta = \frac{\Omega_b+\Omega_a}{2\omega_n}\frac{\Omega_b-\Omega_a}{\omega_n}}$ And define $\boldsymbol{\Omega_b}$ and $\boldsymbol{\Omega_a}$ accordingly;
3) Discuss the pros and cons of the method.

3. FOURIER TRANSFORM

A rectangular function $f(t)$ is defined as follows:

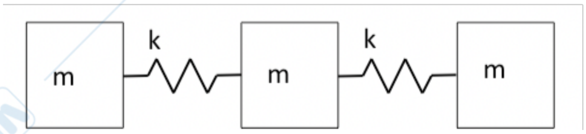
$$f(t) = F_0 \quad with -\frac{T}{2} < t < \frac{T}{2}$$

$$f(t) = 0 \quad elsewhere$$

1) Compute its Fourier Transform;
2) Plot the transform and find its zeros;
3) Show how the result can be used to define the Fourier Transform of a constant.

QUESTIONS COLLECTING---4 (18.09.2014)

1. OSCILLATIONS OF A MDOF SYSTEM



The three degrees of freedom system sketched in the figure is free to oscillate in a horizontal plane.

1) Plot the free body diagram of the three masses;
2) Write the equations of motion;
3) Express the equations in matrix notation;
4) Compute the eigenvalues $\omega_1^2 < \omega_2^2 < \omega_3^2$ as a function of m and k;
5) Compute the mode shapes $\{\psi_1\}$, $\{\psi_2\}$and $\{\psi_3\}$, assuming that ψ_{1r} =1, r =1,2,3;
6) Show that $\{\psi_1\}^T$ [m] $\{\psi_2\}$=0.

2. DUNCAN METHOD

Consider a n degrees of freedom system with **non-proportional** viscous damping; symmetric matrices [m], [k] and [c] are given.

1) Express the equations of motion in the state space according to the Duncan's method;
2) Derive the corresponding eigenvalue problem;
3) Proof that the eigenvectors are orthogonal to the resulting matrices.

3. Extraction of modal parameters

Assume that the FRF $\alpha_{jk}(\boldsymbol{\Omega}) = \boldsymbol{H}(\boldsymbol{\Omega})$ of a structure is given, for example it is the result of an experimental test. Also assume that the FRF may be expressed in the following form

$$\alpha_{jk}(\boldsymbol{\Omega}) = \sum_{r=1}^{n} \frac{a_r + i\boldsymbol{\Omega} b_r}{\omega_r^2 - \boldsymbol{\Omega}^2 + i2\zeta_r\omega_r\boldsymbol{\Omega}}$$

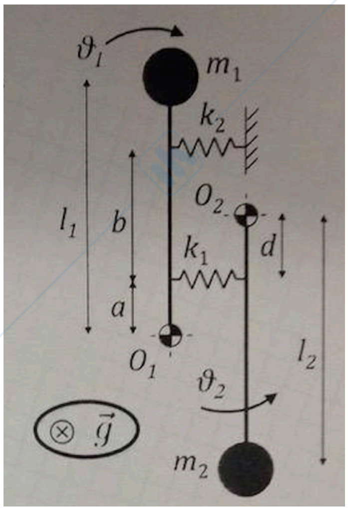
By considering a **single mode S** and using the above expression

1) Derive the necessary equations to determine the unknown modal parameters a_S, b_S, ω_S, ζ_S;
2) Discuss the pros and cons of the method.

QUESTIONS COLLECTING---5 (21.02.2015)

1. OSCILLATION OF A 2 DOFS SYSTEM

Two messes m_1 and m_2 are connected to their hinges O_1 and O_2 by rigid beams. Mass and moment of inertia of each beam are negligible. Gravity is perpendicular to the plane so that it can be neglected. Consider SMALL oscillations only, $\vartheta_1 \ll 1$ *and* ϑ_2 *are both* $\ll 1$



According to the sketch:

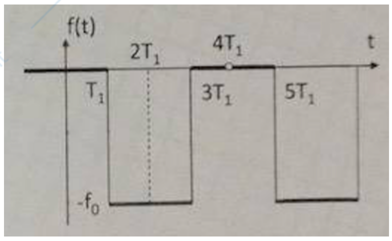
1) Plot the free body diagram of the two beams
2) Write their equations of motion
3) express the equations in matrix notation

