2.6 Time variant channels

When we remove the time invariancy assumption of the channels, we have to include the time variable t in the model parameters. Therefore the channel response (2.3.13) becomes

$$r_L(t) = \sum_n \alpha_n(t) e^{-j2\pi f_C \tau_n(t)} \cdot s_L(t - \tau_n(t)) = \sum_n c_n(t) \cdot s_L(t - \tau_n(t)) \qquad (2.6.1)$$

and the channel impulse response

$$h_L(\tau; t) = \sum_n \alpha_n(t) e^{-j2\pi f_C \tau_n(t)} \cdot \delta(\tau - \tau_n(t)). \qquad (2.6.2)$$

It is important to notice the double role of the variables τ and t: τ is associated to the propagation delays while t refers to the absolute time.

Also the transfer function turns out to be time dependent (the Fourier transform is applied to the τ variable),

$$H_L(f; t) = \int h_L(\tau; t) e^{-j2\pi f \tau} d\tau, \qquad (2.6.3)$$

and the received signal is computed either by

$$r_L(t) = \int h_L(\tau; t) s_L(t - \tau) d\tau \qquad (2.6.4)$$

or by means of the Fourier transform

$$r_L(t) = \int H_L(f; t) \cdot S_L(f) e^{j2\pi f t} df. \qquad (2.6.5)$$

2.6.1 The Doppler effect

In order to understand more deeply the impact of time-variancy on the signal, let us restrict our attention to a single, direct path between a fixed and a mobile station. The mobile station is, at time $t = 0$, at d [m] from the fixed one and it is moving towards it with a velocity v; the transmitted signal $s_L(t)$ is a sinusoid. Under these assumptions, the received signal can be written as

$$r_L(t) = \alpha \cdot e^{-j2\pi f_C (d - v \cdot t)/c} = \alpha \cdot e^{-j2\pi f_C \tau_0} \cdot e^{j2\pi f_C \cdot v/c \cdot t} \qquad (2.6.6)$$

since the amplitude α is considered constant in a time interval sufficiently small around $t = 0$ and $\tau_0 = d/c$. From (2.6.6), it is clear that $r_L(t)$ turns out to be a sinusoid shifted by a frequency offset $f_D = f_C v/c$ and this phenomenon is the well-known Doppler effect. As a consequence we can state that, given a couple of transceivers situated in two points A and B, the received signal does not depend only on their positions but also on their istantaneous velocities v_A and v_B.

When the propagation direction is not parallel to the velocity v but forms an angle θ, it is enough to consider the component of v along this direction and the Doppler shift is

$$f_D = f_C \cdot \frac{v}{c} \cdot cos(\theta) = \frac{v}{\lambda} \cdot cos(\theta). \tag{2.6.7}$$

If we generalize the propagation environment to a multipath scenario, at the receiver we will observe a superposition of sinusoids, each characterized by its Doppler offset, determined by the angle of the propagation path w.r.t. the velocity direction. So the transmitted sinusoid distributes its power on a frequency interval, which corresponds to the set of Doppler shifts imposed on the different signal paths. This power spectral density is called Doppler spectrum $S(\nu)$ (or Doppler power spectral density), it is usually limited between two symmetric extremes $-f_{D,MAX}$ and $+f_{D,MAX}$ (w.r.t. the center frequency f_C) and it returns the distribution of the Doppler spread due to mobility. Sect. 2.6.3 reports a diffused example of $S(\nu)$. Here, the new variable ν (denoting a general frequency variation due to the Doppler effect and related to the absolute time t of the process) is used to differentiate it from f, already associated to the Fourier transform done w.r.t. propagation delay τ.

The extension of the previous discussion to the case of a wide-band signal is not difficult, at least in principle, since each signal can be thought as a sum (or integral) of a set of sinusoids or spectral components; this means that, in a wide-band signal, the spectral components could be affected by different Doppler shifts, causing signal distortion as time evolves.

2.6.2 The coherence time

The coherence time T_C of a channel represents the time interval in which the channel remains approximately constant or quasi-static. This is related to the spectral dispersion given by the Doppler effect, which depends on the movements in the propagation environment. If we consider a generic spectral component in presence of a Doppler shift f_D like (2.6.6), we can observe that, in a time interval $1/(2f_D)$, the phase variation is relevant and equal to π. So the channel response will not change significantly only in a time much smaller than a fraction of $1/(f_D)$ (again the small scale impact of the phases). If we consider a channel with $S(\nu)$ between $-f_{D,MAX}$ and $+f_{D,MAX}$ and denoting as $\Delta f_D = 2f_{D,MAX}$ the maximum frequency dispersion (Doppler dispersion), we can assume that its inverse represents a measure of the time interval in which the channel characteristics do not vary significantly:

$$T_C = \frac{k}{\Delta f_D} \tag{2.6.8}$$

with $k = 1/5, 1/10$. As an example of time variability, if we consider a cellular system with carrier frequency equal to 900 MHz and a relative velocity equal to 60 km/h, the Doppler dispersion turns out to be 50 Hz, and hence the coherence time is a fraction of 20 ms.

