# 3. Optimization of the TX spectrum with the water filling technique.

- Consider a set of *N* Gaussian additive channels, each of them characterized by a different noise to gain ratio

$$\eta_i = \frac{2\sigma_i^2}{c_i^2}$$

"badness" of i-th channel

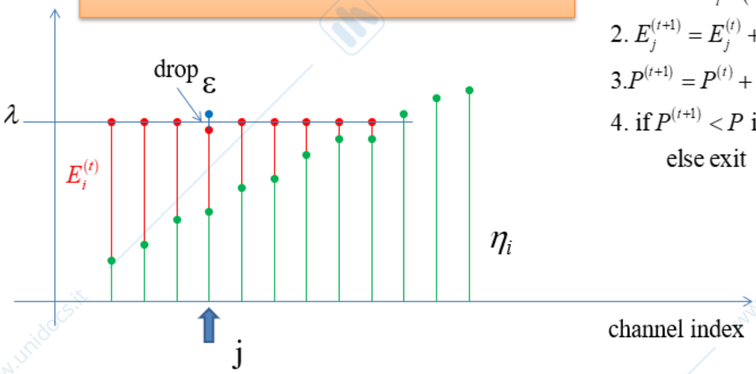
- The Shannon capacity associated to each channel is $C_i = \log\left(1 + \frac{E_i}{\eta_i}\right)$
- The water filling algorithm provides the optimal way to allocate energy to the different channels so as to maximize the total capacity
- **Each energy increment ("drop of water") must maximize the throughput increment (capacity)**

-Constrain:

$$\sum_{i=1}^{N} E_i \leq P$$

Associate each **energy increment** ("drop") to the channel whose current sum of already **allocated energy and channel badness is minimum**.

...the algorithm naturally equalize the "water" level
..and keep going until no more
available energy to be allocated

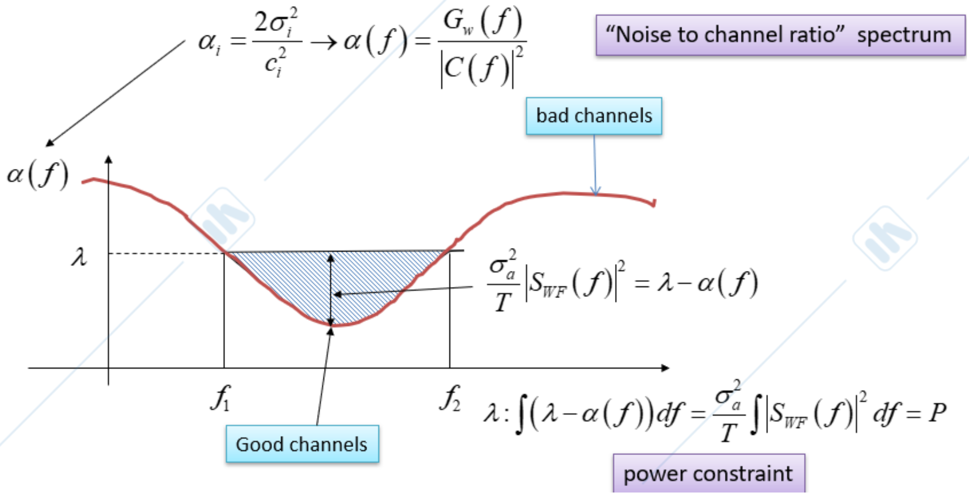



$t = 0, \forall i \quad E_i^{(0)} = 0, P^{(0)} = 0$

1. $j = \arg\min_i \left(E_i^{(t)} + \eta_i\right)$
2. $E_j^{(t+1)} = E_j^{(t)} + \varepsilon$
3. $P^{(t+1)} = P^{(t)} + \varepsilon$
4. if $P^{(t+1)} < P$ increment $t$ and goto 1
   else exit

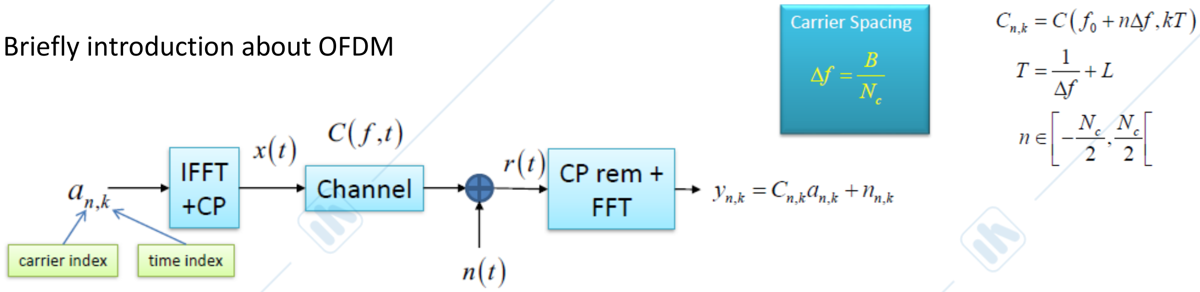
*en la gráfica donde pone t debería ser n, ya que este es el caso discreto, lo mismo en la siguiente gráfica.