Under the hypothesis that $m_1=m_2=m$, $I_1=I_2=I$, $a=b=d$, $k_2=0$

4) Compute the eigenvalues as a function of m, k_1, I, a
5) Compute the second mode shape
6) Compute the second modal mass

2. FOURIER SERIES

The periodic function $f(t)$, period T_0 can be expressed in The form:



$$f(t) = a_0 + \sum_{k=1}^{n}\left(a_k \cos(k\Omega_0 t) + b_k sin(k\Omega_0 t)\right)$$

Where:

$$a_0 = \frac{1}{T_0}\int_0^{T_n} f(t)dt$$

$$a_k = \frac{2}{T_0}\int_0^{T_n} f(t)\cos(k\Omega_0 t)\, dt$$

$$b_k = \frac{2}{T_0}\int_0^{T_n} f(t) sin(k\Omega_0 t)dt$$

1) Express the fundamental frequency Ω_0 as a function of T_1;
2) Compute a_0;
3) Compute b_k;
4) Evaluate a_k with k=1 and k=2.

3. EIGENPROBLEM

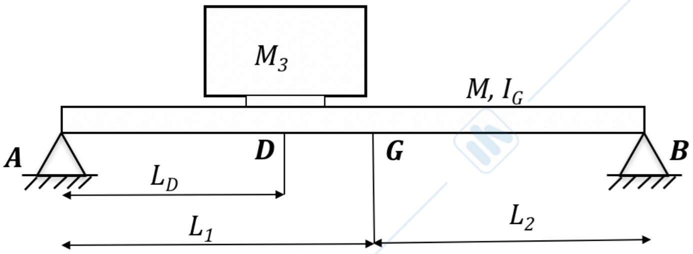
Consider a n degress of freedom system with non proportioanl viscous damping and its equation of motion (Duncan;s formulation) $[A]\{\dot{y}\} + [B]\{y\} = \{F_0\}e^{i\mathbf{\Omega t}}$ With given symmetric matrics $[A]\ and[B]$.

Define the eigenprobelm associated to the system;
By assuming that the resulting eigenvectors $\{\vartheta\}_r$, are orhogonal to $[A]\ and[B]$, proof that the receptance $\alpha_{pq}(\mathbf{\Omega}) = \frac{X_{0p}}{F_{0q}}$ maybe writeen as a function of the r-th eigenvector $\{\vartheta\}_r$
And the constant $\alpha_r\ and\ b_r$.
Notice that $X_{0p}$is the amplitude of the reponse at point p, F_{0q} is the amlitude of the imput at point q.

QUESTIONS COLLECTING---6 (02.02.2017)

1. OSCILLATIONS OF A THREE DEGREES OF FREEDOM SYSTEM

A motor (with mass M3) is fixed in D to an infinitely rigid beam AB (with mass M and moment of inertia I_G) and its unbalance generates a periodic vertical force $f(t) = m\varepsilon\Omega^2 e^{i\Omega t}$. Supports A, B and link D are modelled by springs and dampers, respectively named k1, c1, k2, c2, k3, c3.



Assumed degrees of freedom: vertical displacement of mass M, rotation (small) of beam AB and vertical displacement of M_3.

$$[m] = \begin{bmatrix} 200 & 0 & 0 \\ 0 & 37.5 & 0 \\ 0 & 0 & 20 \end{bmatrix} \qquad [k] = 10^6 \begin{bmatrix} 5.0 & -0.075 & -1.0 \\ -0.075 & 2.2556 & 0.075 \\ -0.1 & 0.075 & 1.0 \end{bmatrix}$$