When a channel varies too fast w.r.t. the signal transmission time, represented by the symbol time T_S in a digital modulation, the received signal might be severely distorted. Similarly to the concept of coherence bandwidth, here we distinguish two situations as well:

- Time selective channel: $T_S > T_C$
- Non time selective channel: $T_S < T_C$.

When the channel approaches the time selectivity, a reliable reception might be really difficult since the receiver could have not time enough for compensating (equalizing) practically the distortion introduced by the channel.

It is interesting to notice the duality between the concepts of coherence bandwidth and coherence time: the former is inversely proportional to the delay spread (and it is measured by means of a wide-band signal) and the latter is inversely proportional to the Doppler spread (and it is measured by means of a narrow-band signal).

2.6.3 Uniform scattering model

A classical and widespread model of Doppler dispersion assumes a situation in which the receiver intercepts echoes from any direction and these reflections are concentrated in an area close and around it (known also as Clarke's model). So let us assume to transmit a carrier at frequency f_c in an environment with intense multipath diffusion, which causes echoes uniformly distributed from all the directions (Fig. 2.15). It is possible to derive the power spectral density $S(f)$ of the signal imposing a constant angular power density C (this step can be done limiting θ between 0 and π and then repeating exactly the same for $-\pi < \theta < 0$) so that

$$S(\nu)|d\nu| = C|d\theta| \tag{2.6.9}$$

and exploiting the relation between angles and frequency shift in (2.6.7), or $\nu = f_C \cdot v/c \cdot cos(\theta) = v/\lambda \cdot cos(\theta)$. We have

$$\left|\frac{d\nu}{d\theta}\right| = v/\lambda \cdot \sqrt{1 - cos^2(\theta)} \tag{2.6.10}$$

and, expressing $cos(\theta) = \nu/(v/\lambda) = \nu/f_{D,MAX}$, we obtain

$$S(\nu) = \frac{C}{f_{D,MAX}\sqrt{1 - \left(\frac{\nu}{f_{D,MAX}}\right)^2}}. \tag{2.6.11}$$

Fig. 2.16 shows the resulting Doppler dispersion spectrum, which occupies the bandwidth $[-f_{D,MAX}, +f_{D,MAX}]$. This means that a transmitted sinusoid at frequency f_C generates a received signal with the power spectrum given in Fig. 2.16 and centered in f_C. Please notice again that, similarly to the double time variable t and τ, here we have used the frequency variable ν, which corresponds to the Fourier transform made on the variable t, while f represents the variable of the transform made w.r.t. τ (as in Sect. 2.6).

Notice also that, w.r.t. the simple scenario represented by (2.6.7) and (2.6.6), here we have a continuous Doppler spectrum instead of the discrete one

$$S(\nu) = \alpha^2 \delta\left(\nu - \frac{v}{\lambda}cos(\theta)\right). \tag{2.6.12}$$

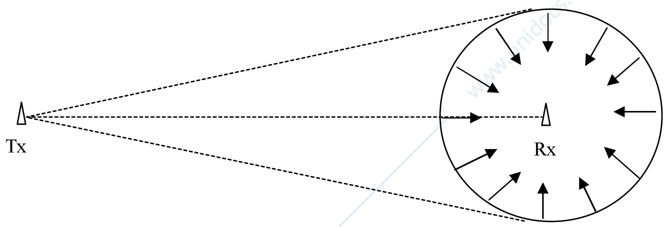

Figure 2.15: Propagation scenario corresponding to the Clarke's model.

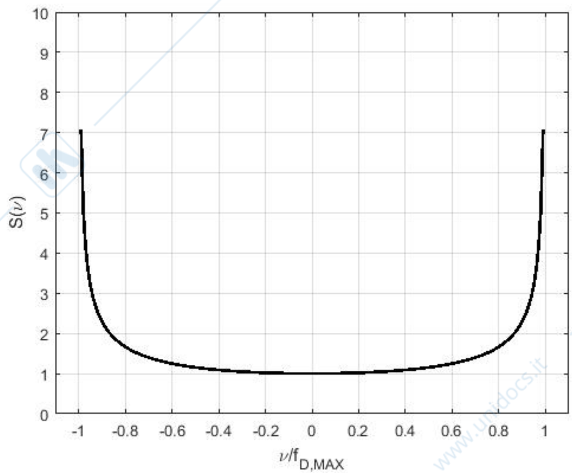

Figure 2.16: Doppler spectrum resulting from Clarke's model.

2.7 The model of a wireless channel

Before concluding this section, let us summarize the main elements composing the model of a wireless channel:

- Multipath intensity profile $P(\tau)$. From $P(\tau)$ we can derive the powers and the delays associated to the paths, also in a tapped equivalent delay line model.
- Doppler spectrum $(S(\nu)$. From $(S(\nu)$ we can derive the information for simulating the time evolution of the channel.
- Statistics of the paths powers or amplitudes. They can be different for each path in $P(\tau)$ and the most classical examples are Rice and Rayleigh; the phases are assumed uniformly distributed between 0 and 2π.