- Continuous case: each tone (f) is ideally associated to an independent channel.



## 6.Channel estimation and equalization with OFDM.

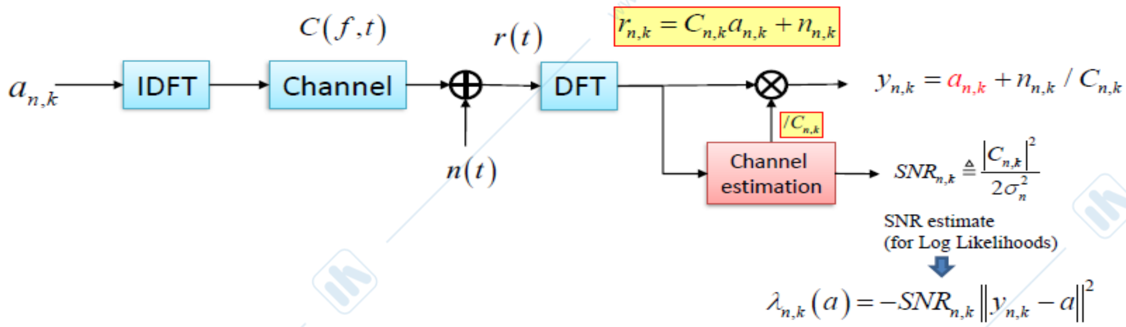
Briefly introduction about OFDM




- OFDM transforms the channel into a set of narrowband, non frequency selective, Gaussian channels.

- ISI is eliminated, but channel estimation is still needed to compensate phase rotation and amplitude scaling (complex AGC).

- As there is no presence of ISI the equalization process is much more simpler (Single tap equalization)




•The channel estimation exploits the correlation of the channel coefficients $C_{n,k}$ both in the time and frequency domain

• How to perform **channel estimation for OFDM**:

- Send pilots data $p_{n,k}$ on a subset of carriers and time intervals
  - Overhead: fraction of pilot symbols w.r.t. total transmitted symbols
- Estimate channel gains at the pilot positions.
- Interpolate the available estimates to obtain estimates of channel gain at all positions
  - Optimal solution: **Discrete Wiener Interpolation**
    This algorithm is based on the estimation of the channel gain at the data position as a linear combination of the observations of the channel gain at the pilot position

## 9. Models of power amplifiers and effects of non linearities on linear modulations.

Power amplifiers (PA) are main transmitter subsystems producing interference and in-band distortion.

Non-linear behaviour: the I/O characteristics are the phase and amplitude.

Strongly depend on the amplitude on the input signal

- only instantaneous amplitude: memoryless
- amplitude variation over time: with memory

Equivalent baseband model of the PA


The output complex envelope only depends on the instantaneous input power (AM)

$\rho_y(t)=G[\rho_x](t)$ AM/AM

$\varphi_y(t)=\varphi_x(t)+\phi(\rho_x(t))$ AM/PM

The AM/AM curve shows a <u>saturation point</u>: input point ($P_{\text{in,sat}}$) that corresponds to the maximum output envelope ($P_{\text{out,sat}}$)

We define the input back-off (IBO) ($\boldsymbol{\beta}_{\text{in,}}$): $10\log\left(\frac{P_{\text{in,sat}}}{P_{\text{in}}}\right)$ and is the working-point of the amplifier

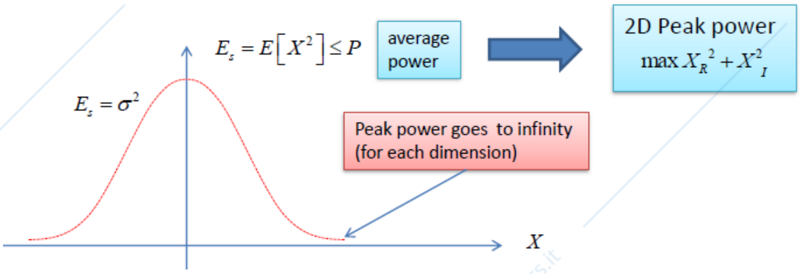
We define the output back-off (OBO) ($\boldsymbol{\beta}_{\text{out,}}$): $10\log(\frac{P_{\text{out,sat}}}{P_{out}})$.

If we select an $\boldsymbol{\beta}_{\text{in}}$ close to zero appears <u>non-linearity effects</u> which are:

- distortion of the signal envelope
- distortion of the received symbol constellation
- generation of ISI
- loss of good properties of the signal shape (e.g., matching)

## 12. Optimal constellations for peak power limited channels

El propósito de este tema es averiguar cómo distribuir los símbolos de una señal 2D (fase y amplitud ¿?) en la constelación. Empieza diciendo que la distribución óptima sería una Gaussiana pero que esta distribución tiene *peak power* infinita (esto supondría una constelación con infinitos puntos) para cada dimensión y que por lo tanto no puede ser una distribución óptima con restricción de *peak power*.