Natural frequencies: $f_1 = 20.96\,Hz,\ f_2 = 37.60\,Hz,\ f_3 = 30.96\,Hz$

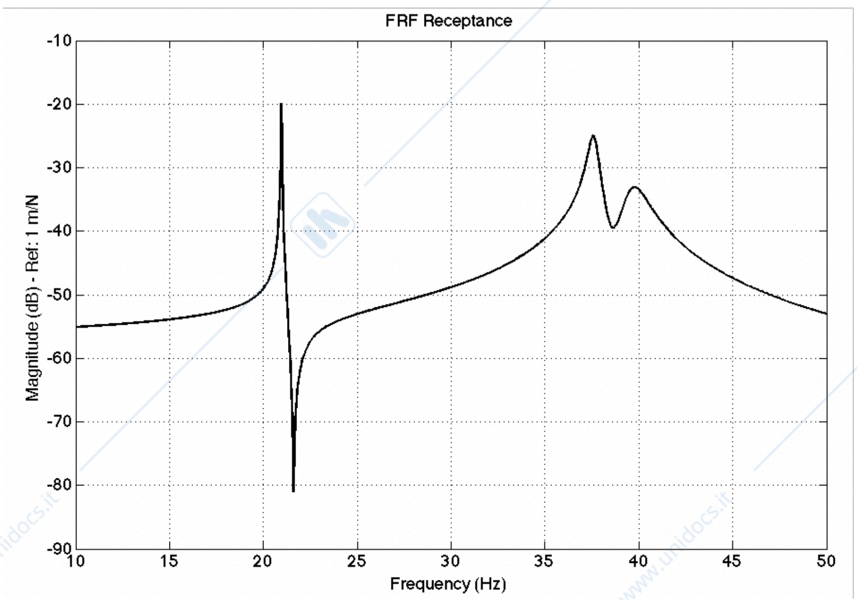
Model shapes: $\{\psi_1\}^T = [1.0\ \ -0.025\ \ 1.534]$; $\{\psi_2\}^T = [1.0\ \ 3.41\ \ -6.42]$

1) Numerically verify the orthogonality of $\{\psi_1\}$ and $\{\psi_1\}$with respect to [m].

2) Compute the third eigenvector $\{\psi_3\}$

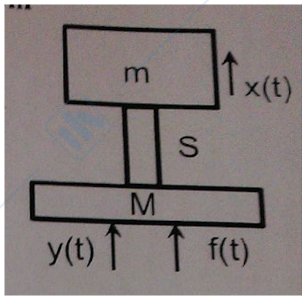
3) Given the generic matrices [m], [k], [Ψ] and the eigenvalues ω_n^2, r=1,...,n, determine the formal expression of the receptance $\alpha_{pq}(\Omega)$

4) Given the following figure $\alpha_{33}(\Omega)$, plot on the same graph the contribution of the second mode.



QUESTIONS COLLECTING---7

1. OSCILLATIONS OF A SINGLE DEGREE OF FREEDOM SYSTEM



in a fatigue test machine, a specimen S is positioned between a moving table with mass M and an upper body with mass m. The moving table undergoes the harmonic notion $y(t) = y_0 e^{i\Omega t}$ whilst $x(t) = x_0 e^{i\Omega t}$ is the absolute displacement of mass m. the specimen can be modelled as a spring with stiffness $k = \frac{EA}{L}$ (E is the Young's modulus of the material, A is the area of the cross section and L is its length) in parallel, with a viscous damper (damping factor $\zeta = 1\%$).

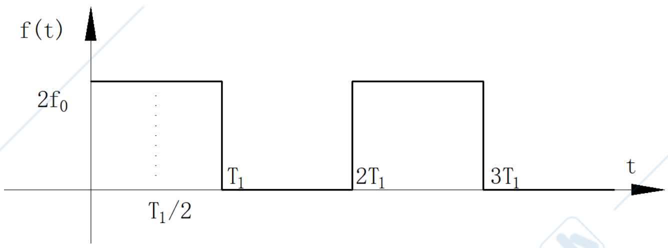
$E = 2.0 \times 10^{11} \frac{N}{m^2} \quad A = 100mm^2 \quad L = 10\ cm \quad m = 2\ kg$

1) Write the equation of motion of mass m;
2) Compute the stiffness k;
3) Compute the nature frequency ω_n;
4) Write the expression of the amplitude A and the phase α of the relative motion

$$z(t) = x(t) - y(t) = z_0 e^{i\Omega t} = Ae^{-i\alpha} e^{i\Omega t} with |z_0| = A$$

5) Compute the value of A when $\Omega = \omega_n$ and y_0=0.1 mm.
6) Write the expression of the force $f(t)$ to be applied at mass M in order to obtain the displacement $y(t)$

2. FOURIER SERIES



Any periodic function $f(t)$, with period T_0, can be expressed in the form

$$f(t) = a_0 + \sum_{k=1}^{n} \left(a_k \cos(k\Omega_0 t) + b_k sin(k\Omega_0 t)\right)$$

Where:

$$a_0 = \frac{1}{T_0}\int_0^{T_n} f(t)dt \qquad a_k = \frac{2}{T_0}\int_0^{T_n} f(t)\cos(k\Omega_0 t)\, dt$$

$$b_k = \frac{2}{T_0}\int_0^{T_n} f(t)sin(k\Omega_0 t)dt$$

1) Express the fundamental frequency Ω_0 as a function of T_1;
2) Compute a_0;
3) Compute b_k;

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4) Evaluate b_k with k=1 and k=2;
5) Plot $g(t) = a_0 + b_1 sin(\Omega_0 t)$ on the same graph of $f(t)$

3. EIGENPROBLEM

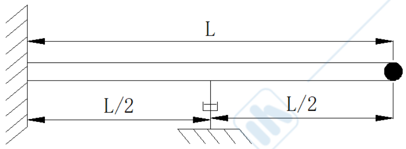
Consider a n degree of freedom system with non proportional viscous damping and its equation of motion $[m]\{\ddot{x}\} + [c]\{\dot{x}\} + [k]\{x\} = \{F_0\}e^{i\Omega t}$

Given symmetric matrices [m] [k] [c]

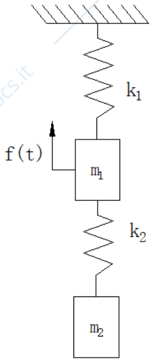
1) Write the equation of motion in the state space according to Duncan's formulation
2) Define the eigenproblem associated to the resulting system of equations;
3) Proof the property of orthogonality of the eigenvectors;

QUESTIONS COLLECTING---8

1. In the figure, determine the natural frequency of the system and damping factor based on LOG DECREMENT METHOD.
   Data:
   beam length L;
   body with mass m is connected to the beam;



2. Determine the amplitude and forc e transmited from the mass m1 to the whole system.



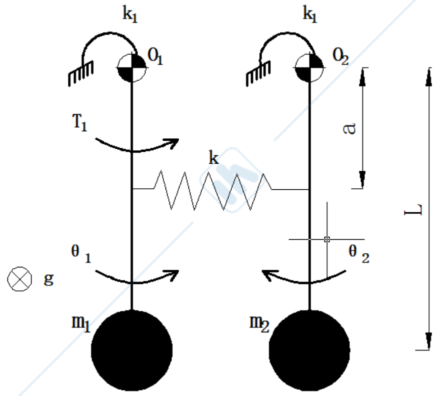
3. Determin the equitions of WIENER-KHINCHIN and express its effect on power spectural density.(show the passages)

QUESTIONS COLLECTING---9(31.01.2018)

1. Oscillations of a two degrees of freedom system

Two rigid beams, with length l, rotate about hinges O_1 and O_2. and carry two point masses m_1 and m_2. k is the stiffness of a linear spring while k_1 and k_2, are those of the torsional springs. T is the external torque. Gravity is orthogonal to the plane containg the oscillating beams.

Angles $\vartheta_1 \ll 1$ *and* ϑ_2 *are both* $\ll 1$.



1) Plot the free body diagrams of the two beams, using the given directions of ϑ_1 *and* ϑ_2, *and* T.

The equation of motion of the system are:

$$m_1 l^2 \ddot{\vartheta}_1 + (k_1 + ka^2)\vartheta_1 + ka^2\vartheta_2 = T_1$$
$$m_2 l^2 \ddot{\vartheta}_2 + (k_2 + ka^2)\vartheta_2 + ka^2\vartheta_1 = 0$$

2) Write the eaquation in matrix notation;

By assuming that $k_1 = k_T,\ k_2 = 2k_T,\ ka^2 = k_T,\ m_1 l^2 = m_2 l^2 = I$, the digenvalue are :

$\omega_1^2 = \frac{5-\sqrt{5}}{2}\frac{k_T}{I}$ and $\omega_2^2 = \frac{5+\sqrt{5}}{2}\frac{k_T}{I}$

3) Determine the corresponding eigenvectors $\{\psi\}_1$ and $\{\psi\}_2$;