- 2D Constellation points should be uniformly distributed (in phase) over a finite number *(C)* of circles.
- **p** is the vector collecting the *C* probabilities of staying on a given circle
- **A** is the vector collecting the *C* amplitudes of the circles
- the number of circles *C*, as well as both the vectors **p** and **A** are optimized for each target PSNR

If we want to design a constellation with a finite number of points *M* the solution is similar:

- The *M* constellation points must be placed on the *C* circles with radii corresponding to the values *Aj*
- The number of points *M(j)* placed on circle *j* correspond to the probability *pj*:

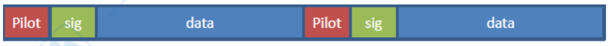
$$M(j) = NINT[p_j M] : \rightarrow \sum_j M(j) = M$$

- Points on each circle must be uniformly spaced in phase

APSK type constellations are used in the Digital Video Broadcasting standard for satellite

## 15. Frame Synchronization.

The transmitted stream is usually organized in frames, each frame contains different info (pilots, signalling and data) thus frame synchronization is required in order to align the received data sequence to the transmitted one.




Frame sync is performed by correlating the input sequence ($y_n$) with all possible delays of the known pilot sequence ($p_k^*$) and find the peak (max correlation). The correlation is shown in the following image where Nf is the length of the frame

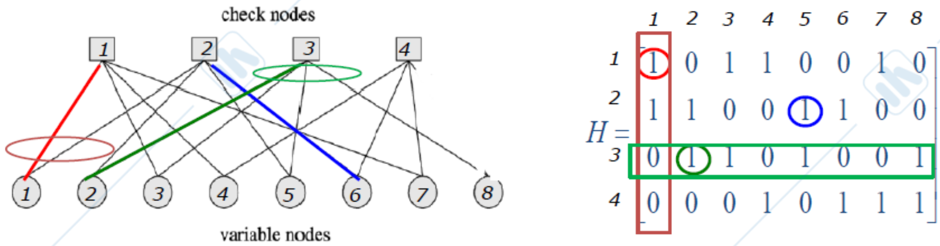
$$\Lambda(\hat{D}) \triangleq \sum_{f} \sum_{k=0}^{N_p-1} \Re\left[ y_{fN_f+k+\hat{D}} \, p_k^* \right] \qquad \hat{D} \in \left[0, N_f-1\right]$$

As frame sync is based on finding the pilot position, all the algorithms based on pilots must be placed after frame sync.

A frame sync robust to frequency offset is required when phase and fine frequency is based on pilots.

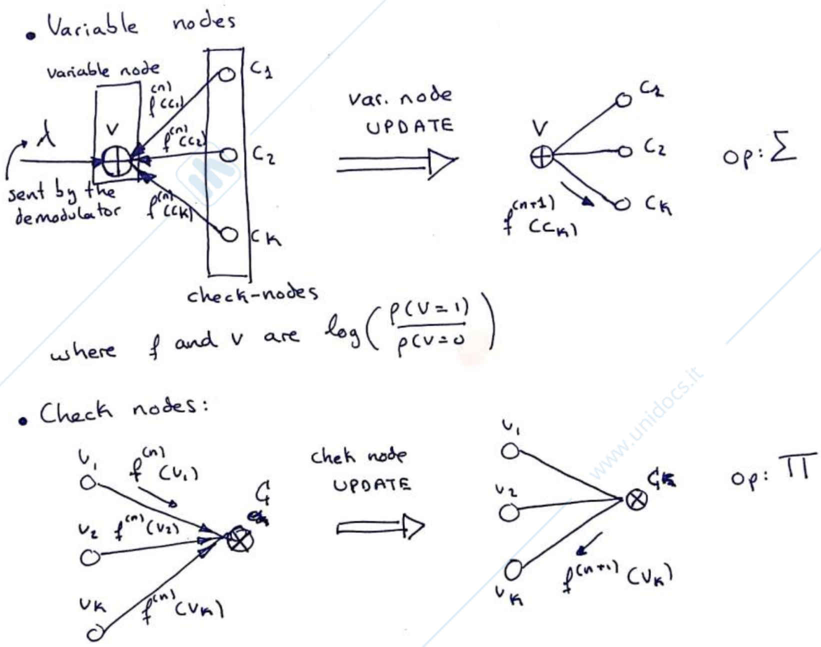
**18. LDPC codes. Description and iterative decoder.**

The Low-density parity-check (LPDC) codes are linear block codes with a very sparse parity-check matrix H, that is, very small number of ones. They are also represented by a Tanner graph, between *m=n-k* check nodes and *n* variable nodes.




They can be regular (same number of edges leaving each node) or irregular (better performance).

The decode of an LDPC code is done by the belief propagation: First, the variables nodes are initialized with the channel messages. Then, the decoding messages are iteratively computed by all the nodes and exchanged through the edges. The message sent by each node is the update of the state of each node computed by the following operations:



www.unidocs.it - Appunti e dispense per superare i tuoi esami universitari