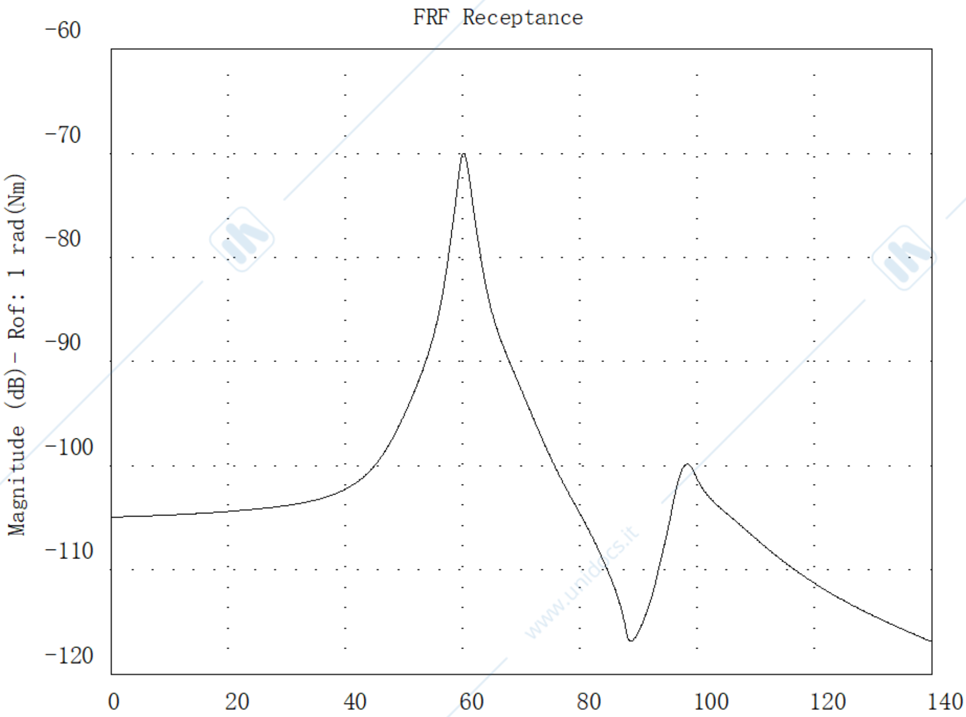
4) With $\psi_{11} = \psi_{12} = 1$ give a pictorial representation of the two mode shapes;

5) Verify numerically that $\{\psi\}_1[m]\{\psi\}_2 = 0$

Use the following plot of a FRF (rotation/torque) to :

6) Indicate if it is related to ϑ_1, *or* ϑ_2

7) Determine the maximum oscillation amplitude with T = 500 Nm



2. Fourrier series

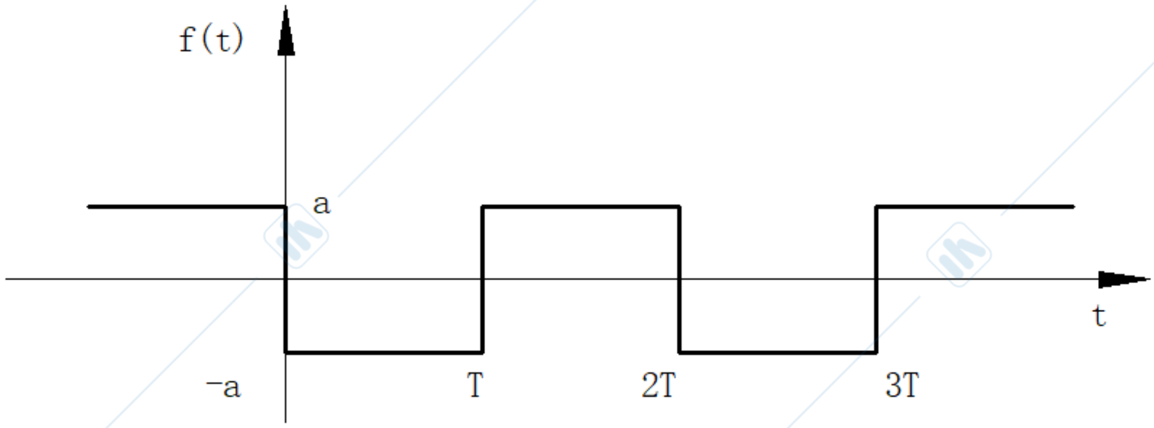
The Fourier series of any periodic function $f(t)$ is expressed by:

$$f(t) = a_0 + \sum_{k=1}^{n} \left(a_k \cos(k\Omega_0 t) + b_k sin(k\Omega_0 t) \right)$$

$$a_0 = \frac{1}{T_0} \int_0^{T_0} f(t)dt \qquad = \frac{1}{T_0} \int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} f(t)dt$$

$$a_k = \frac{2}{T_0} \int_0^{T_n} f(t) \cos(k\Omega_0 t)\, dt = \frac{2}{T_0} \int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} f(t) \cos(k\Omega_0 t)\, dt$$

$$b_k = \frac{2}{T_0} \int_0^{T_n} f(t)\, sin(k\Omega_0 t)\, dt \ = \frac{2}{T_0} \int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} f(t) sin(k\Omega_0 t) dt$$



Given the periodic function in the figure:

1) Verify if the function is even, old or nerither even nor odd;
2) Deterimine the fundamental frequency Ω of the function as a function of T;
3) Determine a_0;
4) Determine a_k;
5) Determine b_k;
6) Plot the firest 4 spectral lines;
7) Plot, on the above figure, the first two non-null harmonics of the series;

3. Modal parameters identification – NbB method

A single degree of freedom system with mass m, stiffness k and loss factor η is forced by harmonic excitation.

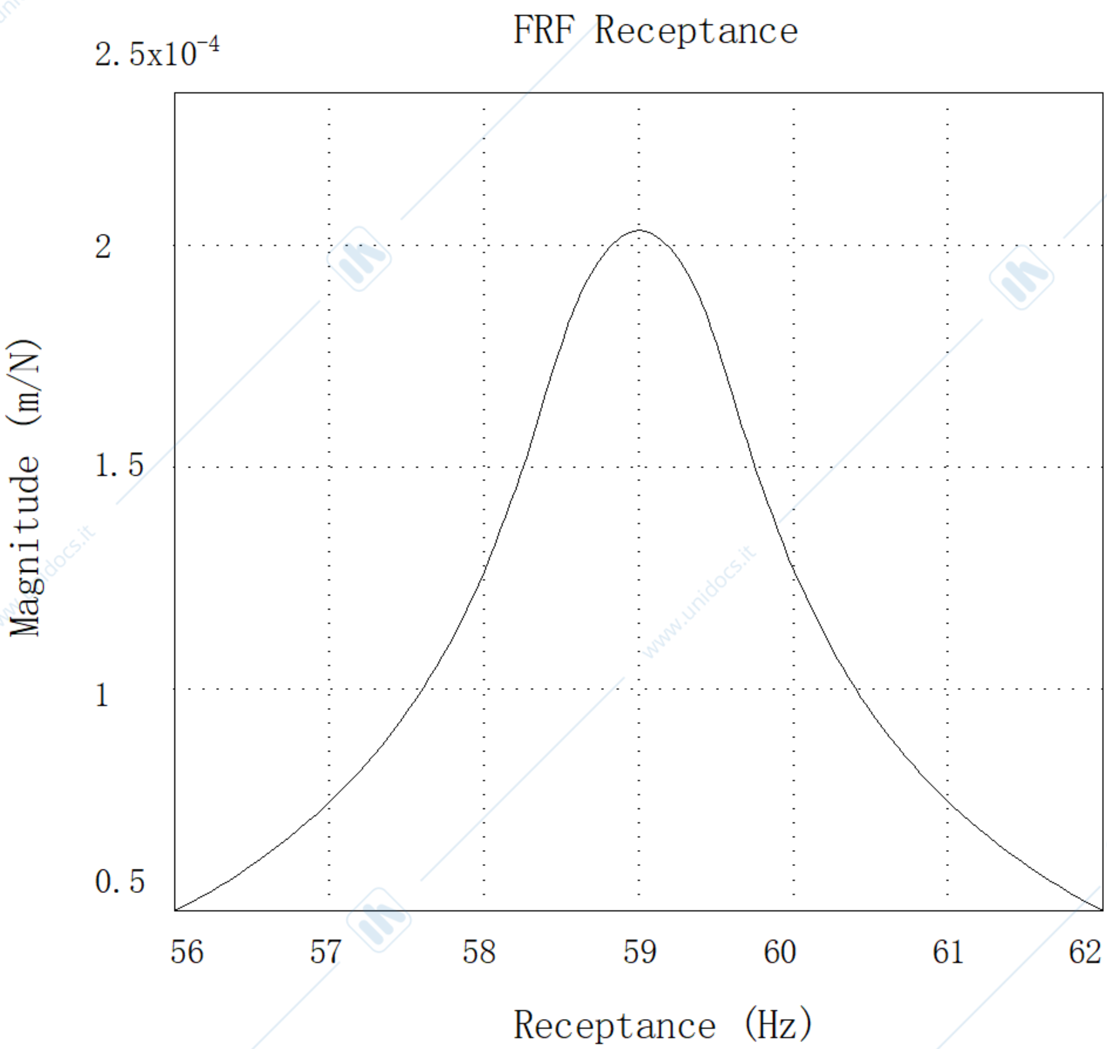
1) Write the equation of motion;
2) Determine the expression of the receptance;
3) Plot the modulus of the receptance;

The resonance of the system is at $\omega_n = \sqrt{k/m}$;

4a) With $20log\sqrt{n} = N$, where N is the number of dBs below the resonance, proof that $\eta = \frac{1}{\sqrt{n-1}} \frac{\Omega_b+\Omega_a}{2\omega_n} \frac{\Omega_b-\Omega_a}{\omega_n}$

4b) Indicate Ω_a and Ω_b on the graph of the receptance;

5) Use the curve in the following page to estimate the loss factor N = 3


FRF Receptance
2.5x10^-4
2
1.5
1
0.5
Magnitude (m/N)
56
57
58
59
60
61
62
Receptance (Hz)

Memo

1. S.D.O.F
   - Fres response underdamped system
   - Impulse function - Dirac's delta
   - Linear (generic) system

     -Convolution intergral;

     -Superposition priciple;
   - Fourier series

     -The Frequency Response Function;

     -modulus and phase;

     -Influence of damping on the resonance;

     -Decibel and dB representation of the gain;

     -Receptance and accelerance;
   - Hysteretic damping

     -Hysteretic damping model;

     -Hysteretical damping with complex notation;

     -Nyqyist plot;
   - Model of acceleromter
   - -3dB method (half-power method)

     -Hysteretical damping model;
   - Log, Dec method

     -S.D.O.F time domain;
2. Fourier series
   - Single processing;

     -Fourier series;
   - Inverse fourier transform;
   - Fourier transform pair;
   - Fourier transform of delta(t-t_0);
   - Rectangle (window function);
   - Fourier transferom of convolution integral;
   - Identification prodedure (inverse problem);

     -Direct parameter estimation(DPE);

     -Singular value decomposition (SVD);
   - Steady state solution harmonic
   - Parseval's identity;

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3. Pendulumn
   - Dynamic of a compond
     -pendulumn in the presente of coulumb friction;
   - Piecewise linear equation;
   - Runge-kutta numerical intergration;

4. Analogue to digital
   - Analogue to digital conversion;
   - Nyquist-shannon sampling theorem;
   - Aliasing error;
     -Analogue filter;
     -Digital filter;
     -Band pass filter;
   - Analogue – digital transformation;
     -high bit n = 24;
     -Alternative coupling/directive coupling;
     -Synchronous Sampling;
     -Multiplexed sampling;
   - ADC/Acqusistion board;
   - Fourier transform;
   - Discrete fourier transform(DFT);
   - Fast fourier transfer (FFT);
   - Leakage;
   - Window technique;
   - Random process;
     -Autocorrection function;
     -Cross – correlation function;
   - Error process;
   - Stationary process;
   - Power spectral density (PSD);
   - Wolch's periodogram;
     -Window;
   - Amplitude of the harmonic function from its PSD;
   - Dynamic sitffness measurement;

5. M.D.O.F system
   - Free body diagram;
   - Charateritic equation;
   - Othogonality of modes;
   - Expansion theroy;
   - Proprotional damping;

     -Harmonic excitation;

     -Receptance condition;

     -To plot a typical FRF;
   - Non – proportional vixcous damping;

     -Duncan's method;

     -State space method;

     -The response (Duncan);

     -Receptance;
   - Root locus of S.D.O.F system;

     -Root locus of M.D.O.F proportional damping;

     -Root locus of M.D.D.F non-proprotional damping;

6. Error analysis
   - Estination of FRF signal input, signal output system SISO;

     -Noise in output;

     -Noise in input;

     -Noise in input and output;

     -Quantity of nosie;
   - Multiple input Multiple output sytsem MIMO;
   - Model parameters extraction methods;
   - Time domain - single degree of freedom;

     -Logrithmic decrement
   - Frequency domain – single degree of freedom;

     -Hysteretical damping;

     --3dB method;

     -NdB method;
   - Kenndy and Pancu method;

     - The K&P method to S.D.O.F systems;

- - Least square solution
  - S.D.O.F frequency domain;
  - Regenerated FRF;
  - M.D.O.F time domain output only;